

Subsidizing Sequential Search

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New ways of searching

best cuisine

X |   

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In the world Restaurant Top 10 Recipes Open now Cuisine Vibe

Results for State College, PA - Choose area

AI Overview

There's no single "best cuisine" as it's subjective, but based on popular rankings and awards, **Greek, Italian, and Mexican cuisines** are frequently cited among the best, often appearing in the top 3 of the TasteAtlas rankings. Other highly-rated cuisines include Spanish, French, Japanese, and Turkish.

Cuisines frequently ranked among the best

Show more

Which Country Has The Best Food? (A Very...
Aug 10, 2025 — I think we need to see if you know v
YouTube - Mythical Kitch

Which is the best cuisine ever tried? - Quora
Jan 11, 2024 — Italian Cuisine and vegetarian/ non vege

New ways of searching

ChatGPT 5 ▾



help me search for a pair of jeans size 32 that are
of good quality and ship for free



Thought for 1m 48s

Got it. To keep "ships free" guaranteed, buy from **Nordstrom** or **Zappos**—both
offer free standard shipping in the U.S. (and easy free returns). [Nordstrom +1](#)

[Zappos +1](#)

Here are dependable, good-quality picks you can grab in **waist 32** (filter to
32×30/32/34 for inseam):

From Zappos (free ship + generous returns)

- Levi's 501 Original Fit – the iconic straight leg; durable non-stretch options
available. Size 32 listed on the product pages. [Zappos +1](#)
- Levi's 512 Slim Taper / 550 Relaxed / 505 Regular – modern fits with lots
of washes; easy to find in 32 waist. [Zappos](#)

From Nordstrom (free ship + free returns)

- Mavi "Matt" Relaxed Straight – soft long-lasting fabric; size grid includes
32×32. [Nordstrom](#)



AI assistants act as *agents*: they process information and deliver a curated short list or even make a purchase.

Traditional advertising targets human attention: clicks, views, impressions.

But assistants, not humans, now search.

How should advertising be reimagined when AI agents control the market actions?



Brand Management

Forget What You Know About Search. Optimize Your Brand for LLMs.

by David Dubois, John Dawson and Akansh Jaiswal

June 4, 2025, Updated June 5, 2025

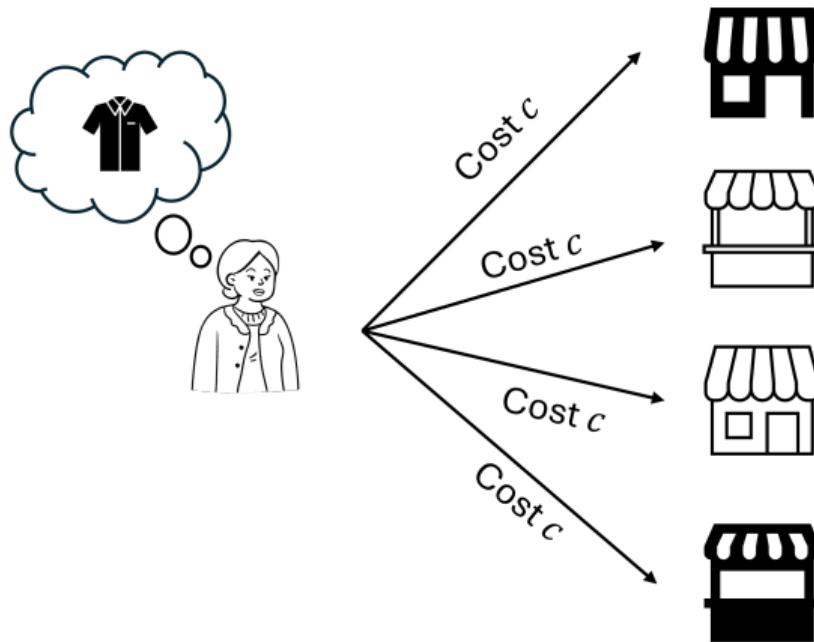
Advertising as Information

- Reveals existence of products.
- Conveys characteristics such as price or features.
- Yet many goods can only be evaluated through inspection.
- Firms can pay to reduce inspection costs \Rightarrow **subsidizing inspection**.

