# Graph Sparsification while Maintaining Cuts

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## The Setting

- Graph $G$ with $n$ nodes and $m$ edges.
- Unweighted for this talk (weighted cases work similarly).
- $m >> n \log n$
- Obtain $G'$ with fewer edges but with all cuts of $G$ preserved approximately.
- $G'$ will be weighted.

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- Two nodes with $m$ edges connected the two.
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## A Randomized Approach: Benczur, Karger

- Sample each edge $e$ with probability $p_e$ and give it weight $1/p_e$.
- For any cut, its expected weight in the new graph $G'$ equals its weight in $G$.
- Do ALL cuts in $G$ have weight in $G'$ that is $(1 \pm \epsilon)$ of the corresponding weight in $G$, w.h.p?
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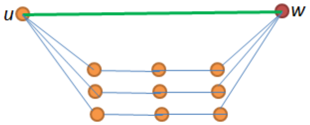
## What should $p_e$ be?

- $p_e \sim \frac{1}{d_e}$? ($d_e$ is min of the degrees of $e$'s endpoints). NO!
- $p_e \sim \frac{1}{k_e} \geq \frac{1}{d_e}$? ($k_e$ is the connectivity of $e$). NO!
- $p_e \sim \frac{\log n}{\epsilon^2} \frac{1}{k_e}$? MAYBE!



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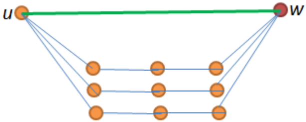

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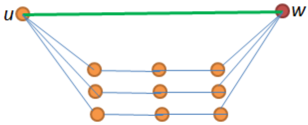

## Chernoff Bound for Sampled Cut Weight.

- Focus on a particular cut of size $\Delta$.
- Group edges in this cut into doubling value categories based on sampling probability.
- Consider one group $\mathcal{S}$ of edges with sampling probabilities $\sim \frac{\log n}{\epsilon^2}\frac{1}{2^i}$.
- For any $\Delta' \geq |\mathcal{S}|$,

$$Pr(|\mathcal{S}_{samp} - |\mathcal{S}|| \geq \epsilon\Delta') \leq e^{-\Theta(\epsilon^2 \frac{\log n}{\epsilon^2 2^i}\Delta')} = n^{-\Theta(\frac{\Delta'}{2^i})}$$

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- We need $\epsilon\Delta'$ to add up at most $\epsilon\Delta$ over all groups.
- And we need $n^{-\Theta(\frac{\Delta'}{2^i})}$ to be small enough to offset the number of such groups $\mathcal{S}$ over all cuts.

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## How many cuts?

- Focus on a particular cut of size $\Delta$, and the subset of edges with connectivity $\sim 2^i$.
- How many such distinct sets of edges exist, over all cuts?
- $n^{O(\Delta/2^i)}$! Will show later.

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# Putting it Together

- Focus on a particular cut of size $\Delta$, and the group $\mathcal{S}$ of edges with sampling probabilities $\sim \frac{\log n}{\epsilon^2 2^i}$.
- For this group,

$$Pr(|\mathcal{S}_{samp} - |\mathcal{S}| \geq \epsilon\Delta) \leq n^{-\Theta(\frac{\Delta}{2^i})}$$

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- The number of distinct groups of edges $\mathcal{S}$ over all cuts of size $\Delta$ is $n^{O(\frac{\Delta}{2^i})}$ (to be shown).
- So in every cut of size $\Delta$, the corresponding group contributes a deviation of $\epsilon\Delta$.
- There are at most $\log n$ groups in each cut.
- So every cut has deviation at most $\epsilon\Delta \log n$. But we need $\Delta$!

