Dynamic Power Allocation For Maximizing Throughput in Energy Harvesting Communication System

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Abstract

The design of online algorithms for maximizing the achievable rate in a wireless communication channel between a source and a destination over a fixed number of slots is considered. The source is assumed to be powered by a natural renewable source, and the most general case of arbitrarily varying energy arrivals is considered, where neither the future energy arrival instants or amount, nor their distribution is known. The fading coefficients are also assumed to be arbitrarily varying over time, with only causal information available at the source. For a maximization problem, the utility of an online algorithm is tested by finding its competitive ratio or competitiveness that is defined to be the maximum of the ratio of the gain of the optimal offline algorithm and the gain of the online algorithm over all input sequences. We show that the lower bound on the optimal competitive ratio for maximizing the achievable rate is arbitrarily close to the number of slots. Conversely, we propose a simple strategy that invests available energy uniformly over all remaining slots until the next energy arrival, and show that its competitive ratio is equal to the number of slots, to conclude that it is an optimal online algorithm.

I. Introduction

We consider the energy harvesting paradigm for powering wireless communication, where the source harvests energy from natural renewable sources, such as solar cells, windmills, etc. for transmitting its data to the destination. Using energy from nature not only improves the lifetime of wireless devices, which are otherwise battery powered, but also provides a means

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of *green* communication. Harvesting energy from natural sources, however, makes the future available energy levels at the source unpredictable and the source has to adaptively choose the transmission power for maximizing its utility function without knowing the future energy arrivals. Another important constraint dictated by harvesting energy from nature is the energy neutrality constraint, i.e. energy spent by any time instant cannot be more than the energy harvested until that time. Energy neutrality and unpredictable energy availability makes the design of optimal algorithms in the energy harvesting paradigm a challenging problem.

In this paper, we consider a wireless communication channel between a single source-destination pair. The source is assumed to harvest energy from renewable sources, and the problem is to maximize the mutual information or the achievable rate between the source and the destination over a fixed number of slots. Each slot corresponds to a coherence interval; time for which the fading coefficients remain constant. The source is assumed to have only causal information about the energy arrivals and fading coefficients. To model the most general energy harvesting paradigm, we assume that the energy arrivals are arbitrarily varying and the source is not assumed to have any information about the future energy arrivals or its distribution. This assumption is valid for the case when energy is harvested from a combination of heterogenous sources such as wind, vibrational source, body strapped devices, for which the distribution of energy arrivals may be time varying and potentially hard to compute.

We consider the scenario when the wireless fading channel is an arbitrarily varying channel (AVC), where the fading coefficients do not follow any distribution and vary arbitrarily over time. AVCs in wireless communication are motivated from the non-stationarities in propagation environment because of mobility, presence/absence of line of sight, Doppler effects etc. In prior work, AVCs have been studied from an information theoretic point of view [1]–[3], however, to the best of our knowledge, AVCs in the energy harvesting paradigm have not been explored before. In any case, assuming arbitrarily varying energy arrivals and fading coefficients, provides a worst case guarantee on the system performance. Therefore, the problem we consider in this paper is to find *online algorithms* (that have access to only causal information about energy arrivals and fading coefficients with no distribution information) that maximize the achievable rate over a fixed number of slots.

In prior work, optimal offline algorithms (that have access to all future energy arrivals instants and amounts) have been derived for maximizing the achievable rate in energy harvesting systems for the wireline Gaussian channel [4]–[6], and for the wireless fading channel [7], [8]. Similar results are available for many other communication channels, e.g. interference channel [9], broadcast channel [10], relay channel [11]. The scope of these algorithms, however, is limited because of unrealistic assumption of non-causal information. Some properties of stochastic online algorithms, where the source has the knowledge of the distribution of energy harvest instants and amounts, have been derived in [12], [13] using results from stochastic control theory. To the best of our knowledge, however, no analysis is known for online algorithms with unknown energy harvest distribution for maximizing the achievable rate.

With arbitrarily varying energy arrivals and fading coefficients, we turn to the competitive ratio analysis of online algorithms that is popular in computer science community [14] to derive ”good” online algorithms for maximizing the achievable rate. With online algorithms, no knowledge of future inputs (energy arrivals and fading coefficients in our case) is assumed and the input can even be generated by an adversary that creates new input portions based on the systems reactions to previous ones. The goal is to derive algorithms that have a provably good performance even against adversarial inputs. The performance of online algorithms is usually evaluated using competitive analysis [14], where an online algorithm A is compared with an optimal offline algorithm O that knows the entire request sequence $\boldsymbol{\sigma}$ in advance and can serve it with maximum profit/minimum cost. In the dynamic programming literature, this framework is known as the *minimax* or *maxmin* control [15], where the objective of the algorithm is to maximize the utility while the nature is assumed to choose parameters to minimize the utility.

In prior work, competitive analysis has been used to design online algorithms for several communication systems, e.g. [16]–[20]. The most related papers to this work are [18], [19], where the problem of dynamic power allocation in an arbitrarily varying wireless fading channel (AVC) under a sum-power constraint is considered. The two fundamental differences between the problem studied in this paper and prior work are : i) future energy availability is unknown, and ii) energy neutrality constraint, and to the best of our knowledge these issues have not been addressed in the literature.

To state our results formally, we define an online algorithm and its competitiveness as follows.

Definition 1: Let $\mathcal{P}$ be an optimization problem that depends on request sequence $\boldsymbol{\sigma} = (\sigma_i),\ i = 1, 2, \ldots,$. An online algorithm A for solving $\mathcal{P}$ is presented with requests $\boldsymbol{\sigma} = (\sigma_i),\ i = 1, 2, \ldots,$ and it has to serve each request without knowing the future requests. In our case $\boldsymbol{\sigma}$ is the sequence of energy arrivals and fading coefficients. Formally, when processing σ_i to solve $\mathcal{P}$, A does not know any requests $\sigma_t, t > i$. Let the profit of the online algorithm A for serving $\boldsymbol{\sigma}$ be $P_A(\boldsymbol{\sigma})$. An optimal offline algorithm O knows the entire request sequence $\boldsymbol{\sigma}$ in advance and serves it with maximum profit $P_O(\boldsymbol{\sigma})$.

Definition 2: Let A be any online algorithm for solving a maximization problem $\mathcal{P}$. Then A is called r_A-competitive or has a competitive ratio of r_A if for all input sequences $\boldsymbol{\sigma}$,

$$\max_{\boldsymbol{\sigma}} \frac{P_O(\boldsymbol{\sigma})}{P_A(\boldsymbol{\sigma})} \leq r_A,$$

and the optimal competitive ratio r is defined as

$$r = \min_{A} \max_{\boldsymbol{\sigma}} \frac{P_O(\boldsymbol{\sigma})}{P_A(\boldsymbol{\sigma})}.$$

The contributions of this paper are as follows.