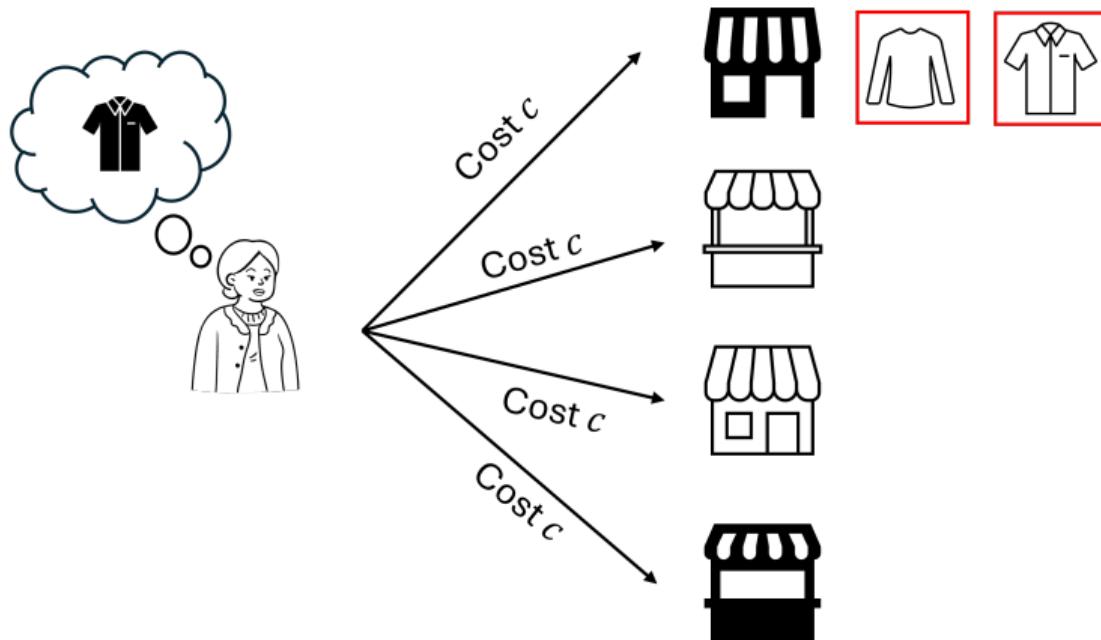
Why Now?

- In the agentic economy, assistants conduct search.
- Attention is metered (tokens, API calls).
- Subsidies can steer which options are inspected first.

