Vigyan Vidushi 2025 (Mathematics)

From card tricks to the Mandelbrot set

(but mostly an excuse to show card tricks)

(which is an excuse for something else...)

Ramprasad Saptharishi (STCS)

I sincerely apologise for...

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- (Replace this placeholder with some useful metaphor about 'youthful curiosity' or something.)

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Also, there will be very little mathematical content in this talk

A brief survey on hobbies

Or go to menti.com

and use the code

2854 1560

What is the Mandelbrot Set?

Bear with me for 2 minutes and we'll move to card tricks!

Some simple iterative processes

$f : \mathbb{C} \to \mathbb{C}$

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$z_0 = 0$

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The Mandelbrot set

(perhaps the most famous "fractal")

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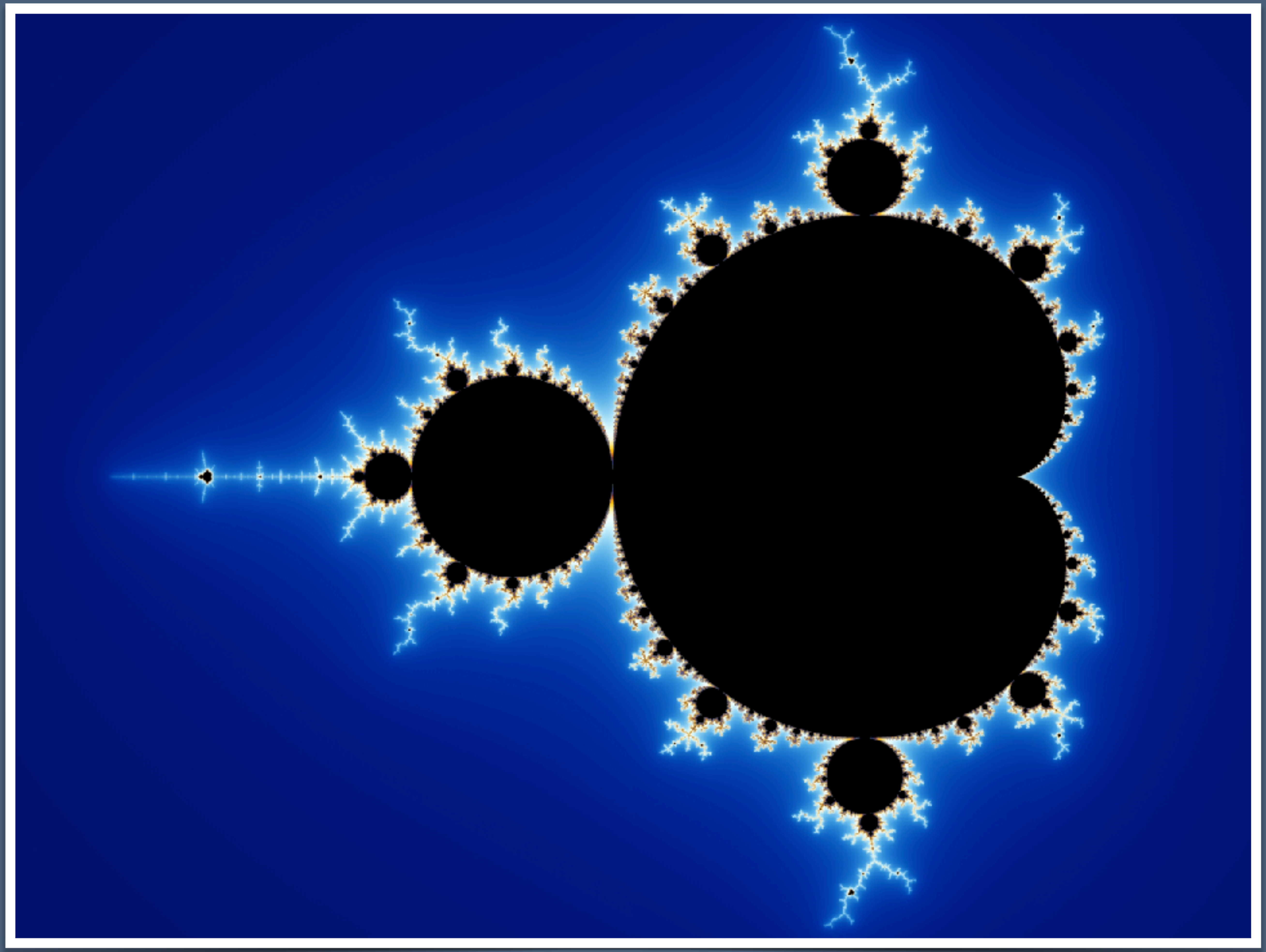

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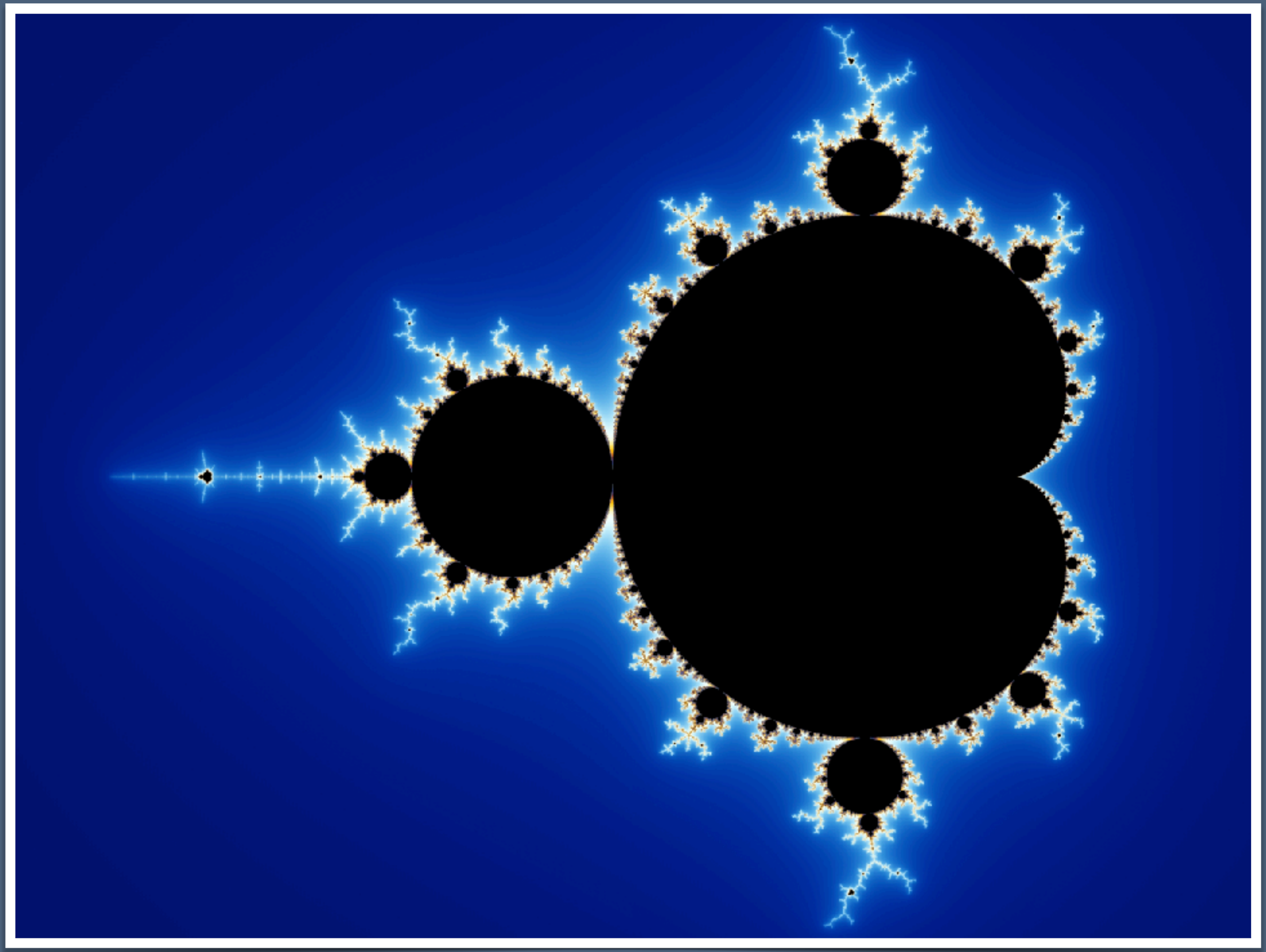
An interesting question:

Which are the c that result in a **periodic** sequence?



Hey, where are the card tricks that you promised?

Up the Ante
(Martyn Smith)

Lie-detector
(Paul Curry)

Memory Opener
(Sal Piacente)

Aunt Mary's Terrible Secret
(David Williamson and John Bannon)

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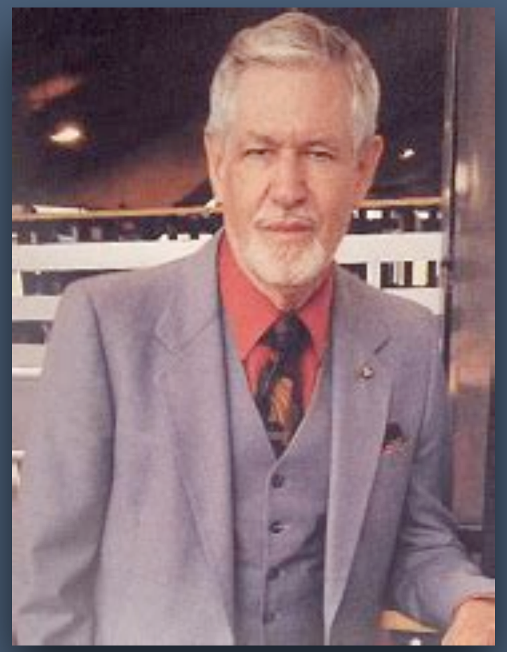
What properties are maintained when we do a standard riffle shuffle?

Norman Laurence Gilbreath

"A mathematician, and a life-long magician"



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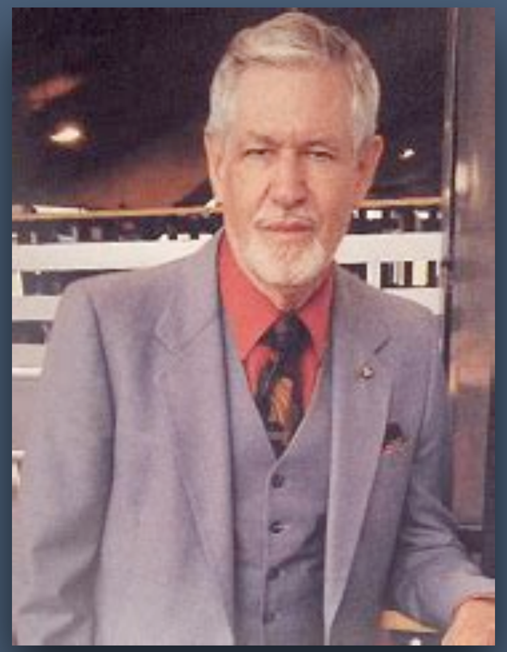

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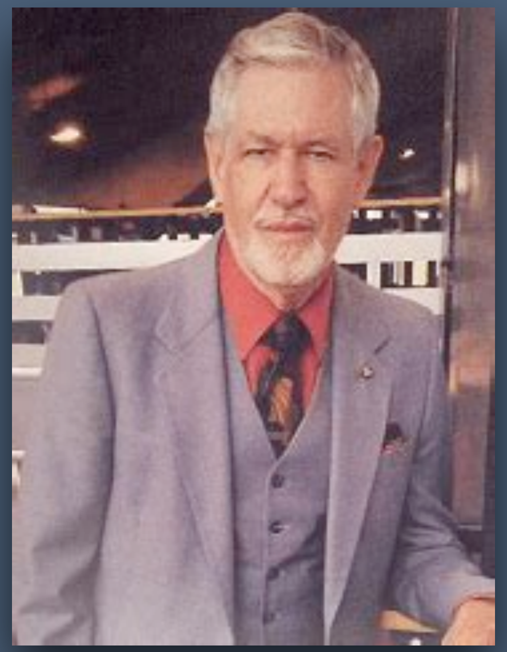

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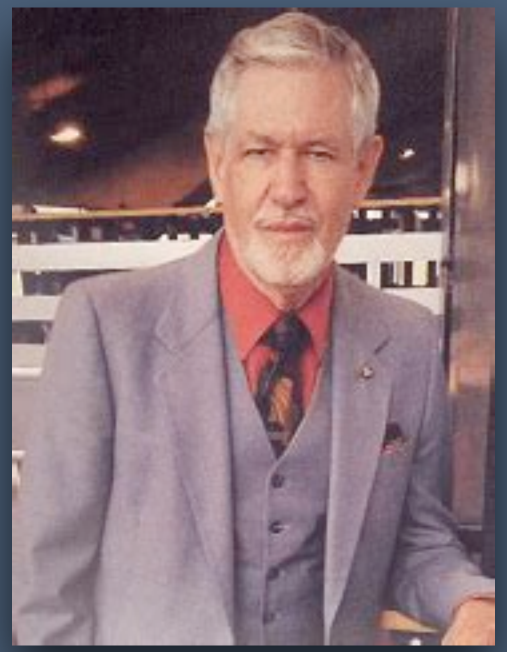

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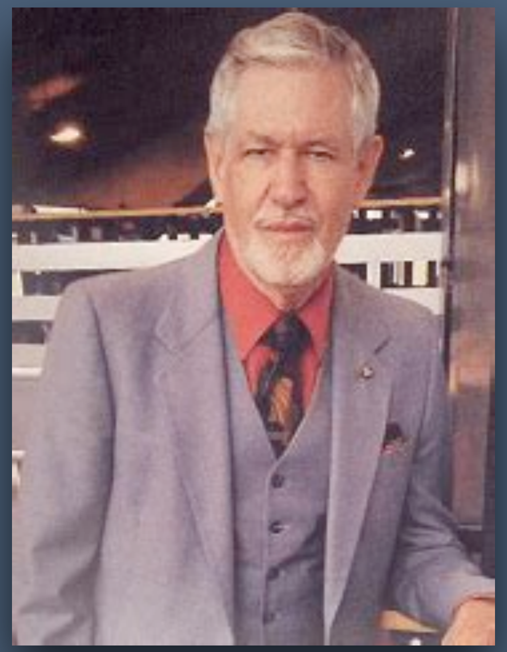