- We first consider the special case when all the energy arrives at the start of transmission, and only the fading coefficients are arbitrarily varying. For this special case, we show that the optimal competitive ratio for solving the achievable rate maximization problem over N slots, is N, and a simple online algorithm that divides the energy equally in all N slots is optimal. This special case setting is equivalent to achievable rate maximization in an AVC with a sum-energy/power constraint [18], where non-matching bounds on the optimal competitive ratio have been derived as a function of the ratio of the maximum and the minimum value of the fading coefficients. The bounds derived in [18], however, are valid for the case when the number of slots N is allowed to be a function of the available energy, and the maximum and the minimum value of the fading coefficients. The bounds [18] are discussed in detail in Remark 3. Our results apply to any fixed number of slots N, where N need not be a function of any other system parameter.
- For the general case of arbitrarily varying energy arrivals and fading coefficients, we show that the optimal competitive ratio is N, and an online algorithm that invests available energy uniformly over all the remaining slots until the next energy arrival is optimal.
- We also consider the problem of minimizing the transmission time of a fixed number of bits when both the energy arrivals and fading coefficients are arbitrarily varying, that is related to the achievable rate maximization problem. We show that the competitive ratio of any online algorithm for minimizing the transmission time of a fixed number of bits is lower

bounded by infinity. This is a negative result that shows that there exist input sequences for which an optimal offline algorithm can finish transmission in finite time, however, no online algorithm can. For the case of minimizing the transmission time of a fixed number of bits under a wireline Gaussian channel, where all fading coefficients are equal to unity, a simple online algorithm has been proposed in [21] whose competitive ratio is less than 2. Thus, the problem of minimizing the transmission time of a fixed number of bits critically depends on the arbitrarily varying nature of fading coefficients.

Notation: Let $f(n)$ and $g(n)$ be two function defined on some subset of real numbers. Then we write $f(n) = \Omega(g(n))$ if $\exists\ k > 0,\ n_0,\ \forall\ n > n_0$, $|g(n)|k \le |f(n)|$, $f(n) = \mathcal{O}(g(n))$ if $\exists\ k > 0,\ n_0,\ \forall\ n > n_0$, $|f(n)| \le |g(n)|k$, and $f(n) = \Theta(g(n))$ if $\exists\ k_1,\ k_2 > 0,\ n_0,\ \forall\ n > n_0$, $|g(n)|k_1 \le |f(n)| \le |g(n)|k_2$. We use the symbol $:=$ to define a variable.

II. System Model

Consider a wireless communication channel between a source and a destination, where the received signal at the destination at time t is given by

$$y_t = \sqrt{P_t} h_t x_t + n_t, \tag{1}$$

where x_t is the signal transmitted by the source with power P_t, h_t is the fading coefficient, and n_t is the additive white Gaussian noise, that is assumed to have zero mean and unit variance, and is independent across time t. We assume a block fading model [22], where the fading coefficients h_t remain constant for w time units. We call each such block as a *slot* of width w, where in the n^{th} time slot, the fading coefficient is denoted as h_n for $n = 1, 2, \dots, N$, where N is the total number of slots of interest. Throughout the rest of this paper we work with slots rather than actual time instants. We assume that the source is powered by a renewable energy source and receives E_n amount of energy at the start of the n^{th} slot.

As discussed before, we consider an arbitrarily varying fading channel and energy arrivals, where at slot n no information (not even the distribution) about the fading coefficients or the energy arrivals of the future slots $h_m, E_m, m > n$ is known. We assume that at the beginning of each slot n, the source obtains the information about fading coefficient h_n of slot n, and the energy E_n that arrives at slot n. The source at slot n can use information about the fading coefficients and energy arrivals till slot n, i.e. $h_i, E_i, i \leq n$ for making transmission decisions (e.g. power to transmit) to maximize its utility. We call this the *causal information*.

In this paper, we consider that the source is interested in maximizing the mutual information or the achievable rate. Let the source use energy UE_n in slot n, then from (1), the achievable rate in slot n is given by [23],

$$R_n = w \log_2 \left(1 + \frac{|h_n|^2 UE_n}{w}\right), \tag{2}$$

since throughout the slot n of width w, the fading coefficient is h_n, and for which equally distributing the energy over all w time units maximizes the achievable rate [23]. Throughout the rest of the paper we consider $\log$ with base 2, and drop the subscript 2 from here onwards. The overall rate accumulated over N slots is

$$R = \sum_{n=1}^{N} R_n, \tag{3}$$

and the total energy consumed is $\sum_{n=1}^{N} UE_n$.

The optimization problem $\mathcal{R}$ of interest is

$$\begin{aligned} \max_{UE_n, n=1,\dots,N} \quad & R = \textstyle\sum_{n=1}^{N} w \log\left(1 + \frac{|h_n|^2 UE_n}{w}\right) \\ s.t. \quad & \textstyle\sum_{n=1}^{m} UE_n \leq \sum_{n=1}^{m} E_n, \forall\ m \leq N, \end{aligned} \tag{4}$$

with only causal information about h_n and E_n. The constraint in (4) $\sum_{n=1}^{m} UE_n \leq \sum_{n=1}^{m} E_n$ represents the fact that the energy used by slot m is less than the energy arrived till slot m, which is popularly known as the *energy neutrality constraint*. We are interested in finding the optimal online algorithm $A^\star$ that achieves the best competitive ratio, i.e.

$$A^\star = \arg \min_{A} \max_{\boldsymbol{\sigma}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}, \tag{5}$$

where $\boldsymbol{\sigma} = ((|h_1|^2, E_1), (|h_2|^2, E_2), \dots, (|h_N|^2, E_N))$ is the sequence of fading coefficients and energy arrivals for N slots, and the optimal competitive ratio is

$$r = \min_{A} \max_{\boldsymbol{\sigma}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}. \tag{6}$$

A related problem to (4) is, given a fixed number of bits B at the beginning of communication, minimize the time by which all B bits are sent to the destination with causal information about h_n and E_n. Note that here we do not have any restriction on the number of slots, i.e. the total slots used to transmit B bits need not be less than N. With the previous definition of UE_n being the energy spent in slot n, and R_n as the bits transmitted in slot n using energy UE_n, the number of bits transmitted until slot m is

$$B(m) = \sum_{n=1}^{m} R_n.$$

Then the optimization problem $\mathcal{T}$ to find the optimal total transmission time is

$$T^\star = \min_{\substack{UE_n \\ B(T) \geq B, \sum_{n=1}^{m} UE_n \leq \sum_{n=1}^{m} E_n, \ \forall \ m}} T. \tag{7}$$

Similar to (4), we are interested in finding online algorithms to solve $\mathcal{T}$ with the best competitive ratio

$$r = \min_{A} \max_{\boldsymbol{\sigma}} \frac{T_A(\boldsymbol{\sigma})}{T_O(\boldsymbol{\sigma})},$$

where we have inverted the ratio in comparison to (6), because (7) is a minimization problem.