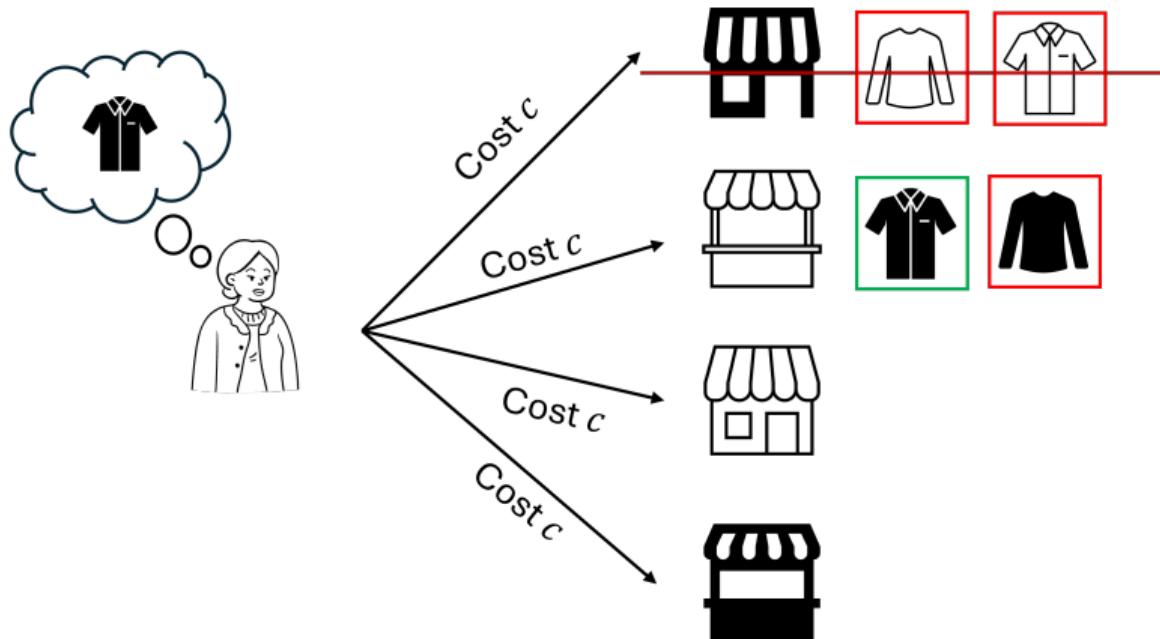
Search with Vertical Uncertainty



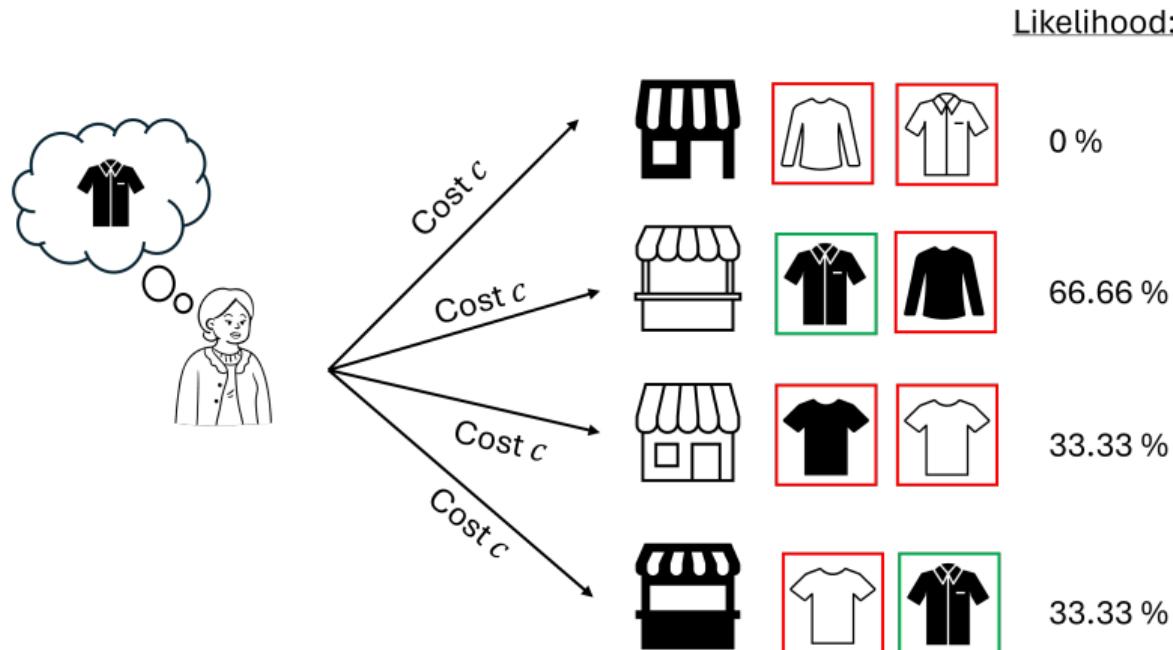
Search with Vertical Uncertainty



Search with Vertical Uncertainty



Search with Vertical Uncertainty



Keyword search: black shirt

But How to Search?