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Also known for the 'Gilbreath Conjecture' on difference sequence of prime numbers

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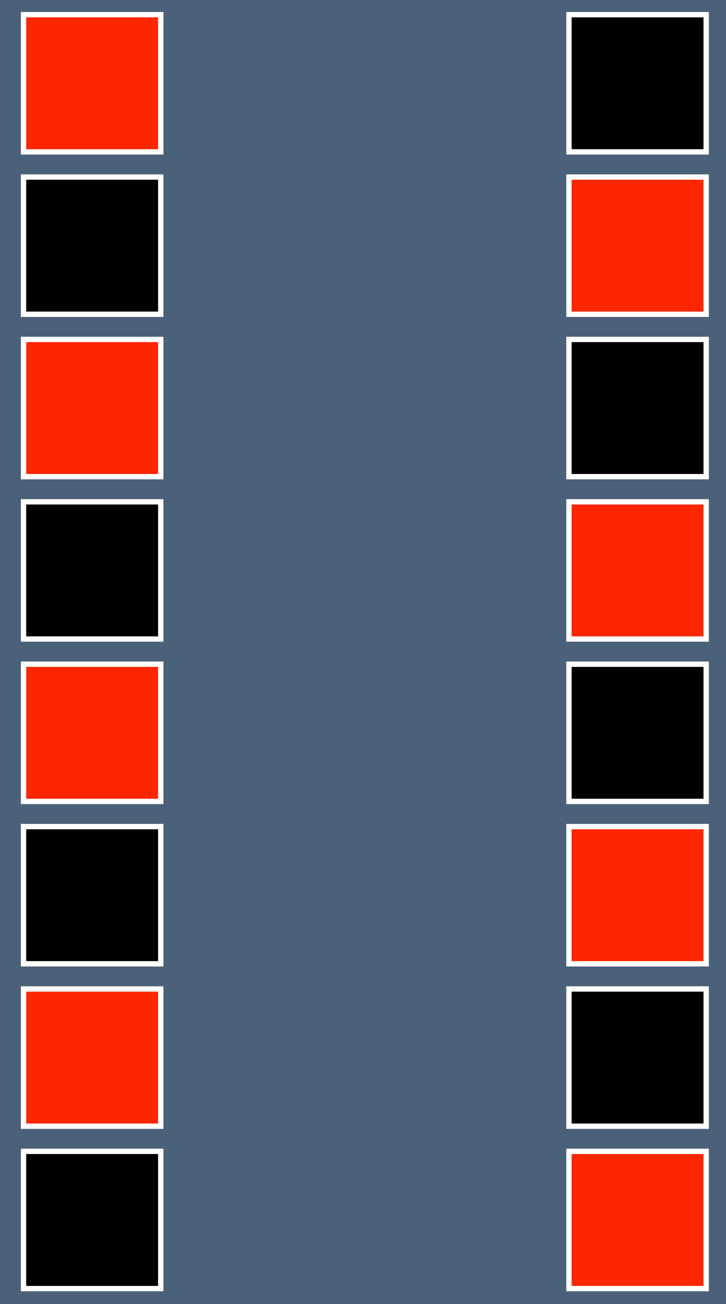

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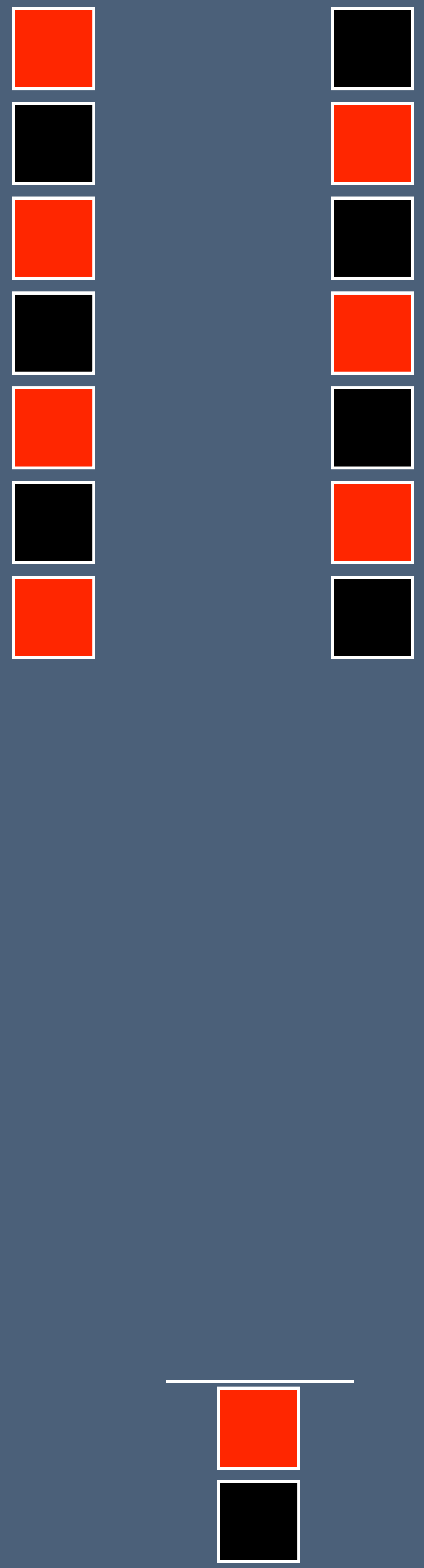

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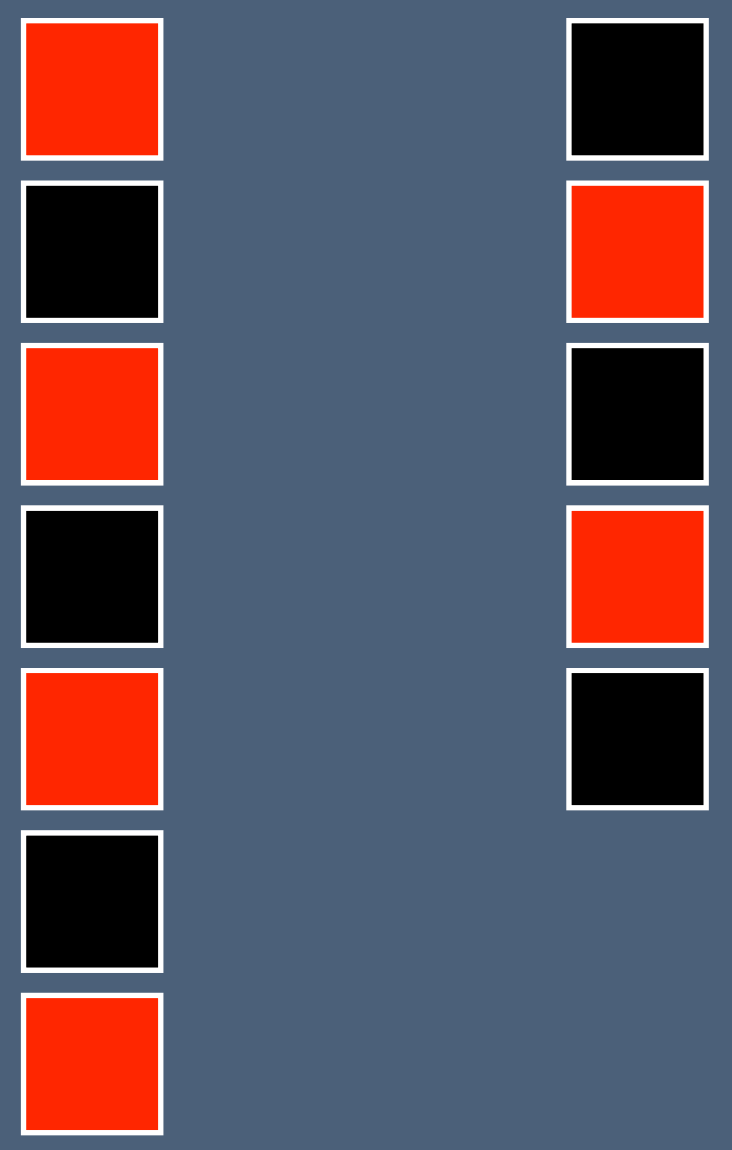

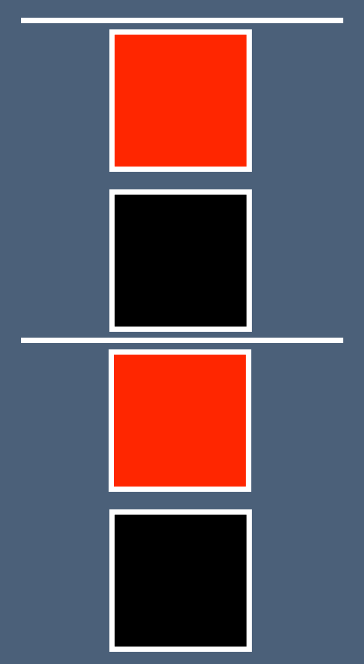

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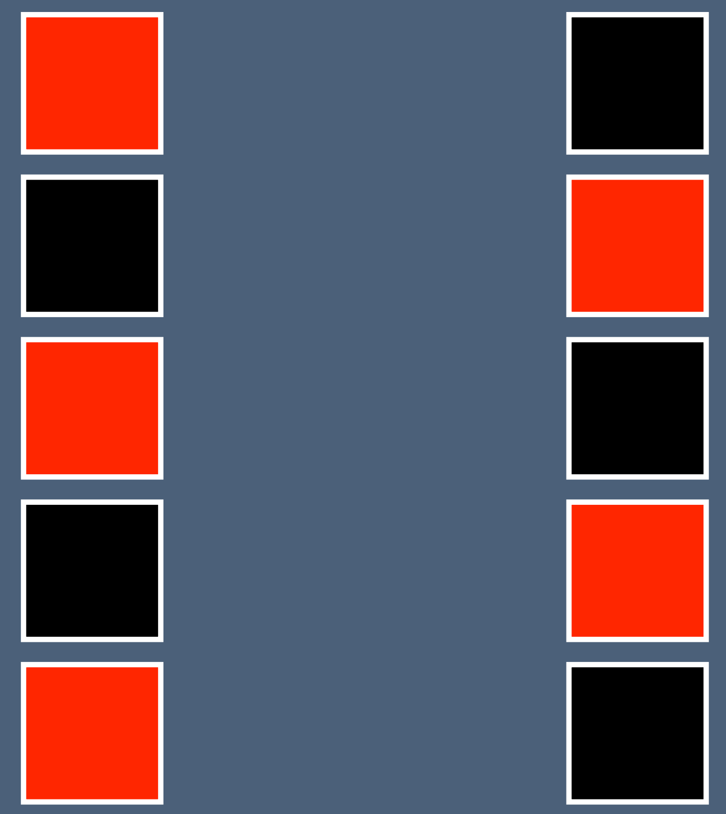

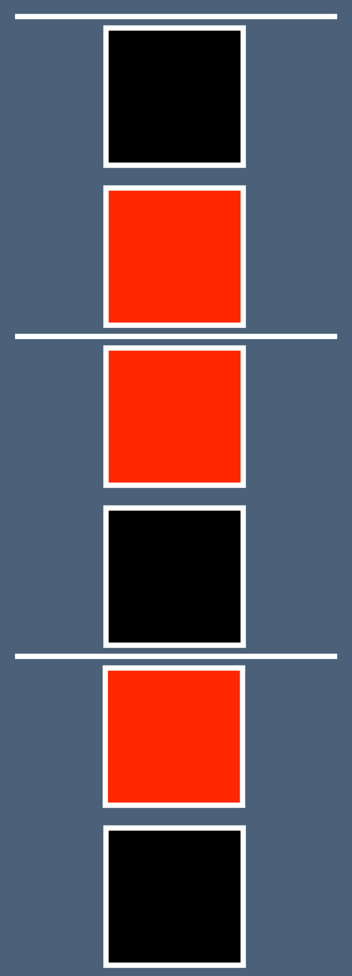

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Exercise:
Figure out why
this works.