For both the optimization problems (4) and (7), the optimal offline algorithm has been characterized in [4]. However, the structure of the optimal offline algorithm does not directly lead to the evaluation of $R_O(\boldsymbol{\sigma})$ or $T_O(\boldsymbol{\sigma})$ that is required for computing the competitive ratio. For analytical tractability of the competitive ratio, we will typically use an upper bound (lower bound) on $R_O(\boldsymbol{\sigma})$ ($T_O(\boldsymbol{\sigma})$).

Remark 1: A typical strategy to find the optimal competitive ratio (e.g. (6)) involves two steps. In the first step, we construct a set of adversarial sequences $\boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_M$ (typically M finite for analytical tractability) to lower bound the optimal competitive ratio r by $r_{LB}(M)$, where $r_{LB}(M) := \min_A \max_{\boldsymbol{\sigma} \in \{\boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_M\}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}$. In the second step, we find an online algorithm A and upper bound r by $r_A := \max_{\boldsymbol{\sigma}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}$. Then if $r_{LB}(M) = r_A$, the optimal solution is found. Finding the choice of sequences such that $r_{LB}(M) = r_A$, however, is often quite difficult.

Before going to the competitive ratio analysis of online algorithms where both energy arrivals and fading coefficients are arbitrarily varying and are only known causally (4), in the next section we consider the case when energy arrives only at the beginning of communication, and only the fading coefficients are arbitrarily varying and known causally. This restricted case is equivalent to sum-rate maximization in an arbitrarily varying fading channel with a sum-power constraint (energy available at the beginning of the communication), that has received recent attention in

[18], from the competitive ratio point of view. Moreover, the competitive ratio analysis of this restricted case acts as a building block for the competitive ratio analysis of the general case.

III. Fixed Energy with Arbitrary Fading Coefficients

Consider the case when energy only arrives at the beginning of transmission at slot 1 given by E_1, and no energy arrives after that, while the fading coefficients are arbitrarily varying and only causally known. In this case, any input sequence is of the form

$$\boldsymbol{\sigma} = \left((|h_1|^2, E_1), (|h_2|^2, 0,), \dots, (|h_N|^2, 0)\right).$$

With this model, the optimization problem (4) specializes to $\mathcal{R}^s$

$$\begin{aligned} \max_{UE_n, n=1,\dots,N} \quad & R^s = \textstyle\sum_{n=1}^{N} w \log\left(1 + \frac{|h_n|^2 UE_n}{w}\right) \\ s.t. \quad & \textstyle\sum_{n=1}^{N} UE_n \le E_1, \end{aligned} \tag{8}$$

where compared to (4), the energy constraint is only a sum-energy constraint of E_1 throughout the N slots.

Remark 2: Under a sum-energy/power constraint, the optimal offline strategy (if the source knows the sequence $\boldsymbol{\sigma}$ at the beginning of transmission) to solve (8) is the well known water-filling strategy [23]. For the special case, when all the fading coefficients are identical, the water-filling strategy is to transmit equal energy/power in each slot. We will use this fact at multiple instances to upper bound the rate achieved by the optimal offline algorithm.

Remark 3: In prior work, bounds on the competitive ratio of $\mathcal{R}^s$ (8) have been derived in [18] as a function of h_{max} and h_{min}, where $h_{min} \le |h_n|^2 \le h_{max}, \ \forall\ n$. Assuming that the number of slots N are a function of initial energy E_1, h_{max}, and h_{min}, a lower bound on the competitive ratio of any online algorithm A has been shown to be $r = \Omega\left(\log\left(\frac{h_{max}}{h_{min}}\right)\right)$, and an online algorithm A is proposed for which the competitive ratio is $r_A = \mathcal{O}\left(\left(\frac{h_{max}}{h_{min}}\right)^2\right)$. Thus, there is a large gap between the lower and upper bound on the competitive ratio.

In this section, we consider the general case, where the number of slots N is fixed, and is not a function of any other system parameter, and show that the optimal competitive ratio $r = N$.

A. Lower Bound on the Competitive Ratio

In this section, we show that the competitive ratio of any online algorithm solving $\mathcal{R}^s$ is lower bounded by $N - \epsilon$ for arbitrarily small $\epsilon > 0$. We first discuss the simple case of $N = 2$ to illustrate the proof idea, and then generalize it for any N.

As discussed in Remark 1, to lower bound the optimal competitive ratio it is sufficient to consider any M input sequences. In this section, we consider only two input sequences ($M = N = 2$), $\boldsymbol{\sigma}_1 = ((1, E_1), (0, 0))$, and $\boldsymbol{\sigma}_2 = ((1, E_1), (\beta, 0))$, where β is the parameter that we will choose to get the largest lower bound on the competitive ratio, and $E_1 = e$. We choose e to be small enough such that for any $|h_n|^2 \in \boldsymbol{\sigma}_i, i = 1, 2$ we consider, the achievable rate obtained in any slot n using energy $\hat{e} \leq e$ is $w \log\left(1 + \frac{|h_n|^2 \hat{e}}{w}\right) \approx |h_n|^2 \hat{e}$ (linear approximation), similar to [18]. Since we are looking for the worst possible input sequence for deriving the lower bound, we can choose any value of e, and in particular e to be small enough. Note that we will prove the upper bound on the competitive ratio for any value of E_1 and w in Subsection III-B.

Following Remark 1, we have the following lower bound on the competitive ratio

$$r \geq \min_{A} \max_{\boldsymbol{\sigma} \in \{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}.$$

With the linear rate approximation in each slot, it immediately follows that the optimal offline algorithm will invest all its e amount of energy in one slot that has the highest fading coefficient. Thus, with $E_1 = e$, $R_O(\boldsymbol{\sigma}_1) = e$, while $R_O(\boldsymbol{\sigma}_2) = \beta e$.

Now, consider an online algorithm A. Note that the input sequence in slot 1 for both $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ is identical, and thus without the knowledge of future fading coefficients of slot 2, A cannot adapt the energy it spends in slot 1 depending on $\boldsymbol{\sigma}_i, i = 1, 2$. Thus, let A spend α fraction of its energy $E_1 = e$ in slot 1 for both $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, and use the rest $(1-\alpha)$ fraction in slot 2. Thus, with parameter α we can index all online algorithms. Note that any online algorithm with choose that α that minimizes the competitive ratio (penalty with respect to an optimal offline algorithm). Next, we show that no matter what α is, the competitive ratio of any online algorithm is at least 2.