- Random search \Rightarrow inefficient.
It's like searching for a needle in a haystack.
- One celebrated solution: ranked list through ad auctions
Google-like: high-type firms bid to be shown first.
- But consumers still pay inspection costs.

What if inspection itself were free / subsidized?

Subsidizing Inspection

Gift cards • Free trials • Try now, pay later

- Firms can directly subsidize inspection costs.
- Higher subsidies \Rightarrow lower search cost, more attention.
- Key questions:
 - How does this shape the search order?
 - Does it lead to efficiency?

① Subsidy–sorting principle

Higher-type firms offer larger subsidies; search follows descending order.

② Efficient equilibrium under refinement

Unique separating outcome minimizes search costs.

③ Platform pricing

Optimal per–token price implements the efficient allocation.

Related Literature

Consumer Search as Optimal Stopping Problem: Weitzmann, 1979; Chade and Smith, 2006; ...

Sponsored Search Auctions: Edelman, Ostrosky, and Schwarz, 2007; Varian, 2007; Athey and Ellison, 2011; ...

Model of Search

Search with Vertical Uncertainty

- A consumer has a need; n firms may or may not match it.
- Firm private type $t_j \in [0, 1]$: probability product satisfies the need.
- Types are i.i.d. from common prior F (full support).
- Consumer searches sequentially.
 - Inspects a product \Rightarrow learns if it matches.
 - Each inspection costs $c > 0$, common and **publicly known**.
- A successful match yields payoff 1 to both consumer and firm.

Environment and Timeline

Environment. Search takes place on a platform. The platform lets firms *subsidize* consumers' inspection cost. Each unit of subsidy costs the firm a price $p \in \mathbb{R}_+$.

Timeline.

- ① Platform sets the per-unit price p (observed by all).
- ② Each firm privately learns its type $t_i \in [0, 1]$.
- ③ Firms simultaneously announce subsidies $s_i \in [0, c]$.
- ④ Consumer observes the subsidy profile $s = (s_1, \dots, s_n)$ and conducts sequential search.

Firms. A (pure) strategy for firm j is a measurable function

$$\sigma_j : [0, 1] \rightarrow [0, c], \quad t_j \mapsto s_j = \sigma_j(t_j).$$

Symmetry. We restrict attention to symmetric profiles in which all firms adopt the same *subsidy policy* σ . The realized subsidy vector is then $\vec{s} = (s_j)_{j=1}^n$, where $s_j = \sigma(t_j)$.

Consumer strategy.

- A search policy $\iota : [0, c]^N \rightarrow \mathcal{R}$ maps subsidy profiles to inspection rules.
- An inspection rule specifies a search path, including stopping, based on the history of inspected firms and outcomes.
- Belief system: $\rho : [0, c]^N \rightarrow \Delta([0, 1]^N)$ assigns posteriors over firm types.

Symmetric Perfect Bayesian Equilibrium (s-PBE)

A symmetric PBE is a triple (σ, ι, ρ) such that:

- ① **Consumer best response.** Given (σ, ρ) , the inspection policy ι maximizes the consumer's expected utility:

$$U_C(\sigma, \iota, \rho) = \Pr\{\text{match}\} - \mathbb{E} \left[\sum_{j=1}^K (c - s_j) \right].$$

- ② **Firm best response.** Given (ι, ρ) , each firm j with type t_j chooses

$$s_j = \sigma(t_j) \in \arg \max_{s \in [0, c]} (t_j - ps) \cdot \Pr\{j \text{ inspected} \mid \iota(s, s_{-j})\}.$$

- ③ **Belief consistency.** ρ is Bayes-consistent on the path of play.

Equilibrium Characterization

The Subsidy–Sorting Principle

Theorem

In any symmetric equilibrium (σ, ι, ρ) :

1 Monotone subsidies.

Higher-type firms never offer smaller subsidies: σ is weakly increasing.