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9

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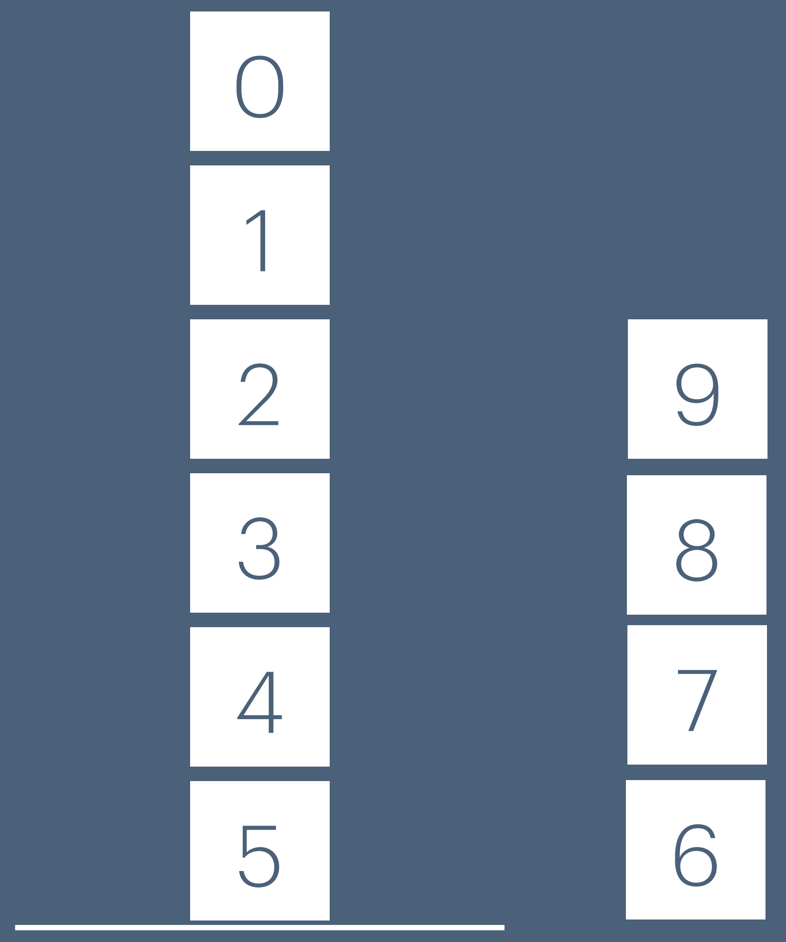

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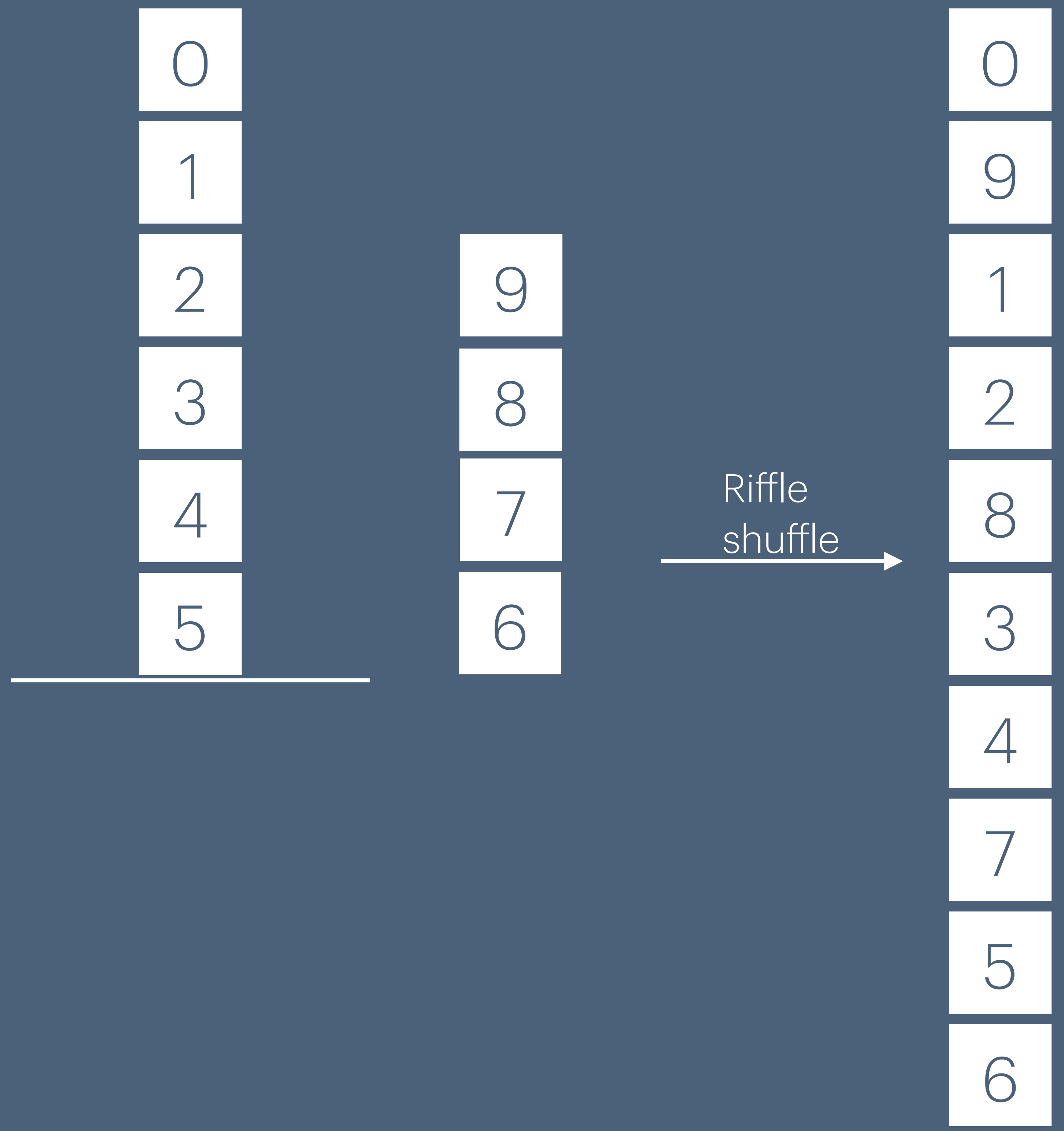

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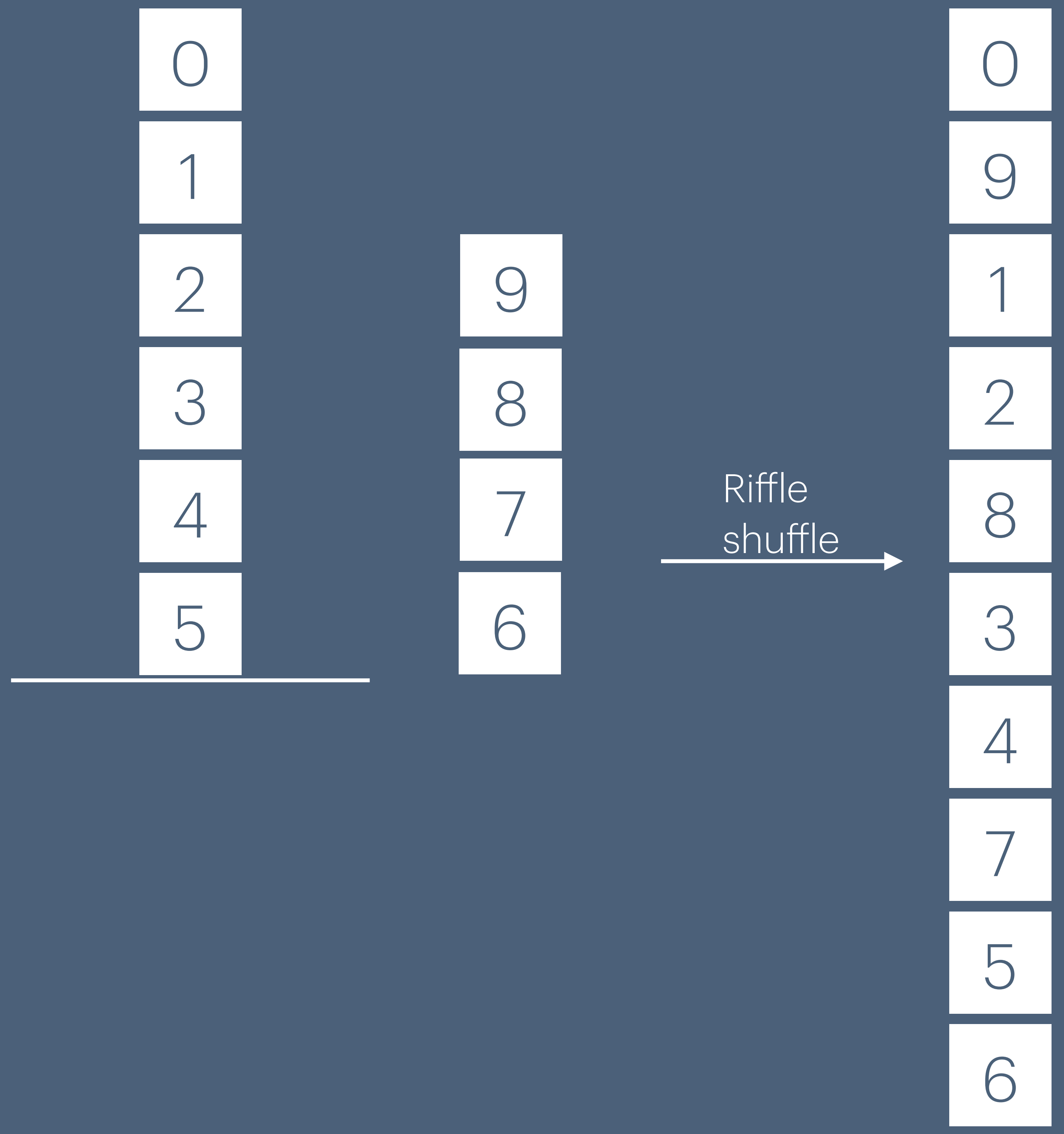

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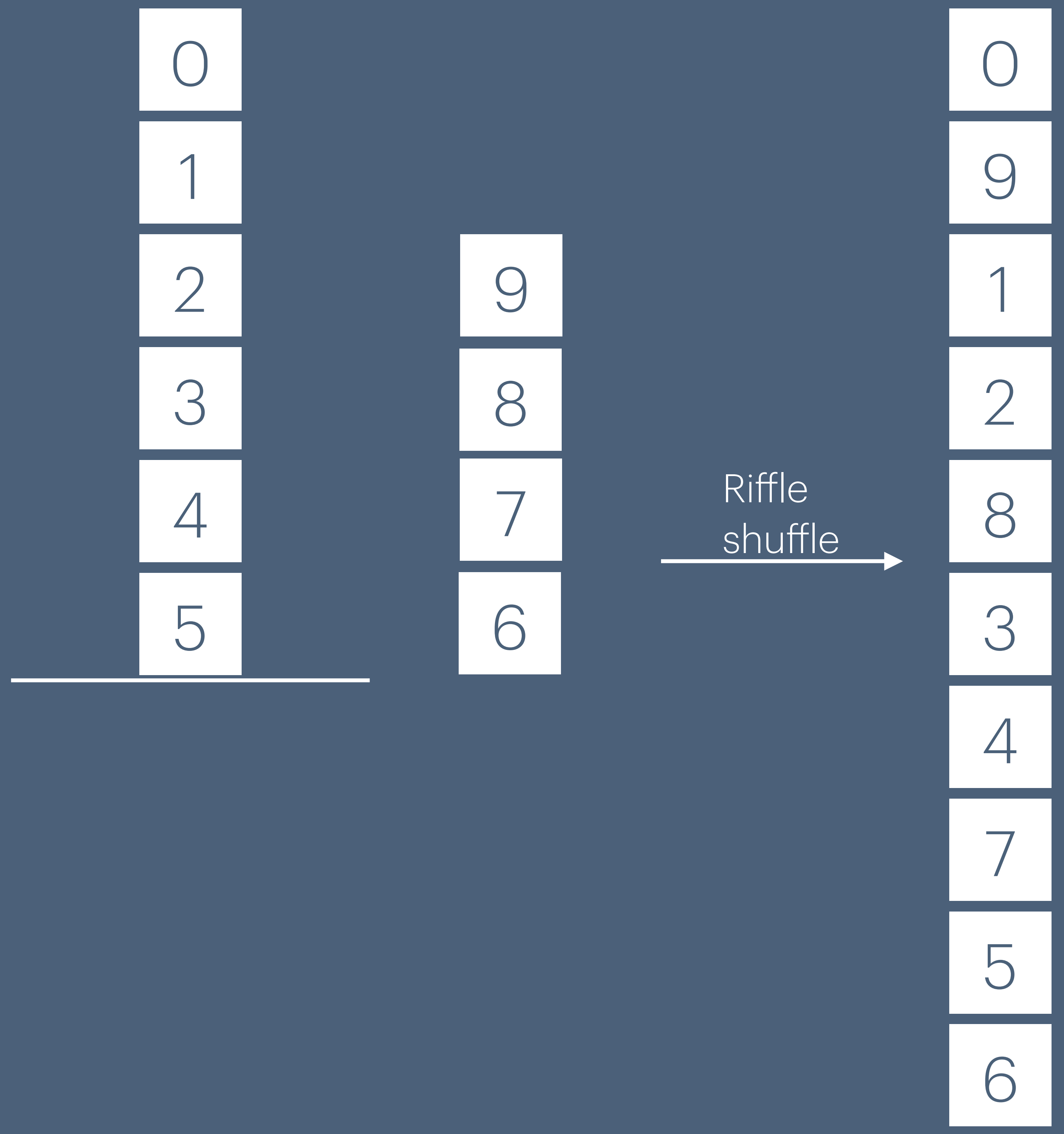

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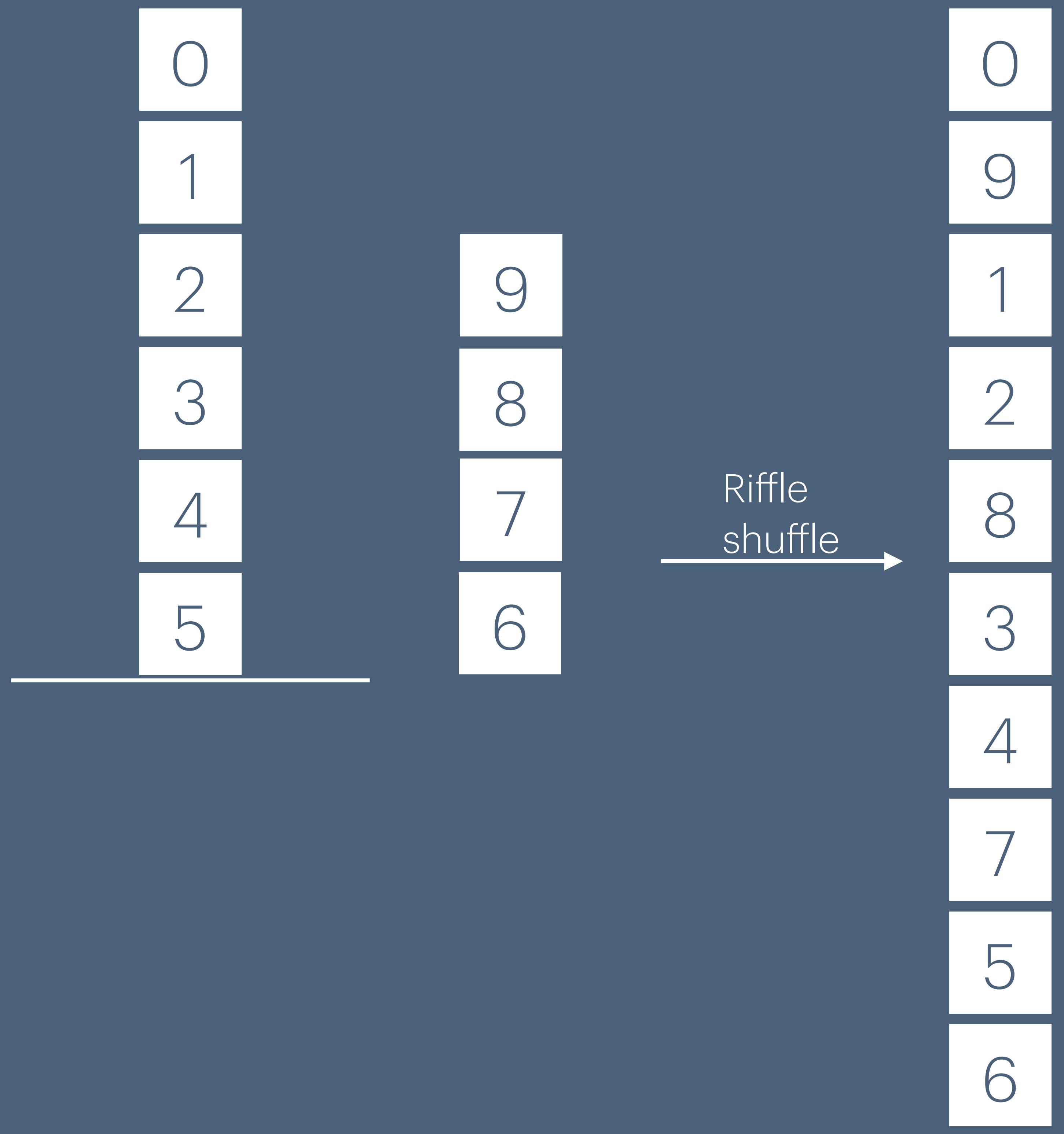