With A spending α fraction of its energy $E_1 = e$ in slot 1 for both $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, $R_A(\boldsymbol{\sigma}_1) = \alpha e$, while $R_A(\boldsymbol{\sigma}_2) = (\alpha + (1-\alpha)\beta)e$. Thus,

$$\begin{aligned} r &\geq \min_A \max_{\boldsymbol{\sigma}\in\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2\}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}, \\ &= \min_{\alpha\in[0,1]} \max\left\{\frac{1}{\alpha}, \frac{\beta}{\alpha+(1-\alpha)\beta}\right\} \end{aligned}$$

At the optimal value of α, $\alpha^\star$, the two terms inside the maximum are equal. Thus, $\alpha^\star$ satisfies $\frac{1}{\alpha^\star} = \frac{\beta}{\alpha^\star+(1-\alpha^\star)\beta}$. Therefore, $\alpha^\star = \frac{1}{2-\frac{1}{\beta}}$, and hence

$$\min_{\alpha\in[0,1]} \max\left\{\frac{1}{\alpha}, \frac{\beta}{\alpha+(1-\alpha)\beta}\right\} = 2 - \frac{1}{\beta},$$

and consequently, $r \geq 2 - \frac{1}{\beta}$. Note that $r \geq \min_{\alpha\in[0,1]} \max\left\{\frac{1}{\alpha}, \frac{\beta}{\alpha+(1-\alpha)\beta}\right\}$ for any value of β. Therefore $r \geq \lim_{\beta\to\infty} \min_{\alpha\in[0,1]} \max\left\{\frac{1}{\alpha}, \frac{\beta}{\alpha+(1-\alpha)\beta}\right\}$. Clearly,

$$\lim_{\beta\to\infty} \min_{\alpha\in[0,1]} \max\left\{\frac{1}{\alpha}, \frac{\beta}{\alpha+(1-\alpha)\beta}\right\} = 2.$$

Thus, $r > 2 - \epsilon$ for any arbitrarily small $\epsilon > 0$.

Working backwards, what we have essentially done is as follows. Lets say we want to show that $r > 2-\epsilon$ for some $\epsilon > 0$. Then we pick β large enough so that $r > 2-\epsilon$ using the linear approximation that $w\log\left(1+\frac{\beta\hat{e}}{w}\right) \approx \beta\hat{e}$ for any $\hat{e} \leq e$. Then pick e small enough so that the linear approximation on achievable rate $w\log\left(1+\frac{\beta\hat{e}}{w}\right) \approx \beta\hat{e}$ for any $\hat{e} \leq e$ is tight. The same technique is applied for obtaining a lower bound for any number of slots N in the following Theorem.

Theorem 1: Let any online algorithm be r_A-competitive for solving R^s. Then $r_A > N - \epsilon$ for any arbitrarily small $\epsilon > 0$.

Proof: Consider N input sequences of length N,

$$\boldsymbol{\sigma}_i = ((1, E_1), (\beta, 0), (\beta^2, 0), \ldots, (\beta^{i-1}, 0), \underbrace{(0,0), \ldots, (0,0)}_{N-i}),$$

$i = 1, 2, \ldots, N$, where $\beta >> 1$. We also fix the total energy available that arrives at slot 1 to $E_1 = e$. As in the case of $N = 2$, we let e small enough such that $w\log(1+\frac{|h_n|^2\hat{e}}{w}) \approx |h_n|^2 e, \ \forall\ n, \hat{e} \leq e, |h_n|^2 \in \boldsymbol{\sigma}_i, i = 1, 2, \ldots, N$. With $E_1 = e$, the achievable rate with the optimal offline algorithm with sequence $\boldsymbol{\sigma}_i$ is $R_O(\boldsymbol{\sigma}_i) = \beta^{i-1}e$, since it spends all its energy in the slot with the largest fading coefficient.

Consider an online algorithm A. Since the input at slot 1 is identical for $\boldsymbol{\sigma}_i, i = 1, 2, \ldots, N$, at slot 1, A does not know which input sequence has actually occurred. Thus, A cannot adapt the amount of energy it spends in slot 1 depending on $\boldsymbol{\sigma}_i, i = 1, 2, \ldots, N$. Thus, let A spend α_1 fraction of its energy $E_1 = e$ in slot 1. At slot 2, if A sees $(0, 0)$ as the input resulting because of $\boldsymbol{\sigma}_1$, it transmits no energy. Otherwise, if $(\beta, 0)$ is the input at slot 2 that is identical for $\boldsymbol{\sigma}_i, i = 2 \ldots, N$, similar to above description, let A spend α_2 fraction of its energy e in slot 2 irrespective of the input sequence, since it has no knowledge of the input sequence among the possible $N-1$ input sequences $\boldsymbol{\sigma}_i, i = 2 \ldots, N$. Carrying this forward, let A spend no energy in any slot for which the input sequence is $(0, 0)$. Otherwise, let A spend α_n fraction of its energy e in slot n, such that $\sum_{n=1}^{N} \alpha_n \leq 1$. Thus, with parameter $\alpha_n's$ we can index all online algorithms. The rate of an online algorithm A with input sequence $\boldsymbol{\sigma}_i$ is $R_A(\boldsymbol{\sigma}_i) = \sum_{j=1}^{i} \alpha_j \beta^{j-1} e$. From Remark 1,

$$\begin{aligned} r &\geq \min_{A} \max_{\boldsymbol{\sigma} \in \{\boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_N\}} \frac{R_O(\boldsymbol{\sigma})}{R_A(\boldsymbol{\sigma})}, \\ &= \min_{\alpha_n, \sum_{n=1}^{N} \alpha_n \leq 1} \max \left\{ \frac{1}{\alpha_1}, \frac{\beta}{\alpha_1 + \alpha_2 \beta}, \right. \\ & \left. \qquad \cdots \frac{\beta^{N-1}}{\sum_{j=1}^{N-1} \alpha_j \beta^{j-1} + ((1 - \sum_{j=1}^{N-1} \alpha_j) \beta^{N-1}} \right\}. \end{aligned} \tag{9}$$

Similar to the case of $N = 2$, at the optimal values of $\alpha_n's$, $\alpha_n^\star$, all the terms inside the maximum are equal. Thus, $\frac{1}{\alpha_1^\star} = \frac{\beta}{\alpha_1^\star + \alpha_2^\star \beta} = \cdots = \frac{\beta^{N-1}}{\sum_{j=1}^{N-1} \alpha_j^\star \beta^{j-1} + ((1 - \sum_{j=1}^{N-1} \alpha_j^\star) \beta^{N-1}}$. Hence

$$r \geq \frac{1}{\alpha_1^\star}, \tag{10}$$

where

$$\begin{aligned} \frac{1}{\alpha_1^\star} &= \min_{\alpha_n, \sum_{n=1}^{N} \alpha_n \leq 1} \max \left\{ \frac{1}{\alpha_1}, \frac{\beta}{\alpha_1 + \alpha_2 \beta}, \right. \\ & \left. \qquad \cdots \frac{\beta^{N-1}}{\sum_{j=1}^{N-1} \alpha_j \beta^{j-1} + ((1 - \sum_{j=1}^{N-1} \alpha_j) \beta^{N-1}} \right\} \end{aligned}$$

Similar to the $N = 2$ case, it can be easily shown that $\lim_{\beta \to \infty} \frac{1}{\alpha_1^\star} = N$. Moreover, since, the lower bound (10) is valid for all β, therefore, $r \geq \lim_{\beta \to \infty} \frac{1}{\alpha_1^\star}$. Thus, $r > N - \epsilon$ for any arbitrarily small $\epsilon > 0$.