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- Up sampling probabilities to $\sim \frac{\log^2 n}{\epsilon^2 k_e}$.
- Focus on a particular cut of size $\Delta$, and the set $\mathcal{S}$ of edges with sampling probabilities $\sim \frac{\log^2 n}{\epsilon^2 * 2^i}$.
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## Expected Size of Sampled Graph

- We show that $\sum_e \frac{1}{k_e} \leq n-1$. So the expected number of edges in the sampled graph is $\leq \frac{\log^2 n}{\epsilon^2}(n-1)$.
- Consider the Gomory-Hu (GH) tree. Each Gomory-Hu edge $f$ has weight $w_f$ equal to the number of graph edges that cross it.
- $e$ crosses a witness Gomory-Hu edge with weight $k_e$.
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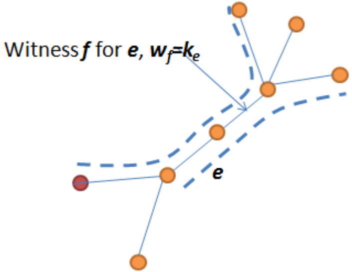

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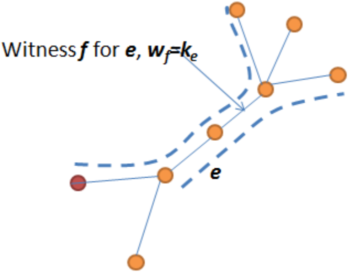

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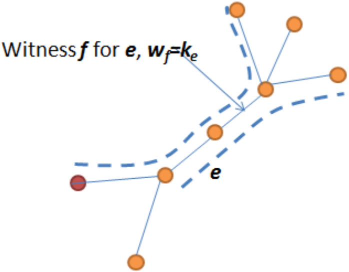

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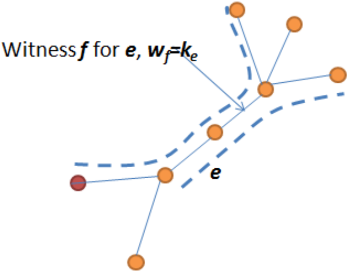

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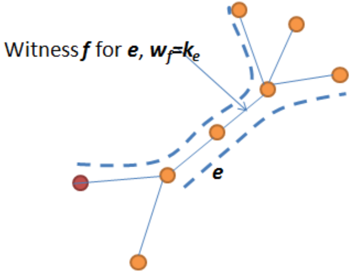

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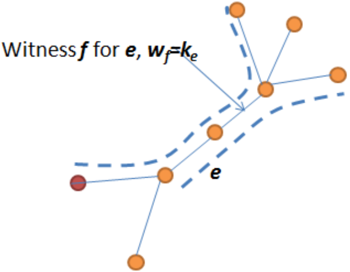

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- Consider a cut of size $\Delta$.
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## Recall Karger's Cut Counting Method

- Randomly choose edges and compress.
- Let $k$ be the min-cut size.
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$$\geq (1 - \frac{\Delta}{nk/2})(1 - \frac{\Delta}{(n-1)k/2}) \cdots (1 - \frac{\Delta}{(\frac{2\Delta}{k}+1)k/2})$$

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- Edges incident on a vertex $v$ can be paired and 'shortcut'.
- So $v$ gets removed from the graph.
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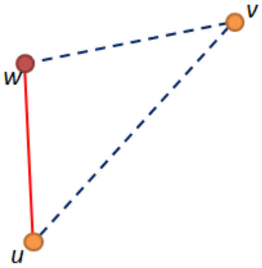

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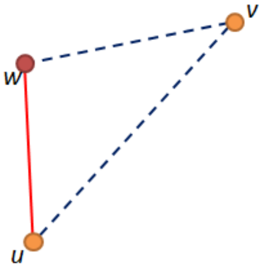

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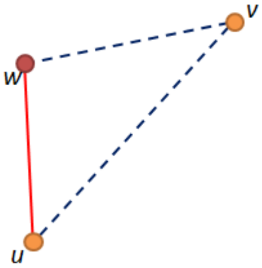

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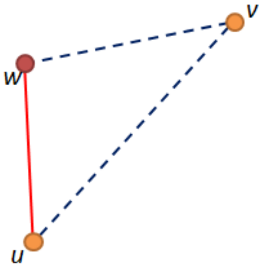

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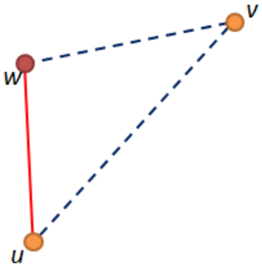