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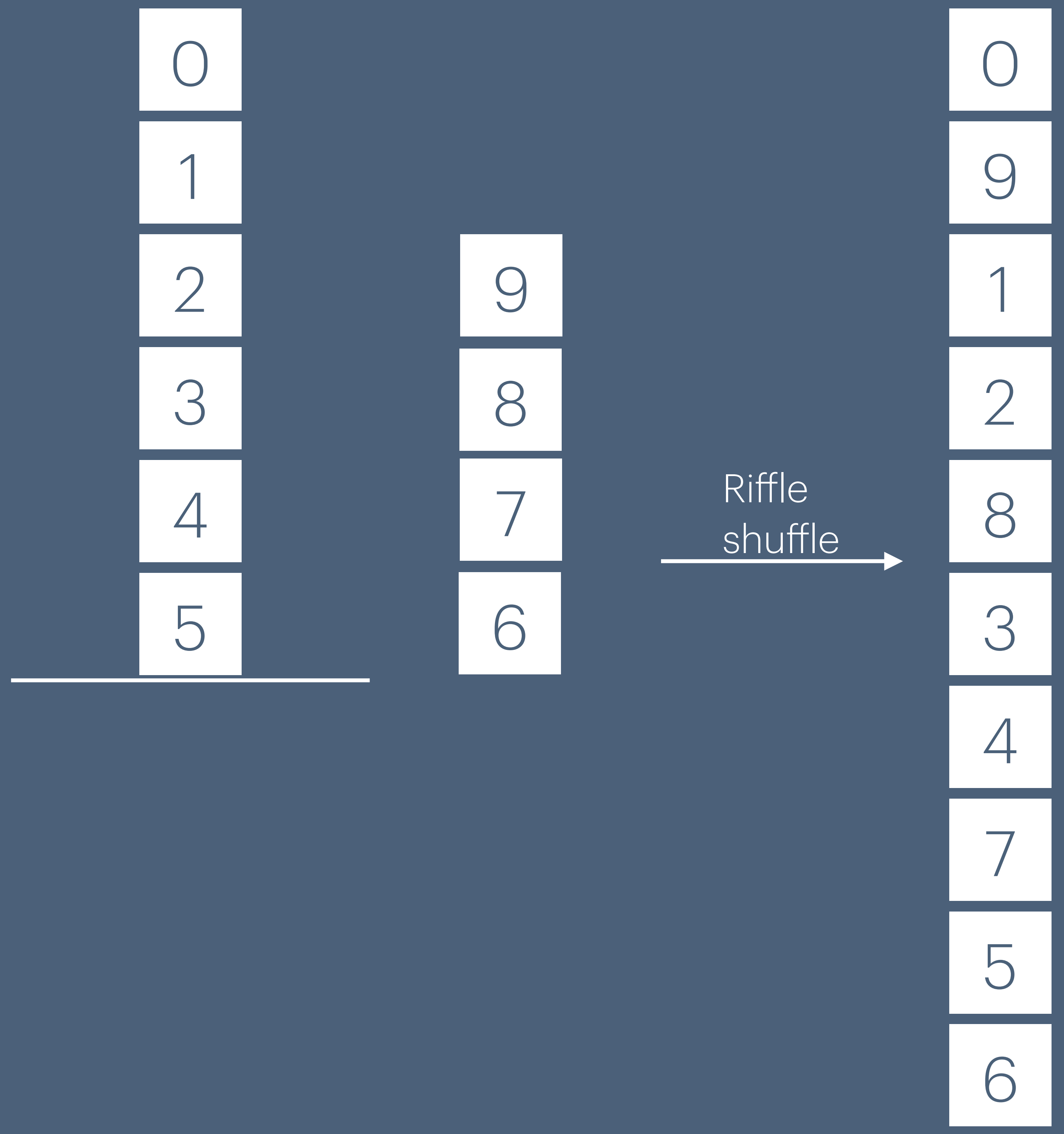

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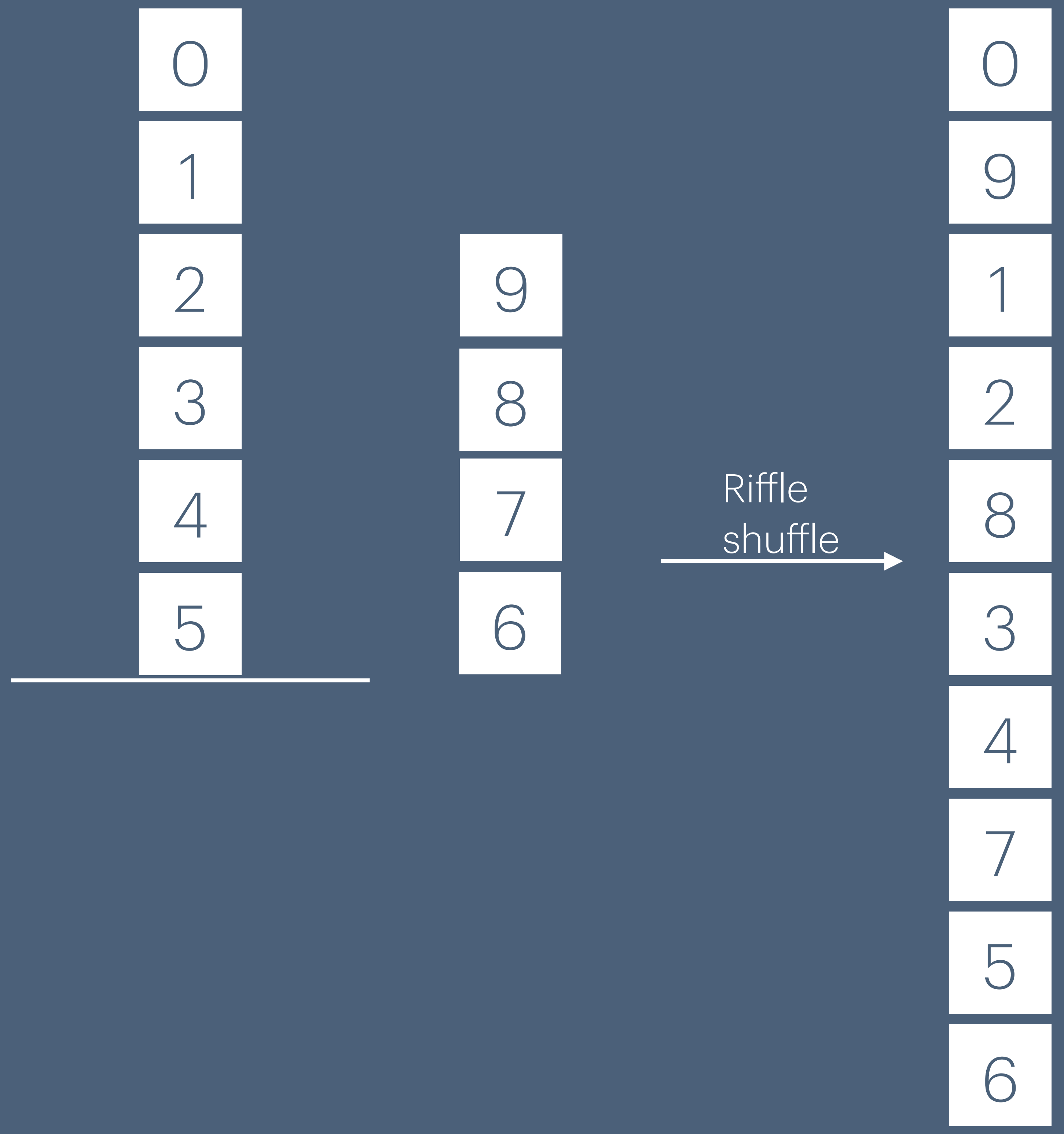

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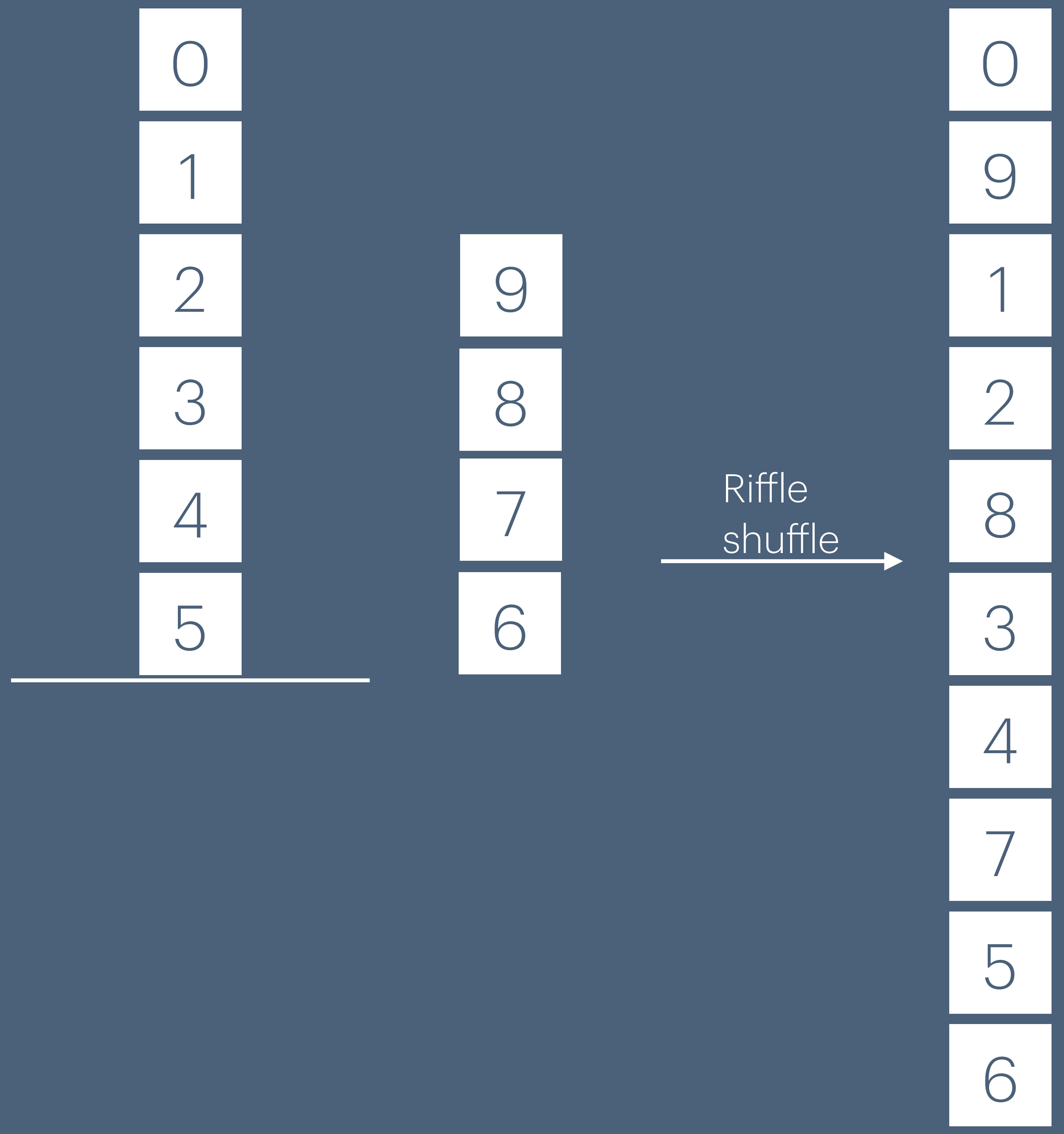