■

In the proof we assumed that $|h_n|^2$ can take any value in $[0, \infty)$, which is consistent with the typical wireless channel modeling [22], where fading coefficients are assumed to have infinite support. In case, the fading coefficients are bounded from below and above, i.e. $h_{min} < |h_n|^2 <$

h_{max}, where $h_{min} > 0$ and $h_{max} < \infty$, then we cannot let any $|h_n|^2 = 0$ and $\beta \to \infty$ in the proof of Theorem 1. However, if $\frac{h_{max}}{h_{min}}$ is large enough, reworking the proof of Theorem 1 by replacing $|h_n|^2 = 0$ by $|h_n|^2 = h_{min}$, and choosing h_{max} to be large enough, we can get $r_A > N - \epsilon$, for small enough ϵ that depends on $\frac{h_{max}}{h_{min}}$. The regime where $\frac{h_{max}}{h_{min}}$ is large enough has been considered previously in [18] to derive a lower bound on the competitive ratio that is given by $r = \Omega\left(\frac{h_{max}}{h_{min}}\right)$.

Remark 4: Note that the lower bound obtained in Theorem 1 also applies to the case when an online algorithm makes decision at slot n with information only about the past h_i and $E_i, i < n$ or no information at all, since knowing the present information can only improve the performance of an online algorithm.

Discussion: In this section, we constructed a lower bound on the competitive ratio of any online algorithm for maximizing the achievable rate under a sum-energy/power constraint. We showed that the lower bound is arbitrarily close to the number of slots N. To derive this bound, we first chose the available energy value to be small enough such that the rate achievable in any slot is well approximated by linear payoff : the product of the fading coefficient and the energy invested in that slot. With the linear payoff, the basic idea behind the lower bound is that if we keep increasing the fading coefficients in subsequent slots, the optimal offline algorithm invests all its energy in the last slot. Any online algorithm, however, has to invest equal energy in all slots since it tries to maintain a minimum ratio between the optimal offline algorithms payoff and its own payoff at each slot without knowing the future fading coefficients.

Compared to [18], we obtained the lower bound as a function of the number of slots N rather than the ratio of the maximum and the minimum fading coefficient magnitudes. The utility of this lower bound is that in the next section we will show that this bound is actually tight, i.e. there exists an online algorithm that can achieve this lower bound. We also note that our lower bound does not contradict the upper bound derived in [18] (Remark 3), since [18] assumes that the number of slots N is a function of the available energy E_1, h_{max} and h_{min}, and the bound is derived for a particular choice of N.

B. Upper Bound on the Competitive Ratio

We propose an equal power allocation (EPA) algorithm to upper bound the competitive ratio of $\mathcal{R}^s$ (8). With EPA algorithm, the available energy at the start E_1 is equally distributed across all the slots, i.e. $UE_n = \frac{E_1}{N}, n = 1, \dots, N$. Next, we show that the competitive ratio of the EPA algorithm is N.

Theorem 2: EPA algorithm is N-competitive for solving R^s. Consequently, the EPA algorithm is an optimal online algorithm for solving $\mathcal{R}^s$ (8).

Proof: Consider any input sequence $\boldsymbol{\sigma} = ((|h_1|^2, E_1), (|h_2|^2, 0) \dots, (|h_N|^2, 0))$. Let

$$m = \arg \max_{j=1,2,\dots,N} |h_j|^2.$$

Since the EPA algorithm invests equal energy E_1/N in each slot, the achievable rate with the EPA algorithm $R^s_{EPA}(\boldsymbol{\sigma}) \geq w \log\left(1 + |h_m|^2 \frac{E_1}{Nw}\right)$, by just counting for the rate obtained in the m^{th} slot.

To upper bound the rate obtained with the optimal offline algorithm, consider an enhanced version of the input sequence $\boldsymbol{\sigma}$ that consists of fading coefficients with all entries equal to $|h_m|^2$, i.e., $\bar{\boldsymbol{\sigma}} = ((|h_m|^2, E_1), (|h_m|^2, 0), \dots, (|h_m|^2, 0))$. Clearly, the achievable rate with $\bar{\boldsymbol{\sigma}}$ is better than $\boldsymbol{\sigma}$. Thus, $R^s_O(\boldsymbol{\sigma}) \leq R^s_O(\bar{\boldsymbol{\sigma}})$. Moreover, with $\bar{\boldsymbol{\sigma}}$, since all fading coefficients are identical, from Remark 2, the optimal offline algorithm (waterfilling) invests equal energy E_1/N in each slot to get $R^s_O(\bar{\boldsymbol{\sigma}}) = Nw \log\left(1 + |h_m|^2 \frac{E_1}{Nw}\right)$. Thus, the competitive ratio of the EPA algorithm is

$$\begin{aligned} r_{EPA}(\boldsymbol{\sigma}) &= \frac{R^s_O(\boldsymbol{\sigma})}{R^s_{EPA}(\boldsymbol{\sigma})}, \\ &\leq \frac{R^s_O(\bar{\boldsymbol{\sigma}})}{R^s_{EPA}(\boldsymbol{\sigma})}, \\ &\leq \frac{Nw \log\left(1 + |h_m|^2 \frac{E_1}{Nw}\right)}{w \log\left(1 + |h_m|^2 \frac{E_1}{Nw}\right)}, \\ &\leq N. \end{aligned} \tag{11}$$

The final conclusion follows by comparing the upper bound (11) with the lower bound on the competitive ratio derived in Theorem 1.

∎

Discussion: In this section, we proposed a simple EPA algorithm that spends equal energy in all slots without using the causal fading coefficient information, and whose competitive ratio is upper bounded by the number of slots N. More significantly, the competitive ratio of the EPA algorithm matches with the lower bound obtained in Theorem 1, and hence we conclude that the EPA algorithm is an optimal online algorithm for solving (8). As described before, upper and lower bounds on the competitive ratio have been derived previously in [18] as a function of the ratio of the maximum to the minimum fading coefficient, however, the bounds do not match. Note that our upper bound on the competitive ratio derived using the EPA algorithm does not contradict the lower bound derived in [18] (Remark 3), since [18] assumes that the number of slots N is a function of the available energy E_1, h_{max} and h_{min}. Using Theorems 1 and 2, we note that the number of slots N is the right quantity of interest in terms of the competitive ratio rather than the ratio of the maximum to the minimum fading coefficient [18].

In light of Remark 4, from Theorems 1 and 2, it also follows that the EPA algorithm is an optimal online information even if only past or no information about the fading coefficients is available at each slot. Thus, the optimality of the EPA algorithm is somewhat a negative result, since the optimal competitive ratio is invariant to the availability of the information about the past/present fading coefficients, and shows that the causal fading coefficient information is actually not useful.