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- Compressing an edge causes potential increase in the cut size $\Delta$.
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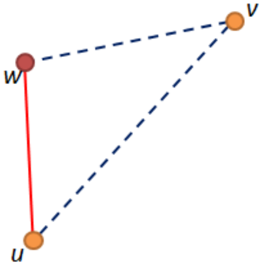

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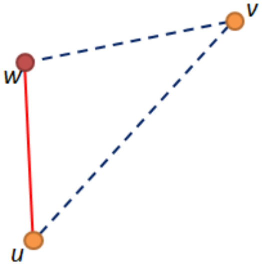

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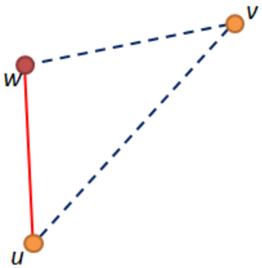

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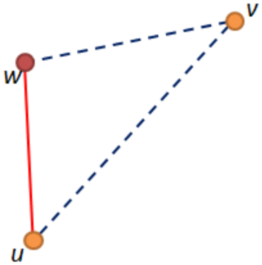

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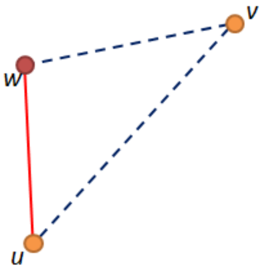

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- Randomly choose edges and compress.
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- Given edge *uv*, there exists *vw* such that:
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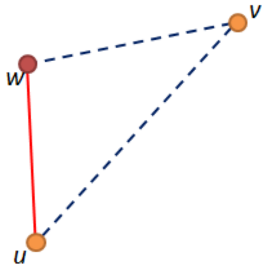

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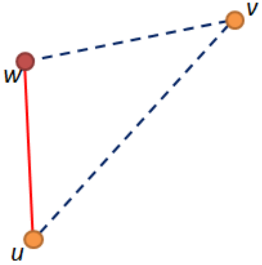

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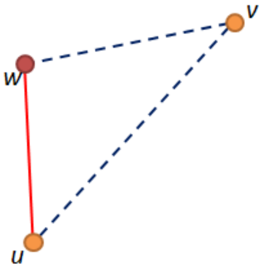

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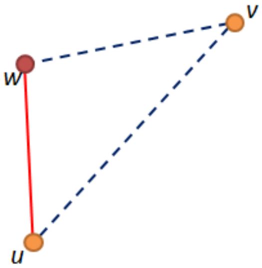

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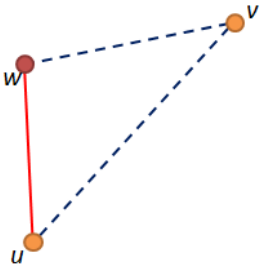

## The Challenge with Splitting Off

- The only cuts that reduce in size are those which split *u* and *v*.
- If there exists such a cut of size $\alpha$ or $\alpha + 1$, it will drop below $\alpha$ iff *w* is on the same size as *u* in this cut.
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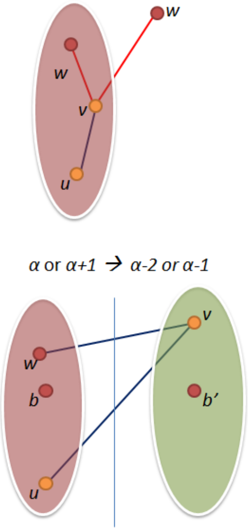

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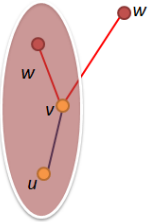
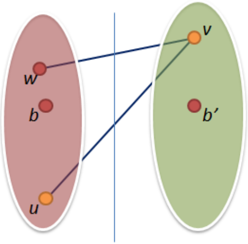



α or α+1 → α-2 or α-1



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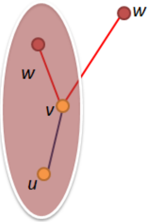