2 Descending–subsidy search.

The consumer follows the descending–subsidy index rule (DSIR):

- **Ordering.** *Inspect firms in descending subsidy order (ties broken at random).*
- **Stopping.** *Continue searching only if the next firm's expected match benefit > net inspection cost.*

Theorem (First part)

*In any symmetric equilibrium, the **subsidy policy** is weakly increasing in type.*

Proof sketch. If there's an inversion, either the lower type or the higher type gains from deviating to the other's subsidy.

Descending-Order Search

Theorem (Second part)

In any symmetric equilibrium, consumer performs search in decreasing-subsidy order.

Proof sketch. A subsidy s reveals that firm's type lies in the level set $\sigma^{-1}(s)$.

- Posterior belief about success from a subsidy s is $\mathbb{E}[t \mid \sigma(t) = s]$
- Monotone subsidy policy implies higher subsidy has weakly higher belief
- Higher subsidy has lower net inspection cost
- Therefore higher subsidy items more attractive
- So search in decreasing subsidy order; stop if for next item $\mathbb{E}[t \mid \sigma(t) = s] < c - s$

Formal proof follows from **Weitzman, 1979**.

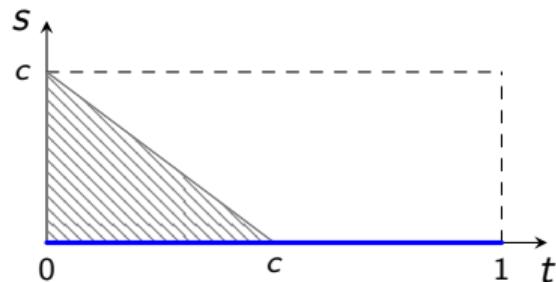
What We Have Learned So Far

In any equilibrium:

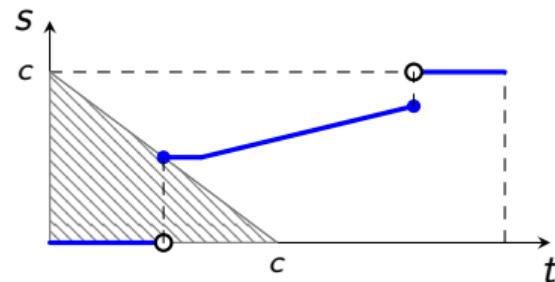
- The subsidy policy is weakly increasing.
- The consumer searches according to the descending–subsidy index rule (DSIR).

What next? We now turn to examples that illustrate the structure of equilibria beyond these basic properties.

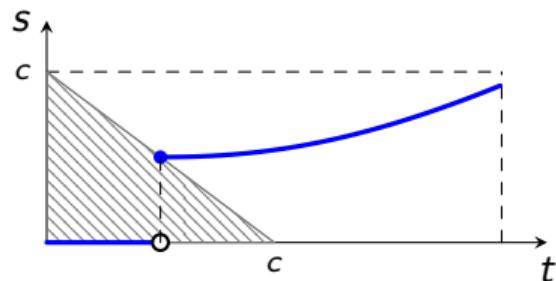
Subsidy policy examples



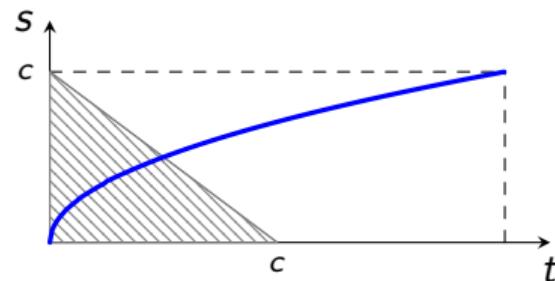
No subsidy



Policy with jumps



Optimal policy

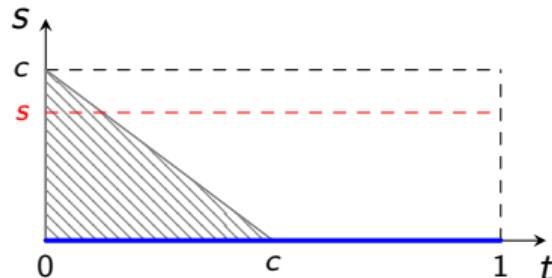


Fully revealing policy

Off-Path Beliefs

Claim. No positive subsidy can be supported in equilibrium.