IV. Arbitrary Energy Arrivals and Fading Coefficients

In this section, we consider the general case, where both the energy arrivals and fading coefficients are arbitrarily varying and only causal information is available about them, i.e., we are interested in solving (4).

A. Lower Bound on the Competitive Ratio

With arbitrarily varying energy arrivals and fading coefficients, the input sequence is $\boldsymbol{\sigma} = ((|h_1|^2, E_1), (|h_2|^2, E_2), \dots, (|h_N|^2, E_N))$. Let us restrict our attention to the case when $E_n = 0, n = 2, \dots, N$. Then, we are in a setting equivalent to Section III, where all the energy arrives at the beginning, and since $E_n = 0, n = 2, \dots, N$ is a special case of input sequences, from Remark 1, it follows that the lower bound on the optimal competitive ratio obtained in Theorem 1, also applies to the general case of arbitrarily varying energy arrivals and fading coefficients. We summarize the result in the following Theorem.

Theorem 3: Let any online algorithm be r_A-competitive for solving $\mathcal{R}$ (4). Then $r_A > N - \epsilon$ for any arbitrarily small $\epsilon > 0$.

Remark 5: Note that from the definition of the competitive ratio, considering a special case of $E_n = 0, n = 2, \dots, N$ is sufficient to derive a lower bound on the optimal competitive ratio. However, since the upper bound has to hold for all input sequences $\boldsymbol{\sigma}$, i.e. all possible values of $E_n, n = 1, \ldots, N$, limiting to special cases of inputs is not sufficient for deriving an upper bound on the competitive ratio. Hence, the results of Subsection III-B, do not apply for arbitrarily varying energy arrivals. In the next section, we propose a modified EPA algorithm that is online, and whose competitive ratio is upper bounded by N.

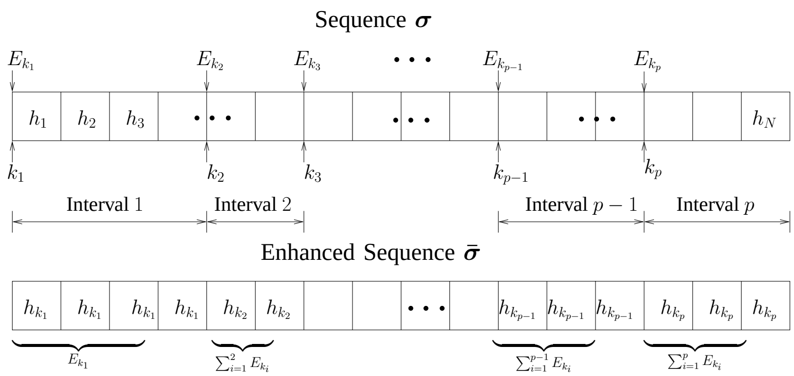

Fig. 1. Illustration of strategy used to upper bound the rate with optimal offline algorithm.

B. Upper Bound on the Competitive Ratio

To upper bound the competitive ratio with arbitrarily varying energy arrivals and fading coefficients, we modify the EPA algorithm as follows, and call it repeated equal power allocation (REPA) algorithm. With REPA, if at slot n the available energy is $\hat{E}_n$, then it equally distributes the available energy over the remaining slots, and uses energy $UE = \frac{\hat{E}_n}{N-n+1}$ in each slot until the next energy arrival. Clearly, REPA algorithm is online, i.e. it does not depend on future, and satisfies the energy neutrality constraint. Note that if the energy only arrives at slot 1, then the REPA algorithm is equivalent to the EPA algorithm. Next, we show that REPA algorithm is N-competitive for solving (4).

Theorem 4: REPA algorithm is N-competitive for solving $\mathcal{R}$ (4). Consequently, the REPA algorithm is optimal for solving (4).

Proof: Consider any input sequence $\boldsymbol{\sigma} = ((|h_1|^2, E_1), (|h_2|^2, E_2), \ldots, (|h_N|^2, E_N))$. Consider the slot indices n for which energy arrivals $E_n \neq 0$, and denote them by $k_1, \ldots k_p$, where $p \leq N$. Without loss, assume that $k_1 = 1$, i.e., non-zero energy arrives in slot 1. Otherwise, we can start from the k_1^{th} slot, and remove the first $N - k_1 - 1$ slots from consideration. This can only improve the upper bound. Let i denote the index of slot interval between energy arrivals at slots k_{i+1} and k_i, $i = 1, \ldots, p-1$. The p^{th} slot interval represents the slots between slot k_p, where the last energy arrival happens, and the last slot N. See Fig. 1 for illustration. For simplicity of exposition, we let the input sequence corresponding to the i^{th} slot interval be $\sigma_i = (|h_{k_i}|^2, E_{k_i}), (|h_{k_i+1}|^2, 0) \ldots, (|h_{k_{i+1}-1}|^2, 0), i = 1, \ldots, p$, where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_p)$.

Let $|h_i^{max}|^2$ be the largest fading coefficient magnitude in slot interval $i = 1, \ldots, p$, i.e., $|h_i^{max}|^2 = \max\{|h_{k_i}|^2, |h_{k_i+1}|^2, \ldots, |h_{k_{i+1}-1}|^2\}$. Then as in the proof of Theorem 2, we enhance the input sequence corresponding to the i^{th} slot interval σ_i as

$$\bar{\sigma}_i = ((|h_i^{max}|^2, E_{k_i}), (|h_i^{max}|^2, 0) \ldots, (|h_i^{max}|^2, 0), i = 1, \ldots, p,$$

and $\bar{\boldsymbol{\sigma}} = (\bar{\sigma}_1, \bar{\sigma}_2, \ldots, \bar{\sigma}_p)$. See Fig. 1 for illustration.

Clearly, the rate achievable with $\bar{\boldsymbol{\sigma}}$ is better than $\boldsymbol{\sigma}$, thus, $R_O(\bar{\boldsymbol{\sigma}}) \ge R_O(\boldsymbol{\sigma})$. Note that because of energy neutrality constraint, the maximum energy spent by the end of slot interval i is $\sum_{j=1}^{i} E_{k_j}, \forall\, i$. Thus, in any slot interval i, the maximum energy that can be spent is $\sum_{j=1}^{i} E_{k_j}, \forall\, i$. Therefore, the maximum rate achievable in any slot interval i is obtained by spending all the energy that has arrived till then in slot interval i.