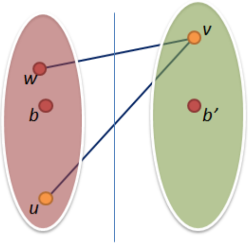

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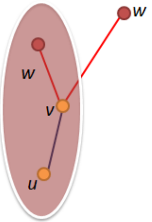


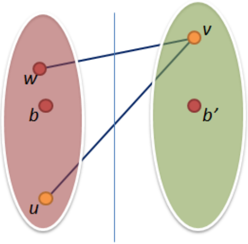

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- $v$ must have a neighbour on the right side of the cut.
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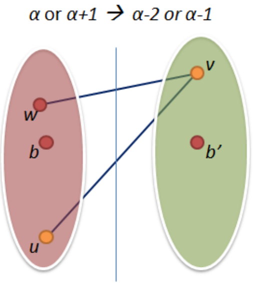

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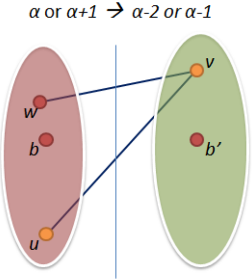

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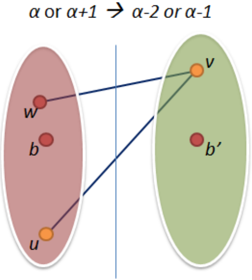

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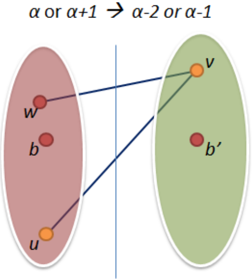

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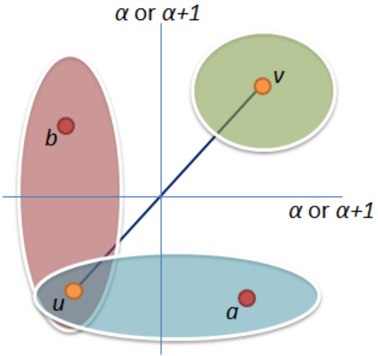

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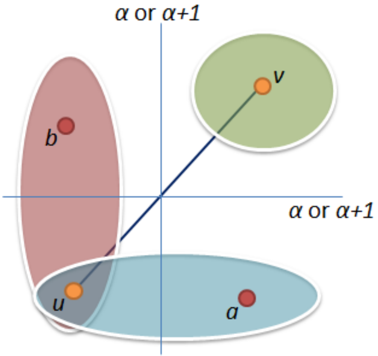

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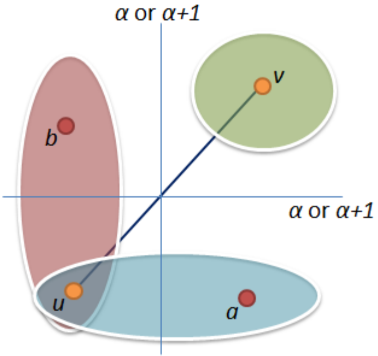

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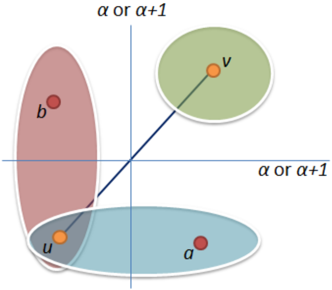


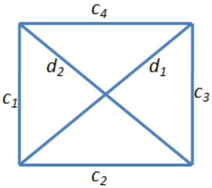


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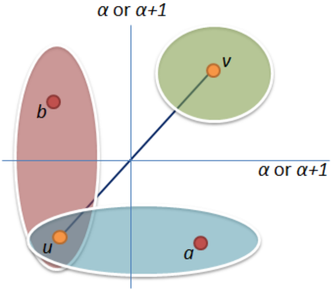


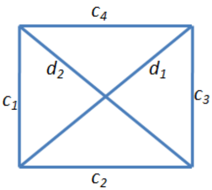


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- How do we handle odd degrees?
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## Computing Sampling Probabilities

