Course proposal

Markov chains and Random walks

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Course outline. This course will be an introduction to Markov chains and their applications to sampling and counting algorithms. It will also emphasize the variety of techniques that have been used in the analysis of Markov chains. If time permits, the course will also provide a glimpse of how these techniques relate to ideas such as geometry of polynomials, decay of correlations in Gibbs distributions, and to measure concentration. An earlier version of this course was offered in the Monsoon, 2017 semester, and its details are available at `https://www.tifr.res.in/~piyush.srivastava/teaching/mon-2017-markov/`.

I expect to cover the following topics:

1. The fundamental theorem of Markov chains.
2. Canonical examples of Markov chains: Glauber dynamics and the Metropolis sampler, Random walks of graphs (especially the Boolean hypercube), etc.
3. Coupling and associated methods for bounding mixing times: examples from hypercube random walks and Glauber dynamics.
4. Coupling and algorithms for "perfect sampling": the Propp-Wilson algorithm.
5. Spectral methods and combinatorial methods for bounding the spectral gap. Examples: Ferromagnetic Ising model, Matching in general graphs and (tentatively) perfect matching in bipartite graphs.

Aside from this, I also plan to cover a selection (based on interest) from the following list of tentative topics.

1. Recent progress using high dimensional expanders and its connection with the geometry of polynomials.
2. Recent new results in perfect sampling of colourings and other spin systems.
3. Sharper mixing time bounds from log Sobolov inequalities and hypercontractivity. Connections with concentration of measure.
4. Sampling from convex bodies: an introduction to Markov chains on continuous state spaces.
5. Local algorithms: the evolving set Markov chain and its use in finding local sparse cuts
6. Concentration inequalities for Markov chains.

Prerequisites. In addition to undergraduate algorithms and discrete mathematics, I expect the only other prerequisites to be some familiarity with linear algebra (the notions of eigenvalues, eigenvectors and eigenvalue decompositions) and analysis (notions of convergence, norms, Hölder's inequality etc.).

References. The main references will be the book *Markov chains and mixing times* by Galvin, Peres and Witmer, and the monograph *Lectures on finite Markov chains* by Saloff-Coste. For some of the topics related to measure concentration in the tentative list, I also plan to refer to the book *Concentration Inequalities* by Boucheron, Lugosi and Massart and the monograph *Concentration of measure and logarithmic Sobolev inequalities* by Ledoux. Apart from these, I also plan to refer to the original papers when needed.