Moreover, since with $\bar{\boldsymbol{\sigma}}$, in each slot interval the fading coefficients are identical, and hence the energy spent by an optimal offline algorithm in any slot interval is spent equally among all the slots in that slot interval (Remark 2), therefore, $R_O(\bar{\sigma}_i) \le (k_{i+1}-k_i)w \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{(k_{i+1}-k_i)w}\right)$, since $(k_{i+1}-k_i)w$ is the width of the i^{th} slot interval. To obtain this upper bound, we have made significant relaxation of energy constraint since we allow spending energy E_1 in slot interval 1, spending energy $E_1 + E_2$ in slot interval 2 and so on such that the energy spent in slot interval p is $\sum_{i=1}^{p} E_{k_i}$, as shown in Fig. 1.

Hence $R_O(\bar{\boldsymbol{\sigma}})$

$$\begin{aligned} &= \sum_{i=1}^{p} R_O(\bar{\sigma}_i), \\ &\le \sum_{i=1}^{p} (k_{i+1}-k_i)w \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{(k_{i+1}-k_i)w}\right). \end{aligned} \tag{12}$$

Thus, (12) serves as an upper bound on $R_O(\bar{\boldsymbol{\sigma}})$, that we will use to upper bound the competitive ratio of the REPA algorithm.

Next, we lower bound the rate with the REPA algorithm by considering the original input sequence $\boldsymbol{\sigma}$, and not its enhanced version. With the REPA algorithm, let the energy used in any slot n of slot interval i be UE_n^i. Then $UE_n^1 = \frac{E_1}{N}$, and $UE_n^i = \frac{E_i + UE_{i-1}(N-(k_{i-1}-k_i))}{(N-k_i+1)}$ for $i = 2, \ldots, p$. Simple algebraic manipulations show that $UE_n^i \geq \frac{\sum_{j=1}^{i} E_{k_j}}{N}$, $\forall\ i$. Then with the REPA algorithm, for each slot interval $i = 1, 2, \ldots, p$, considering only one slot that achieves the fading coefficient $|h_i^{max}|^2$ in σ_i,

$$R_{REPA}(\sigma_i) \geq w \log\left(1 + |h_i^{max}|^2 \frac{UE_n^i}{w}\right).$$

Substituting, $UE_n^i \geq \frac{\sum_{j=1}^{i} E_{k_j}}{N}$, we get

$$R_{REPA}(\sigma_i) \geq w \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{Nw}\right),$$

and the total achievable rate $R_{REPA}(\boldsymbol{\sigma})$

$$\begin{aligned}
&= \sum_{i=1}^{p} R_{REPA}(\sigma_i), \\
&\geq \sum_{i=1}^{p} w \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{Nw}\right), \\
&= \sum_{i=1}^{p} w \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{\frac{N}{(k_{i+1}-k_i)}(k_{i+1}-k_i)w}\right), \\
&\stackrel{(a)}{\geq} \sum_{i=1}^{p} \frac{w}{\frac{N}{(k_{i+1}-k_i)}} \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{w(k_i - k_{i-1})}\right), \\
&= \sum_{i=1}^{p} \frac{(k_{i+1}-k_i)w}{N} \log\left(1 + \frac{|h_i^{max}|^2 \sum_{j=1}^{i} E_{k_j}}{w(k_i - k_{i-1})}\right), \qquad (13)
\end{aligned}$$

where (a) follows from the fact that $\log(1 + \frac{y}{x}) \geq \frac{1}{x}\log(1+y)$ for $x, y \geq 1$. Thus, from (12) and (13), the competitive ratio of the REPA algorithm is

$$\begin{aligned}
r_{REPA}(\boldsymbol{\sigma}) &= \frac{R_O(\boldsymbol{\sigma})}{R_{REPA}(\boldsymbol{\sigma})}, \\
&\leq N.
\end{aligned}$$

■

Discussion: In this section, we proposed the REPA algorithm that spends the available energy equally in all future slots until the next energy arrival, and upper bounded its competitive ratio for solving (4). REPA algorithm is essentially a pessimistic algorithm that assumes that no further energy is going to arrive in future, and spends its energy equally in each slot. Even though the REPA algorithm is pessimistic, and does not depend on the current or past fading coefficients, we showed that the competitive ratio of the REPA algorithm is equal to the number of slot N which matches with the derived lower bound in Theorem 3. Similar to the comment made in Discussion of Subsection III-B, once again we conclude that the value of current or past fading coefficients information is minimal for solving (4), since the competitive ratio of the REPA algorithm that is agnostic to the causal fading coefficient information is optimal.

In the next section, we discuss a related problem of minimizing the transmission time for a fixed number of bits, when both the energy and fading coefficients are arbitrarily varying.

V. Transmission Time Minimization Problem

In this section, we consider the problem of minimizing the transmission time of fixed number of bits B that are available at the beginning of transmission (7), when both the energy arrivals and fading coefficients are arbitrarily varying, and only causal information is known about them. This problem has been previously considered in [7], where the optimal offline algorithm has been derived. For this problem, we will show a negative result that the competitive ratio of any online algorithm is infinity. We would like to note that for the minimum transmission completion time problem in an additive white Gaussian noise (AWGN) channel, where all the fading coefficients are equal to 1, the competitive ratio is upper bounded by 2 [21].

Theorem 5: Let any online algorithm be r_A-competitive for solving $\mathcal{T}$ (7). Then $r_A = \infty$.

Proof: Following Remark 1, to prove the Theorem we will construct two sequences $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, and a value of B such that the time taken to transmit B by the optimal offline algorithm is finite, but the time taken by any online algorithm to transmit B bits with at least one of the two sequences is infinite. As before, any input sequence is of the type $\boldsymbol{\sigma} = ((|h_1|^2, E_1), (|h_2|^2, E_2), \ldots\ldots)$ Let

$$\boldsymbol{\sigma}_1 = ((1,1),(10,0),(0.01,0),\ldots,(0.01,0),\ldots,),$$

while

$$\boldsymbol{\sigma}_2 = ((1,1),(0.38,10),(0.01,0),\ldots,(0.01,0),\ldots,).$$

Let $B = 3$ bits, and slot width $w = 1$.

Consider the optimal offline algorithm. We upper bound the time taken by the offline algorithm to finish the transmission of $B = 3$ bits. With $\boldsymbol{\sigma}_1$, the channel in slot 2 is far better than in slot 1, and if the optimal offline algorithm invests all its energy $E_1 = 1$ in slot 2, then the number of bits transmitted by the optimal offline algorithm is $\log(1+10) > 3$. Hence the optimal offline algorithm finishes transmission of $B = 3$ bits within 2 slots, i.e. $\mathcal{T}_O(\boldsymbol{\sigma}_1) \le 2$. Similarly, with $\boldsymbol{\sigma}_2$, since 10 units of energy arrives in slot 2, it is optimal [7] to invest $E_1 = 1$ in slot 1 and $E_2 = 10$ in slot 2, and the number of bits transmitted by the optimal offline algorithm in two slots is $\log(1+1) + \log(1+3.8)$ which is again greater than 3 bits, and hence $\mathcal{T}_O(\boldsymbol{\sigma}_2) \le 2$.