- It suffices to underestimate edge connectivites, i.e., compute $k'_e \leq k_e$.
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$$Pr(|\mathcal{S}_{samp} - |\mathcal{S}|| \geq \epsilon x) \leq n^{-\Theta(\frac{x}{2^i})}$$

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- A collection of edge-disjoint forests.
- If $u$ and $v$ are connected in forest $i$, they are also connected in forests $1 \dots i-1$.
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- So sampling with probability $\frac{\log^2 n}{\epsilon^2 k'_e}$ preserves all cuts within $1 \pm \epsilon$ w.h.p.
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- Define the $2^i$-projection of a cut to be the subset of its edges with $k'_e \sim 2^i$.
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- Take subgraph $G'$ formed by edges in NI trees $2^{i-2} \dots 2^i$.
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- Process vertices in (a to be specified) order; for the chosen vertex, add all incident edges (these are incident on yet unprocessed vertices).
- For each vertex $v$, define $I(v)$ as the index of the first NI tree where $v$ is singleton.
- For each edge $e = uv$ processed, add $e$ to tree $min(I(u), I(v))$.
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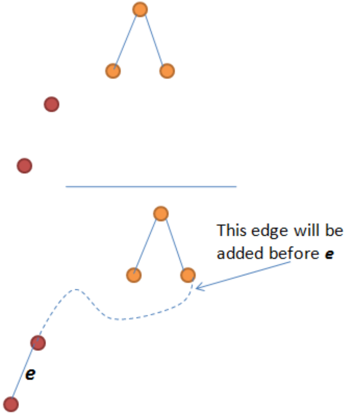
- Key invariant: If a new connected component is created in a tree, it stays separate even after all future edge additions.
- $O(n \log n + m) \sim O(m)$ time.



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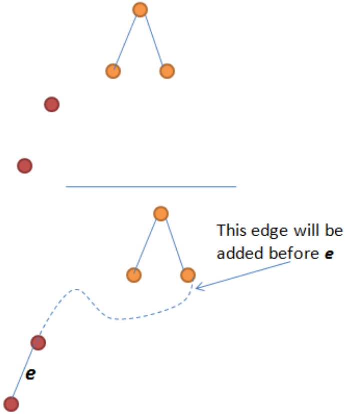

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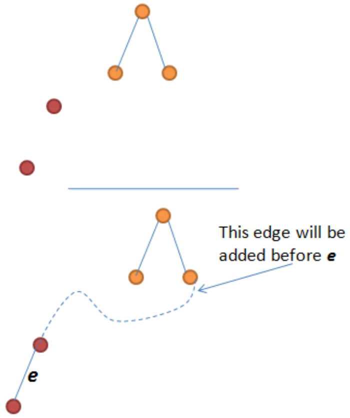

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## Sampling by Effective Conductance

- Sample edge $e$ with probability $\frac{\log^2 n}{\epsilon^2 c_e}$ (where $c_e = 1/r_e$).
- Key Property: $c_e \leq k_e$.
- Recall that underestimating $k_e$'s suffices.
- $\sum_e \frac{1}{c_e} = \sum_e r_e = n - 1$ (use the spanning tree fraction interpretation).
- So sampling with effective conductance yields a graph with $O(n\frac{\log^2 n}{\epsilon^2})$ edges that preserves all cuts within a $(1 \pm \epsilon)$ factor, w.h.p.

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- But there are other edges around.
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- Shrinking edge $f$ is like setting its resistance to 0, so effective resistance of $e$ should only decrease, i.e., conductance increases.
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## Proving Rayleigh's Monotonicity Principle: Energy

- A feasible flow is an assignment of current to the edges satisfying current conservation at each vertex, except the endpoints of $e$ which have a deficit/excess of 1, respectively.
- The energy of a feasible flow is $\sum_f i_f^2$ over all edges $f$.
- The energy of a feasible flow is also the voltage drop across $e$, which is the effective resistance of $e$ (easy proof using current conservation).
- Of all feasible flows, the one that minimizes energy has currents that are differences of endpoint voltages (can be shown using the primal-dual approach, for instance).