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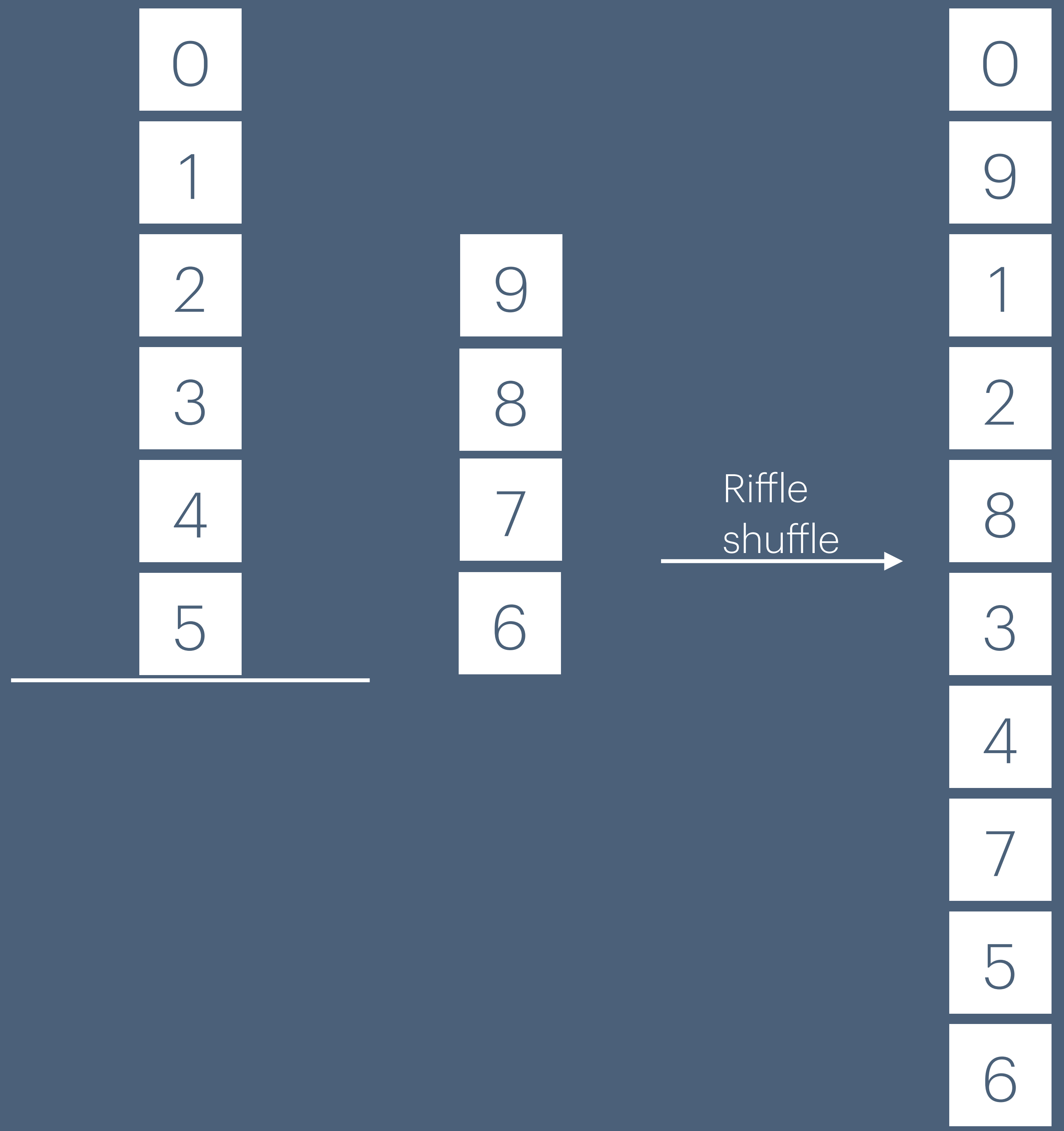

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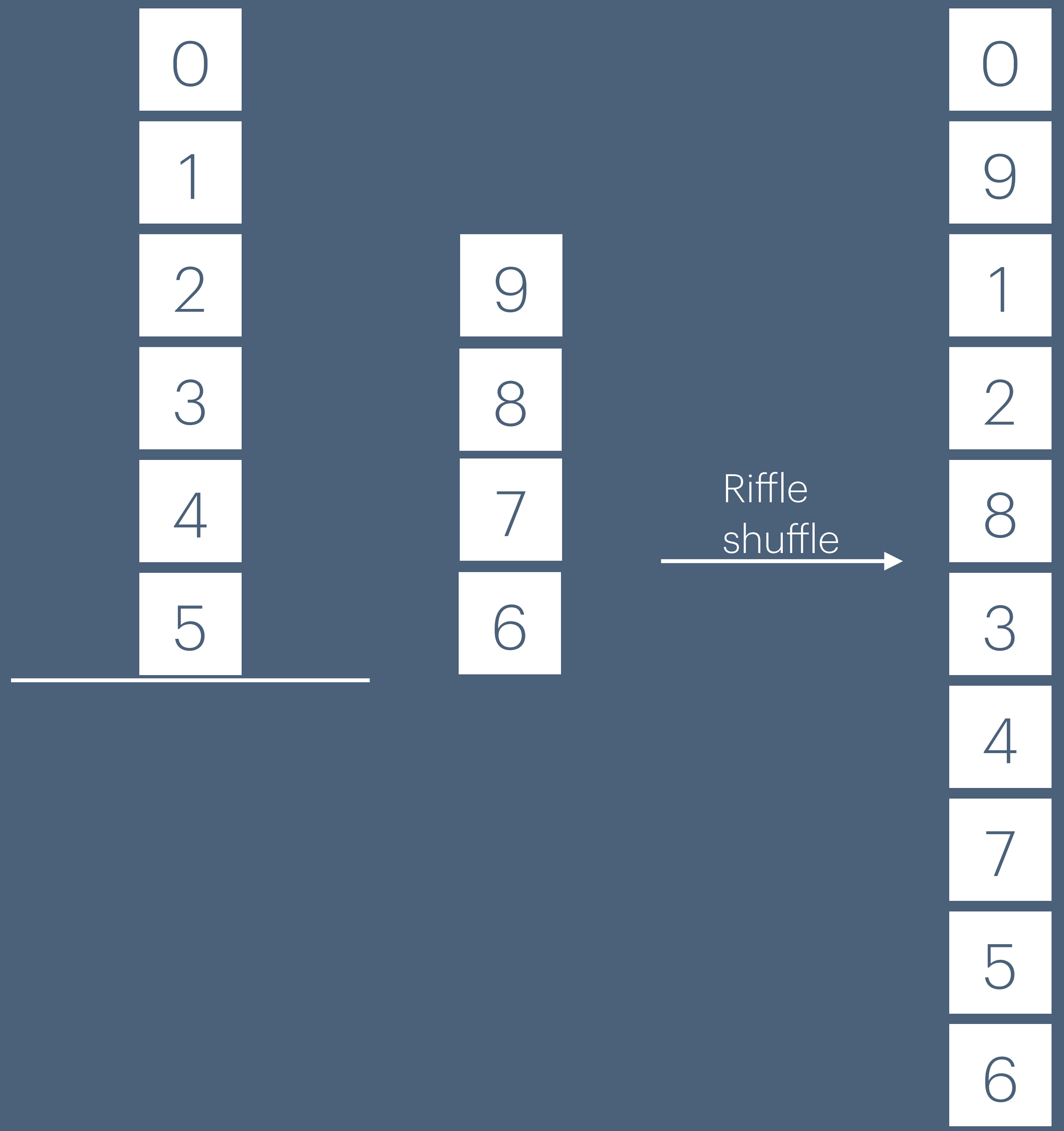

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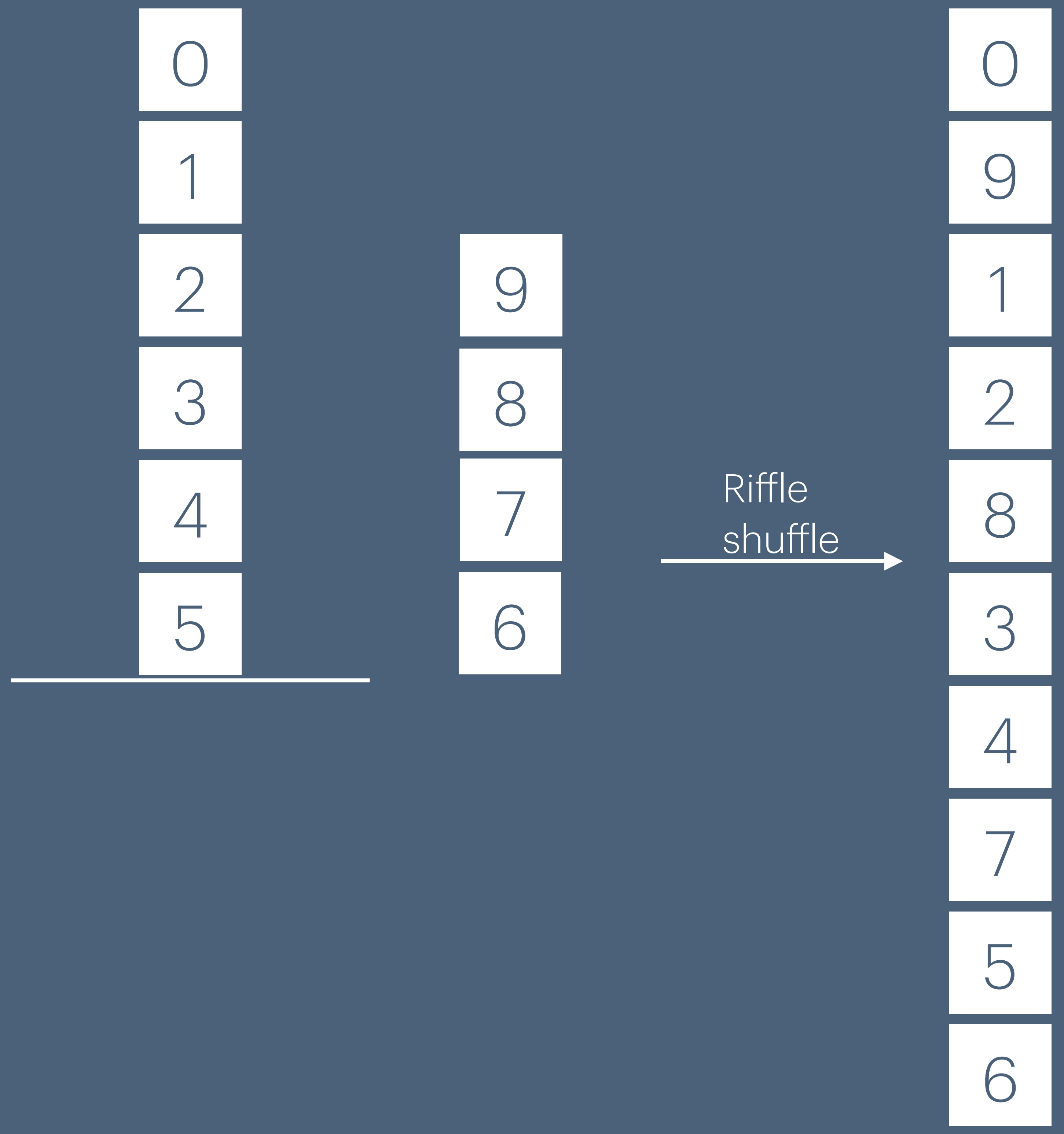

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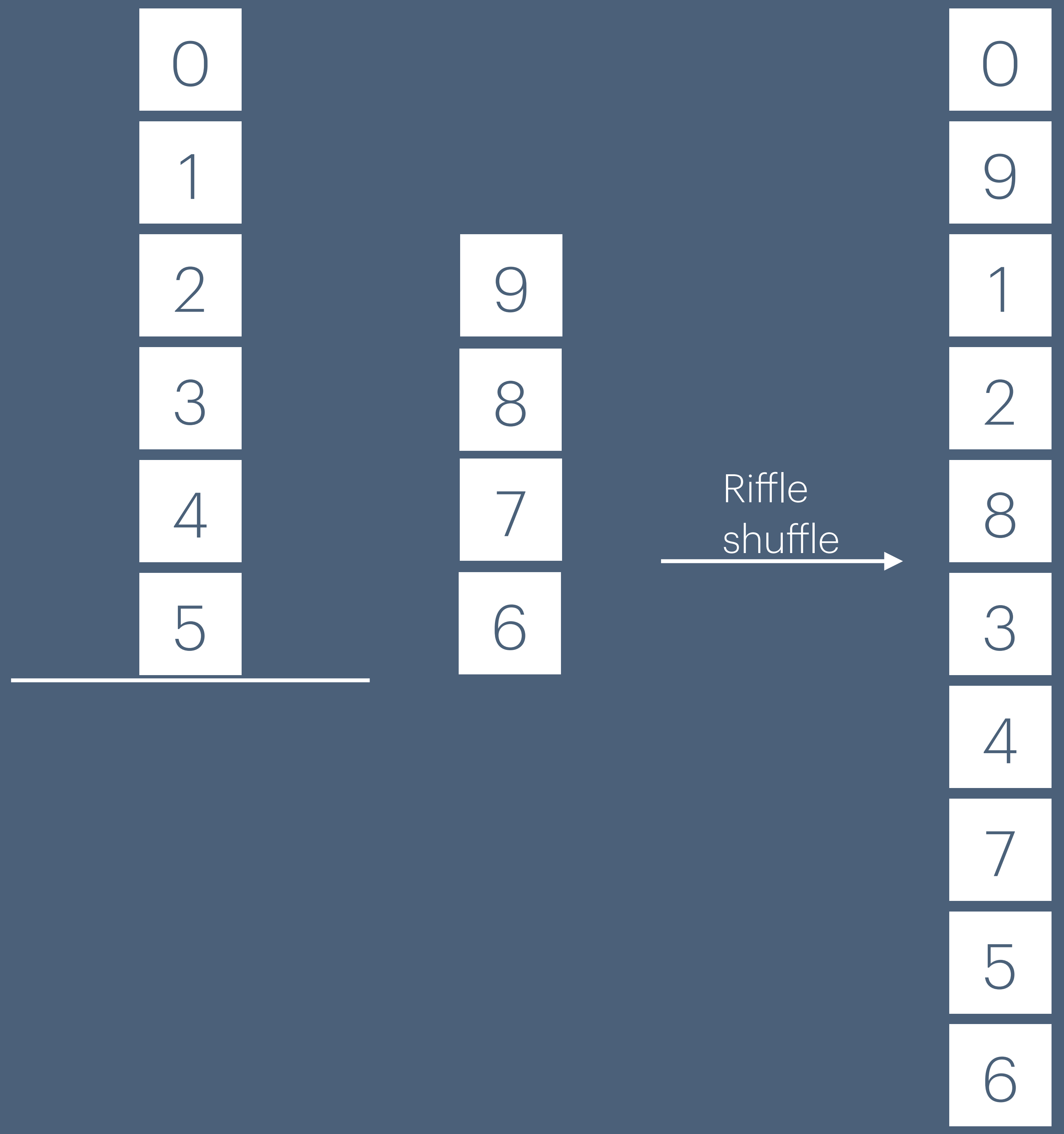