Let any online algorithm A spend α_1 fraction of its energy available in slot 1, and α_2 fraction of its energy available in slot 2. Also, let the online algorithm know the future energy arrivals and fading coefficients from slot 3 onwards. This relaxation can only improve the performance of any online algorithm. Then the number of bits sent by A is

$$\begin{aligned} B_A(\boldsymbol{\sigma}_1) &= \log(1+\alpha_1) + \log(1+(1-\alpha_1)\alpha_2 10) \\ &\quad + \lim_{t\to\infty} t \log\left(1 + \frac{1-\alpha_1-(1-\alpha_1)\alpha_2}{100t}\right), \end{aligned}$$

and $B_A(\boldsymbol{\sigma}_2)$

$$\begin{aligned} &= \log(1+\alpha_1) + \log(1+(1-\alpha_1+10)\alpha_2 0.38) \\ &\quad + \lim_{t\to\infty} t \log\left(1 + \frac{11-(1-\alpha_1+10)\alpha_2-\alpha_1}{100t}\right). \end{aligned}$$

Note that $\lim_{t\to\infty} t\log\left(1+\frac{x}{t}\right) = x\log_2 e$. We have chosen $|h_n|^2 = 0.01$ to be small enough for $n > 2$, so that $\arg\max_{\alpha_2\in[0,1]} B_A(\boldsymbol{\sigma}_1) = 1$, and $\arg\max_{\alpha_2\in[0,1]} B_A(\boldsymbol{\sigma}_2) = 1$, i.e. all the available energy is used up by slot 2 with both $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, knowing the future from slot 3 onwards.

With the optimal choice of $\alpha_2 = 1$, $B_A(\boldsymbol{\sigma}_1)$ is a decreasing function of α_1, while $B_A(\boldsymbol{\sigma}_2)$ is an increasing function of α_1. Moreover, since $\max_{\alpha_1} B_A(\boldsymbol{\sigma}_i) > 3$ and $\min_{\alpha_1} B_A(\boldsymbol{\sigma}_i) < 3$, for $i = 1, 2$, $B_A(\boldsymbol{\sigma}_1) = B_A(\boldsymbol{\sigma}_2)$ for some value of α_1. Let $\hat{\alpha}_1$ be the value of α_1 for which $B_A(\boldsymbol{\sigma}_1) = B_A(\boldsymbol{\sigma}_2) < 3$, then we know that for $\alpha_1 \ge \hat{\alpha}_1$, $B_A(\boldsymbol{\sigma}_1) < 3$, while for $\alpha_1 \le \hat{\alpha}_1$ $B_A(\boldsymbol{\sigma}_2) < 3$. For the choice of input sequences $\boldsymbol{\sigma}_i, i = 1, 2$, we have that $B_A(\boldsymbol{\sigma}_1) \ge 3$ for $\alpha_1 < 0.6$, while $B_A(\boldsymbol{\sigma}_2) \ge 3$ for $\alpha_1 > 0.608$, also illustrated in Fig. 2. Thus, at the intersection point, the value of $B_A(\boldsymbol{\sigma}_1) = B_A(\boldsymbol{\sigma}_2) < 3$. Since α_1 cannot be simultaneously less than 0.6 and more than 0.608, we conclude that any online algorithm will not finish transmission with at least one of the input sequences. Hence we get the following lower bound on the optimal competitive ratio $r \ge \min_{\alpha\in[0,1]} \max\left\{\frac{\mathcal{T}_A(\boldsymbol{\sigma}_1)}{\mathcal{T}_O(\boldsymbol{\sigma}_1)}, \frac{\mathcal{T}_A(\boldsymbol{\sigma}_2)}{\mathcal{T}_O(\boldsymbol{\sigma}_2)}\right\} = \infty$.

Note that this choice of $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ is not unique, and one can easily find many other input sequences for which the competitive ratio of any algorithm is ∞. $\blacksquare$

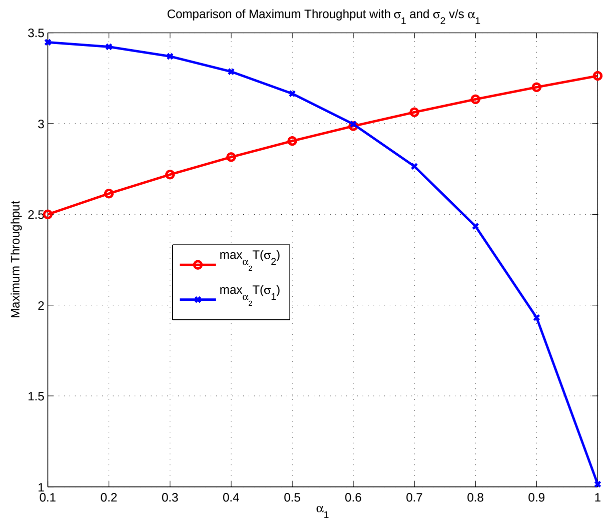

Fig. 2. Comparison of number of bits sent with $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_1$ as a function of α_1.

Discussion: In this section, we constructed two input sequences to show that the competitive ratio of any online algorithm to solve (7) is lower bounded by infinity. The basic idea behind the construction was to find a set of two input sequences and a value of B for which the choice of energy invested in the first slot by an online algorithm so that it can transmit B bits in finite time with the two sequences is contradictory to each other. Therefore, any online algorithm with any choice of energy to transmit in slot 1 can transmit B bits in finite time with only one of the two sequences. This construction is indeed possible since the energy arrivals and the fading coefficients in future can be arbitrarily ordered. For a special case, when all the fading coefficients take a constant value (e.g. unity in case of additive white Gaussian channel), we cannot have such a construction, and there exists an online algorithm for solving (7) for which the competitive ratio is less than 2 [21]. So, really, the lower bound of infinity on the competitive ratio is a manifestation of arbitrarily varying energy arrivals together with arbitrarily varying fading coefficients.

VI. Conclusions

In this paper, we derived the optimal competitive ratio for maximizing the achievable rate in a wireless communication channel over a fixed number of slots, with arbitrarily varying energy arrivals and fading coefficients. The competitive ratio analysis provides strong worst case guarantees on the performance of any online algorithm that has access only to the causal information. We showed that a very simple algorithm that invests equal energy in all future slots and which is agnostic to the current or past fading coefficient realizations is an optimal online algorithm, and has competitive ratio equal to the number of slots of interest. Another important conclusion we drew was that the optimal competitive ratio is invariant to the availability of the current/past fading coefficient information, i.e. with or without the causal information, the optimal competitive ratio remains the same. This is quite a pessimistic result, since one would expect an optimal online algorithm to adapt its transmission power as a function of current/past fading coefficients and provide with a better competitive ratio. We also considered the problem of minimizing the transmission time of a fixed number of bits, and showed that no matter how smart an online algorithm is, the competitive ratio of any online algorithm is infinity, i.e. there exists a set of input sequences for which the online algorithm never finishes transmission of appropriately chosen number of bits.

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