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## Proof of Rayleigh's Monotonicity Principle

- If you shrink an edge $f$, then the least energy flow prior to shrinking $f$ is still a feasible flow after shrinking $f$.
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# Another Nagamochi-Ibaraki Sampling Scheme

- Sample edge $e$ with probability $\frac{\log n}{\epsilon^2 k''_e}$ ($k''_e$ is the index of the first NI tree where the endpoints of $e$ are not in the same connected component).
- Consider the graph $G''$ comprising edges $e$ with $k''_e \geq 2^{i-1}$.
- Any edge $e$ with $k''_e \geq 2^i$ is $\Theta(k''_e)$ connected in $G''$.
- Replicate an edge in $G''$ with $k''_e \sim 2^j, j \geq i-1$, $n/2^j$ times, to obtain graph $H''$.
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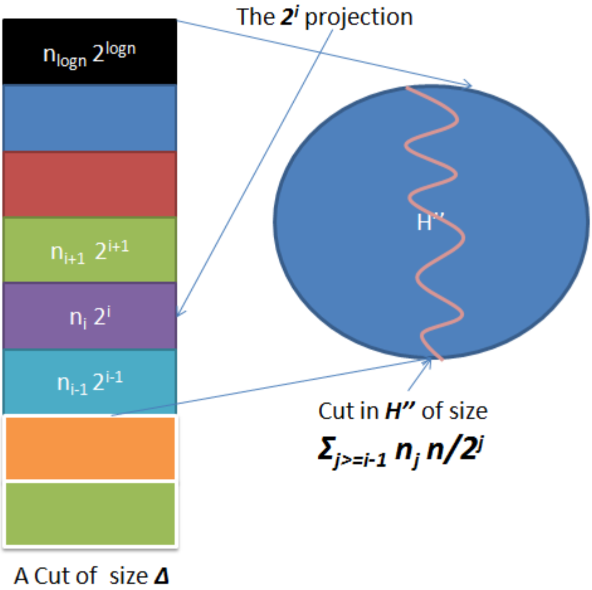

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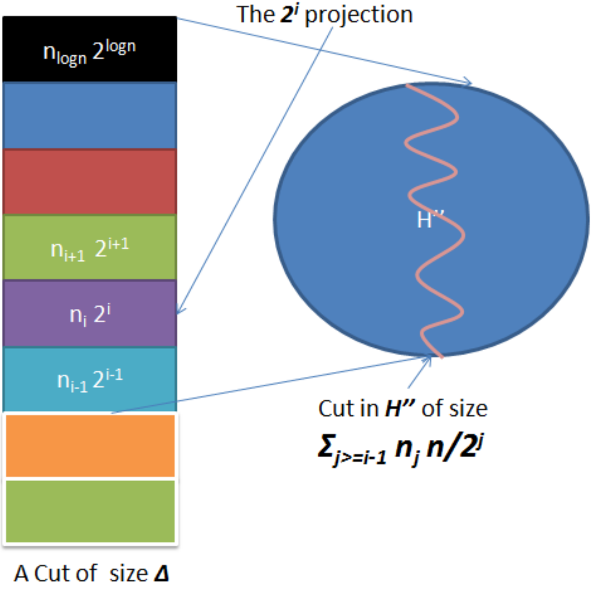

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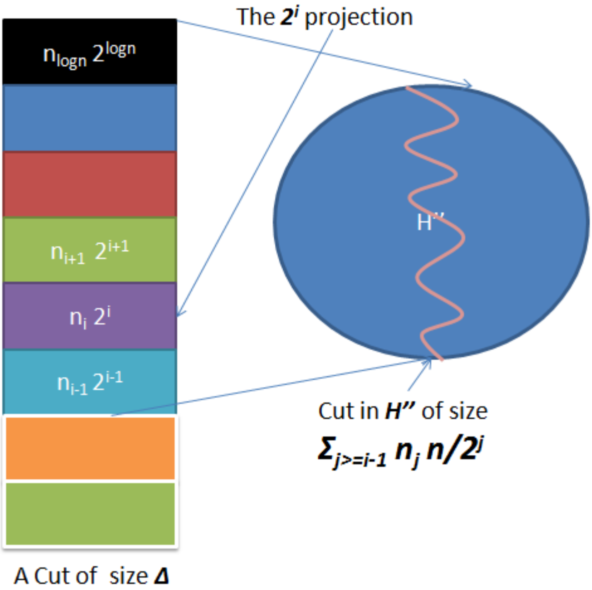