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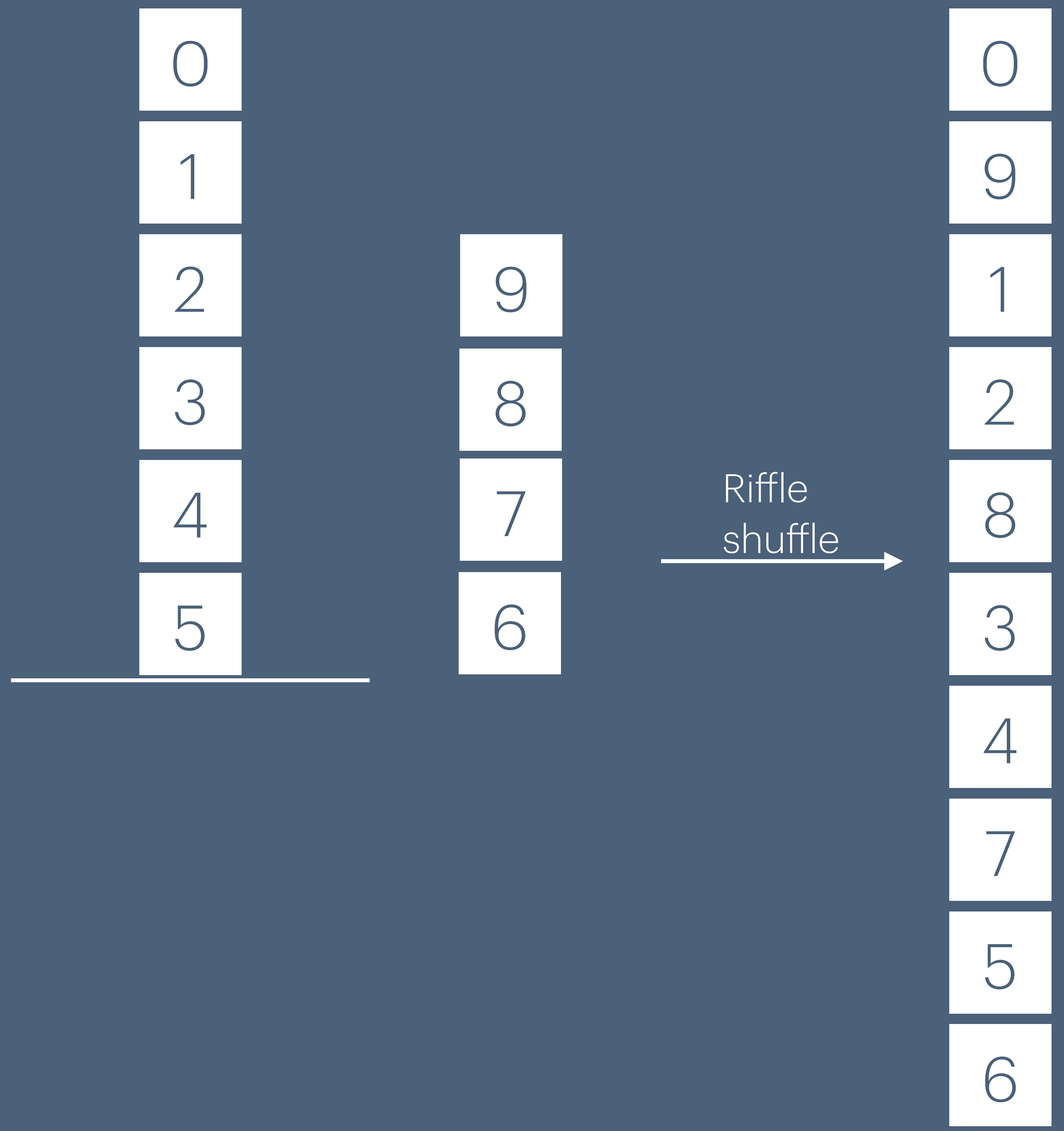
After a Gilbreath Shuffle of **kb cards**, if you take **blocks of b cards** starting from the top, **each block** consists of **exactly 1 card** of type {0 mod b, 1 mod b, ..., (b-1) mod b}

If the cards were originally arranged as
RBRBRBRBRBRBRBRBRBRBRBRBRBRBRB
every block of 2 cards starting from the top will consist of {R,B}.

If the cards were originally arranged as
♣♥♠♦♣♥♠♦♣♥♠♦♣♥♠♦♣♥♠♦
every block of 4 cards starting from the top will consist of {♣,♥, ♠,♦}.

If the cards were originally arranged as
A23456789TJQKA23456789TJQKA23456789TJQK

Defn (Gilbreath Shuffle):
Take a sequence of cards. Cut it into two piles. Reverse one of the piles and riffle shuffle them.



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every block of 13 cards starting from the top

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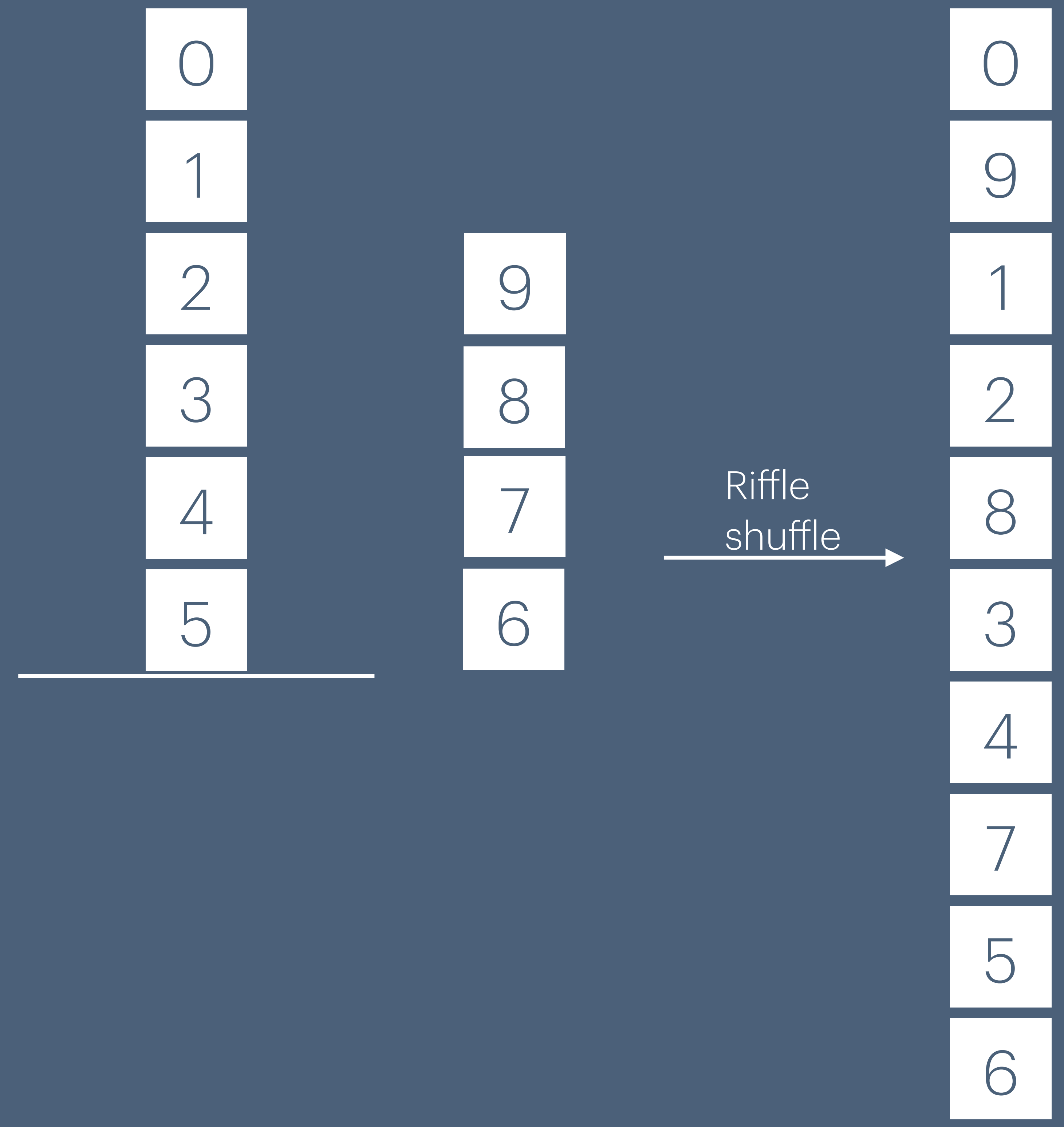

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A23456789TJQKA23456789TJQKA23456789TJQK
every block of 13 cards starting from the top will consist of A23456789TJQK.

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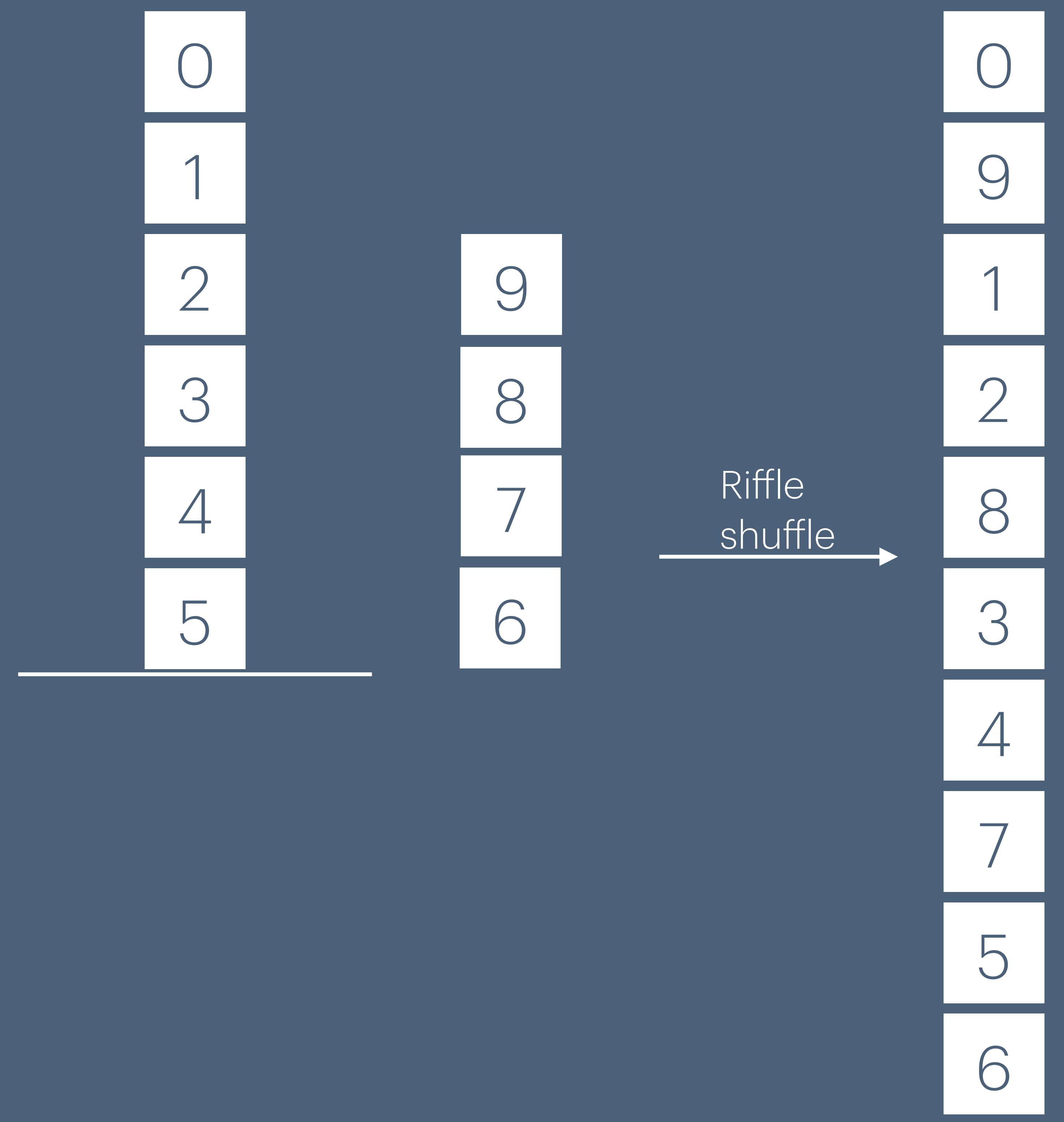

Why does this work?

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Same argument as last time!