Pessimistic beliefs: any off-path subsidy is attributed to type 0.



An **intuitive** consumer might use the following **criterion**: who could profit from deviating?

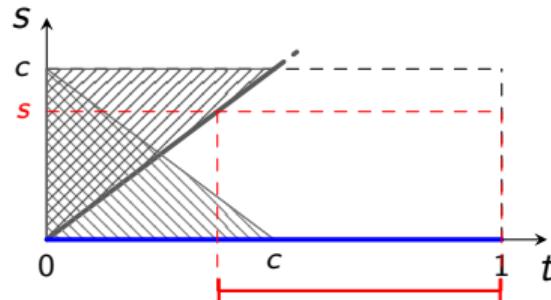
Firms subsidize only if profits stay non-negative:

$$\pi(t, s) = [t - ps] q \geq 0 \implies t/p \geq s.$$

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The Intuitive Criterion

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- Ask: *who could profit from deviating to this subsidy?*
- If only higher types would benefit, assign belief to higher types.
- If no type can profit, put zero belief on the deviation.

Implication here.

- Any interior pooling at $s \in (0, c)$ unravels:
 - A slightly higher subsidy improves rank,
 - Intuitive Criterion attributes this deviation to higher types,
 - \Rightarrow deviation is profitable.
- The only pooling that survives is at the cap $s = c$, where further increases are infeasible.

Consumer Optimal Equilibrium

Theorem

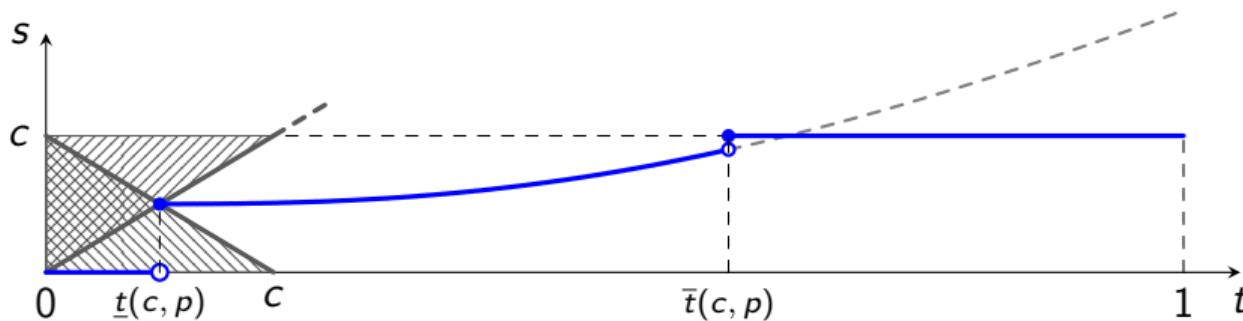
Aside from the no-inspection region, there is a unique subsidy policy that survives the intuitive criterion.

This policy has a simple form:

- **No subsidy.*** Low types do not subsidize and are never inspected.
- **Strictly increasing subsidies.** Intermediate types fully reveal by offering strictly increasing subsidies.
- **Pooling at the top.** High types pool at the full subsidy c , and ties are broken uniformly at random.

This is the consumer optimal equilibrium.

Consumer Optimal Equilibrium



Note: Consumers search optimally as if they knew the types!

Conclusion

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- Advertising creates competition among firms for human attention.
- In agentic markets, subsidized search can replace advertising.
- Consumers benefit from this structure, performing optimal search in equilibrium.
- Platforms provide the service of subsidizing costs at a fee; if they set the fee to maximize revenue this further helps consumers and harms firms as it results in excessive search.