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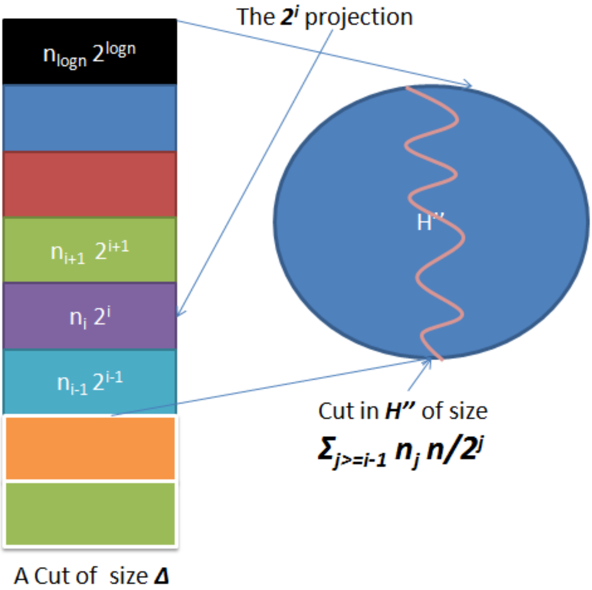

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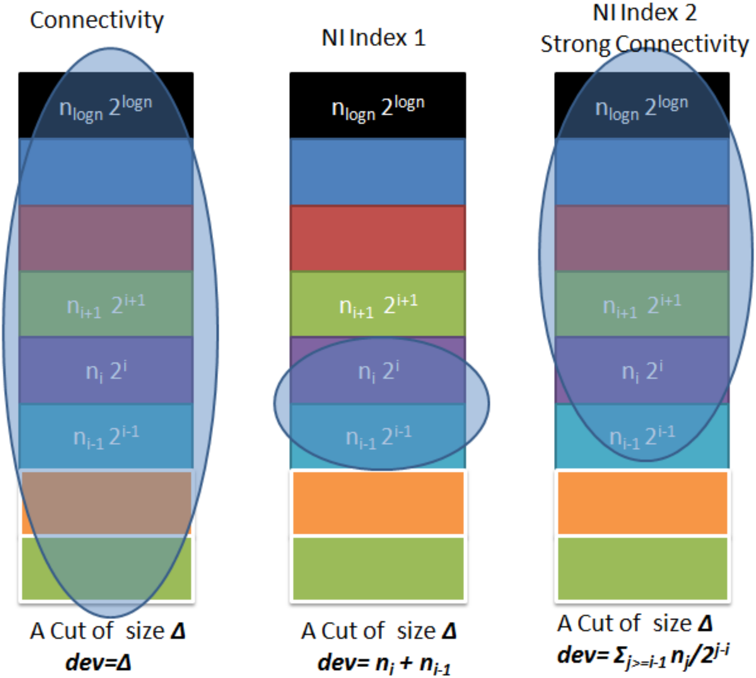
Connectivity
NI Index 1
NI Index 2
Strong Connectivity
$n_{logn}\ 2^{logn}$
$n_{i+1}\ 2^{i+1}$
$n_i\ 2^i$
$n_{i-1}\ 2^{i-1}$
A Cut of size $\Delta$
$dev=\Delta$
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## Other Results

- An $O(n\frac{\log n}{\epsilon^2})$ size sparsifier in time $O(n\log n + m)$ (Hariharan and Panigrahy)
- Sampling by conductance yields an $O(n\frac{\log n}{\epsilon^2})$ size sparsifier (Spielman, Srivastava); this is more general as well, but conductances are more complex to compute.
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## Open Problem

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