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♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️♣️♥️♠️♦️

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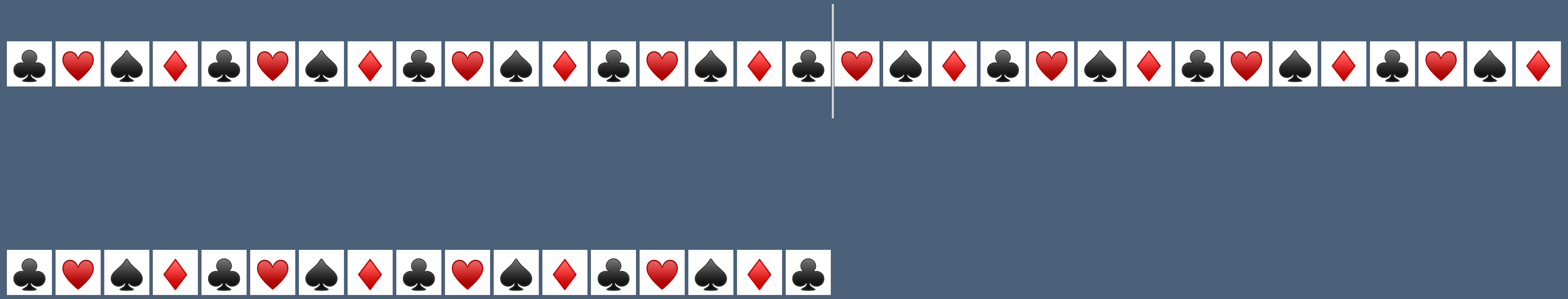

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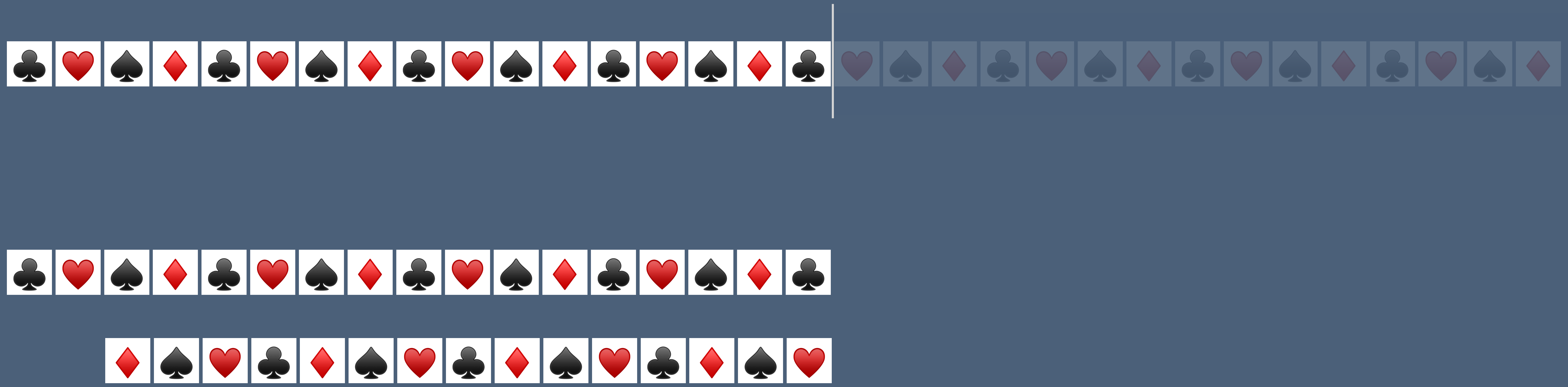

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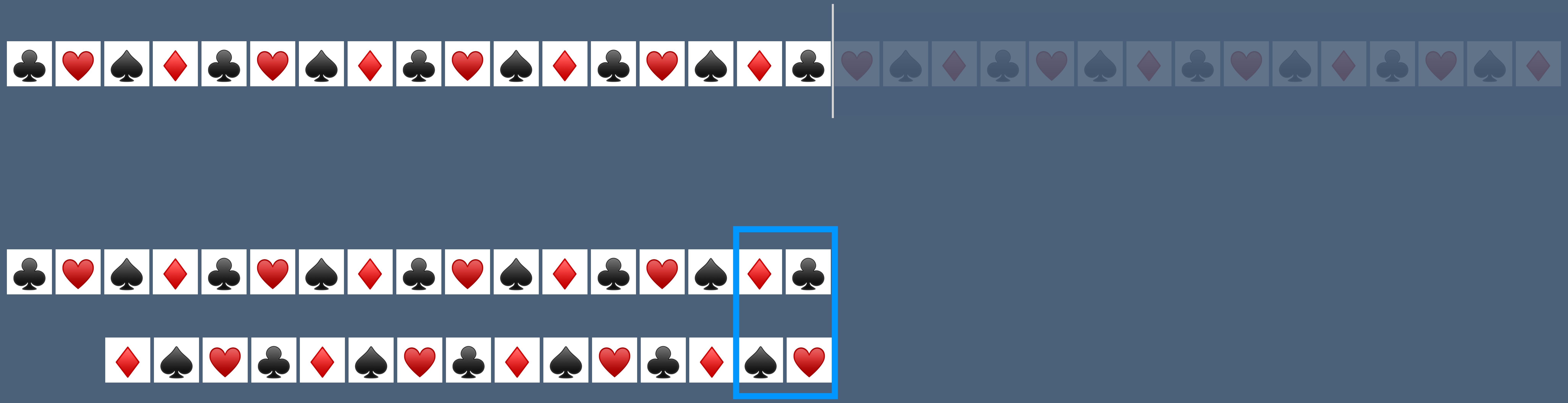

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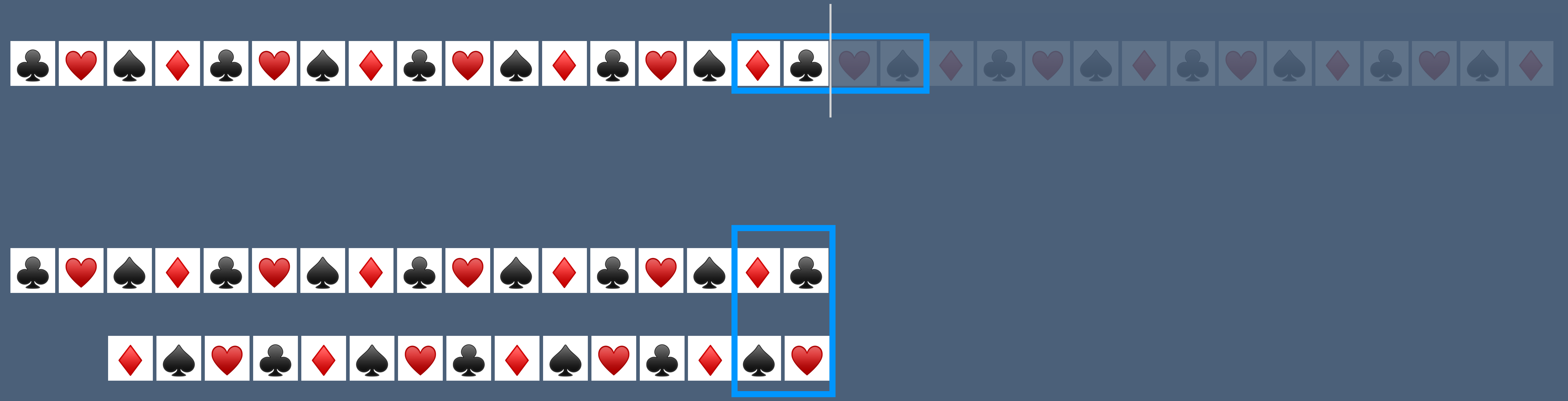

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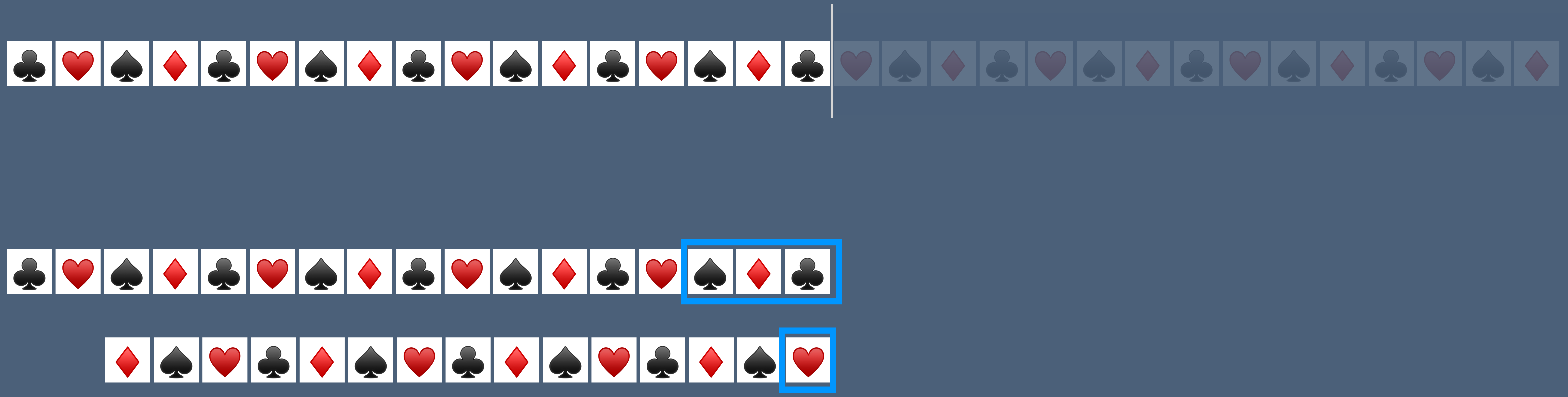

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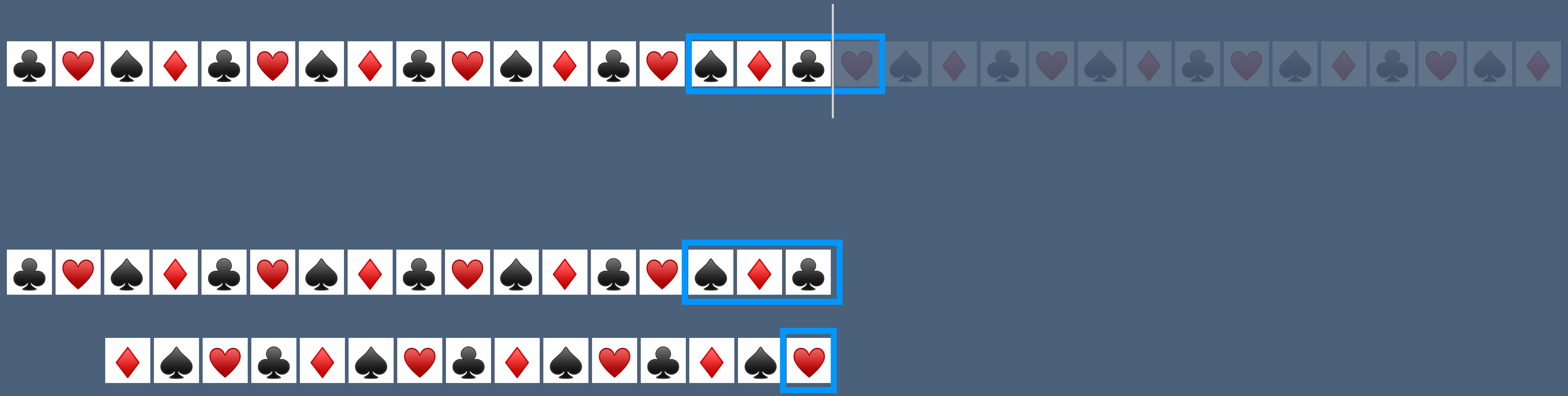

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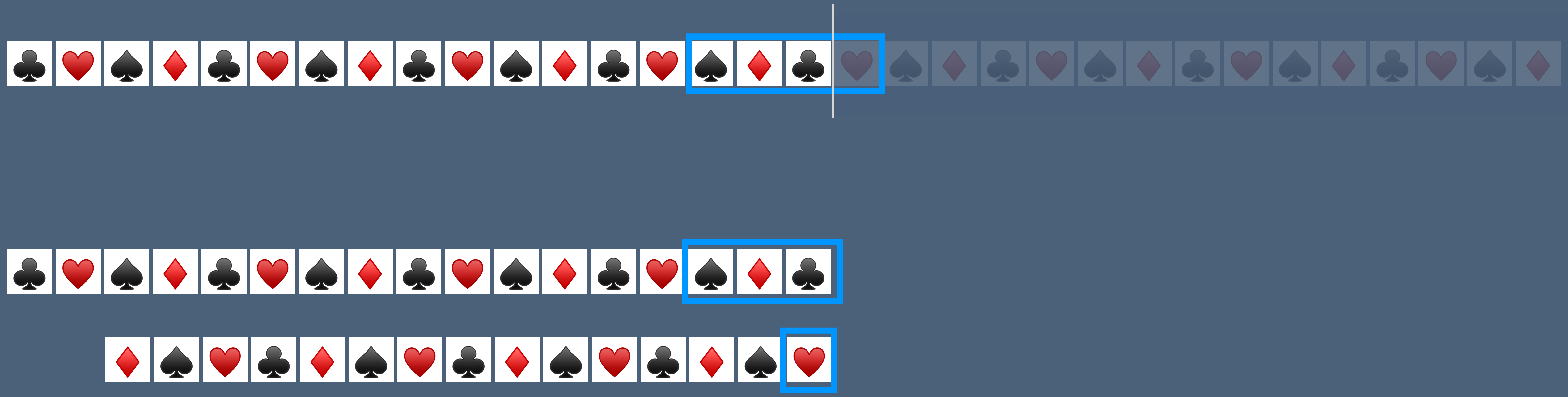

Bottom 4k cards is a 'bubble' of size 4k originally

"Up the Ante" by Martin Smith

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Starting order of deck

A♣ 2♥ 3♠ 4♦ 5♣ 6♥ 7♠ 8♦ 9♣ T♥ J♠ Q♦ K♣
A♥ 2♠ 3♦ 4♣ 5♥ 6♠ 7♦ 8♣ 9♥ T♠ J♦ Q♣ K♥
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black-red x 26

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♣♥♠♦ x 13

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A23456789TJQK x 4

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Gilbreath Shuffle →

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black-red x 26

♣♥♠♦ x 13

A23456789TJQK x 4

Gilbreath Shuffle →

Blocks of 2: {black, red}

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black-red x 26

♣♥♠♦ x 13

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Gilbreath Shuffle →

Blocks of 2: {black, red}

Blocks of 4: {♣,♥,♠,♦}

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	Gilbreath Shuffle →	
black-red x 26		Blocks of 2: {black, red}
♣♥♠♦ x 13		Blocks of 4: {♣,♥,♠,♦}
A23456789TJQK x 4		Blocks of 13: {A23456789TJQK}

"Up the Ante" by Martin Smith

What I did: (Si Stebbins Stack: CHaSeD order, +3)

A♣ 4♥ 7♠ T♦ K♣ 3♥ 6♠ 9♦ Q♣ 2♥ 5♠ 8♦ J♣
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Gilbreath Shuffle →

black-red x 26 → Blocks of 2: {black, red}

♣♥♠♦ x 13 → Blocks of 4: {♣,♥,♠,♦}

A23456789TJQK x 4 → Blocks of 13: {A23456789TJQK}

Does this actually have something to do with the Mandelbrot set?!

Gilbreath permutations of {1,2,3,4}

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1234

2134

2314

2341

3214

3241

3421

4321

Gilbreath permutations of {1,2,3,4}

	4-cycle?
1234	
2134	
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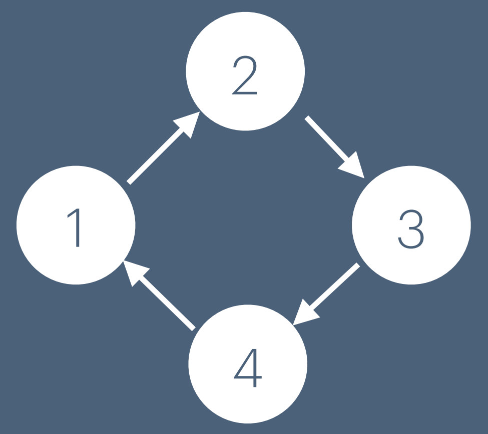
2341 ✓ 

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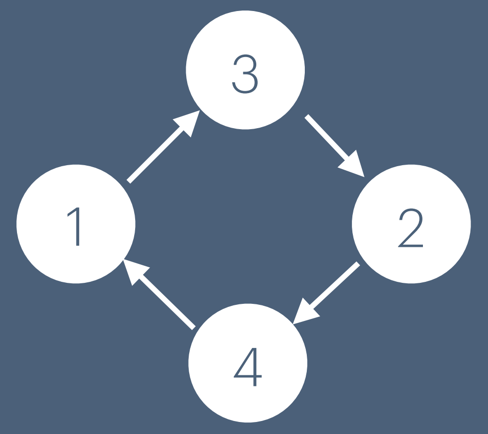
3421 ✓ 

4321

Gilbreath permutations of {1,2,3,4}

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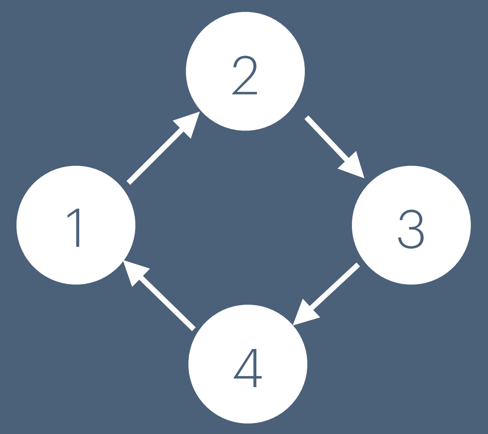

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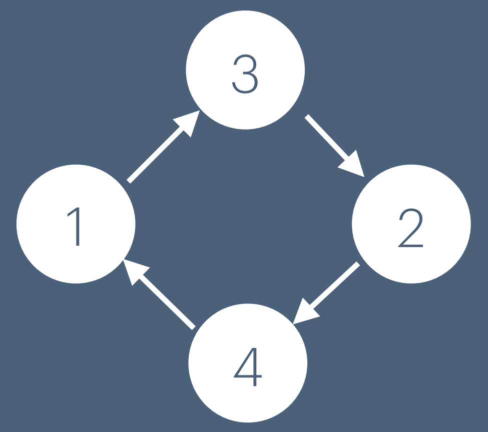

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$z_0(c) = 0$

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$z_2(c) = z_1(c)^2 + c = c^2 + c$

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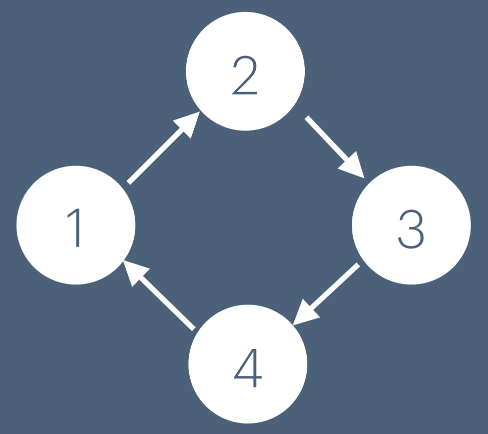

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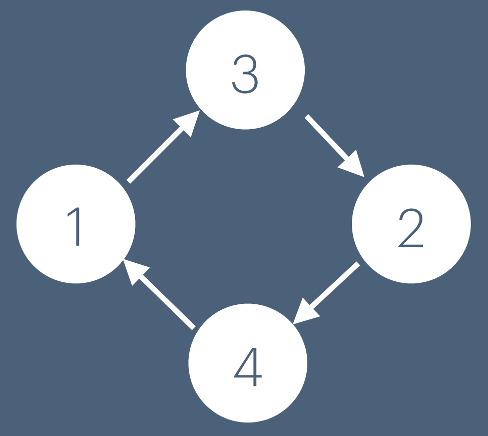

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Real roots of $z_4(c)$:
0, -1, -1.9407... and -1.3107...

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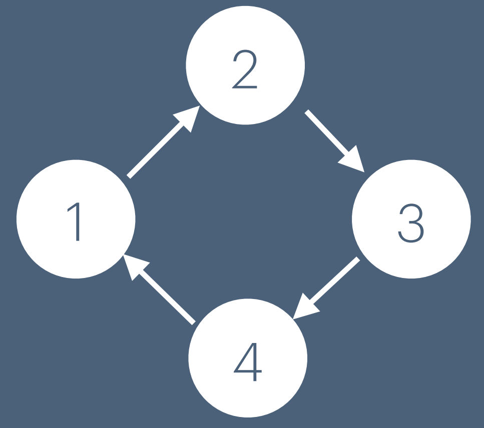

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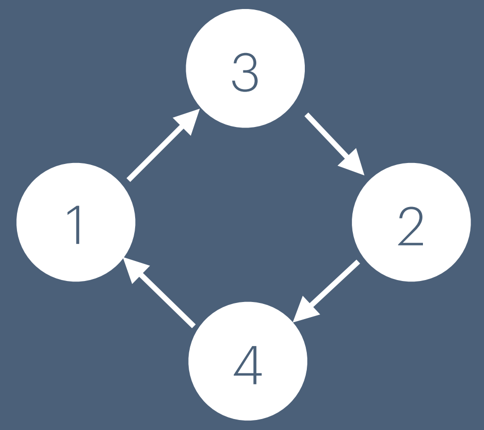

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0→-1.9407...→1.8259...→1.3931...→0 → ⋯

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4-cycle?

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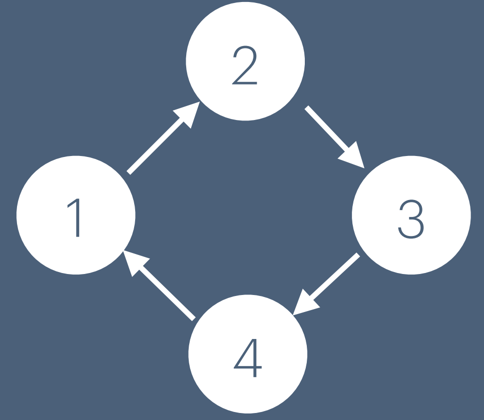

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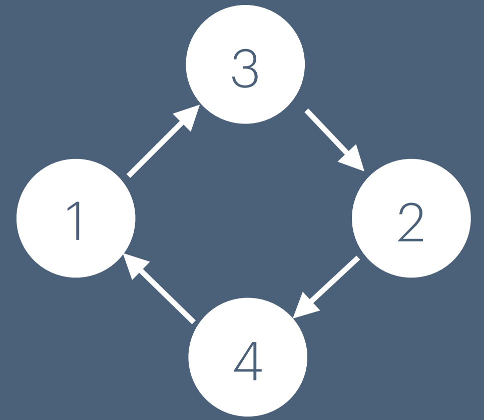

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0→-1.3107...→0.4072...→-1.1448...→0 → ⋯

Gilbreath permutations of {1,2,3,4}

4-cycle?

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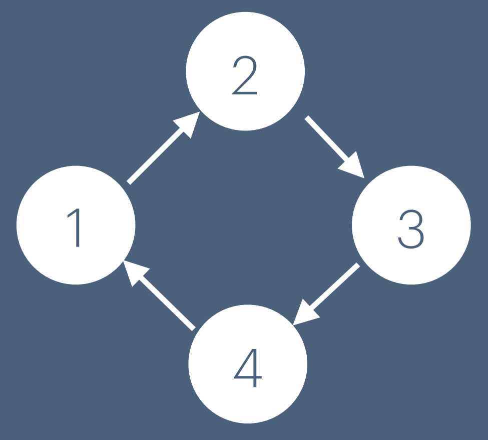
2341 ✓ 

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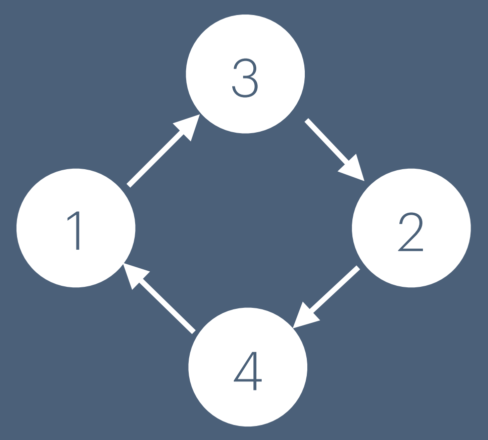
3421 ✓ 

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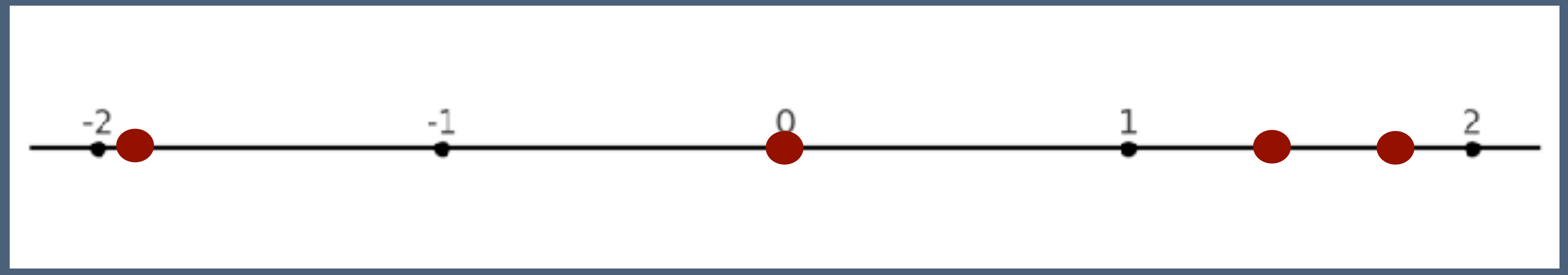

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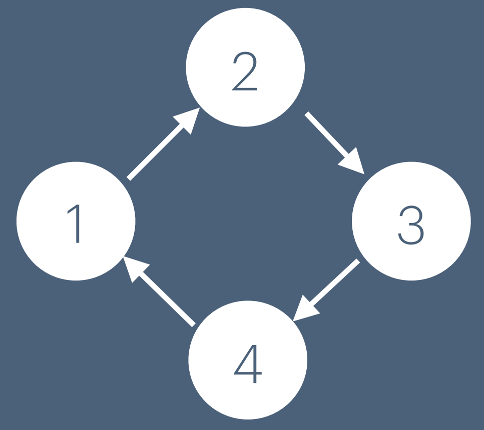

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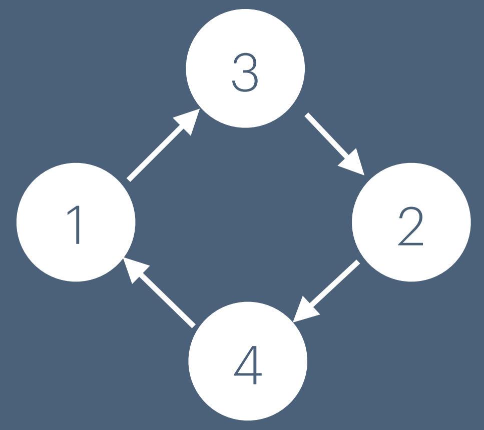

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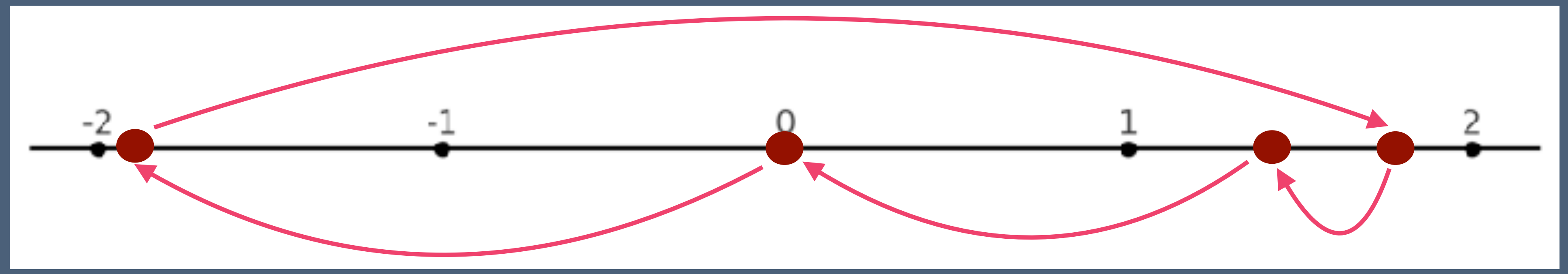

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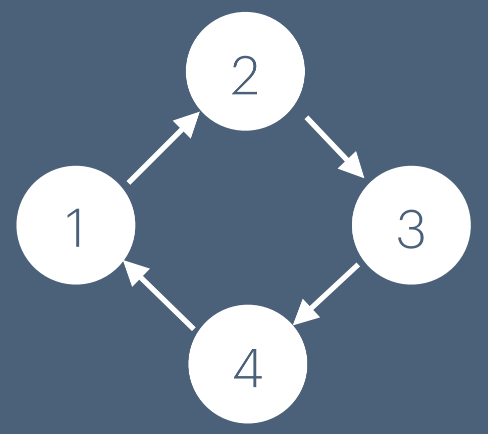

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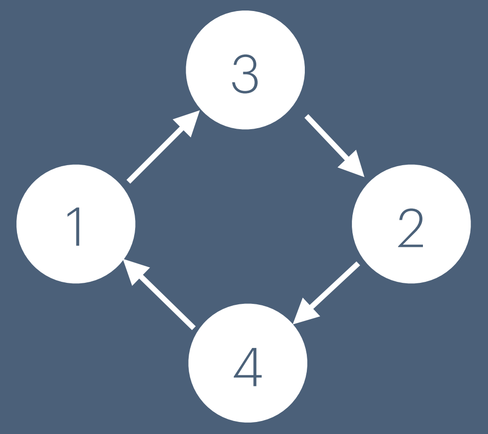

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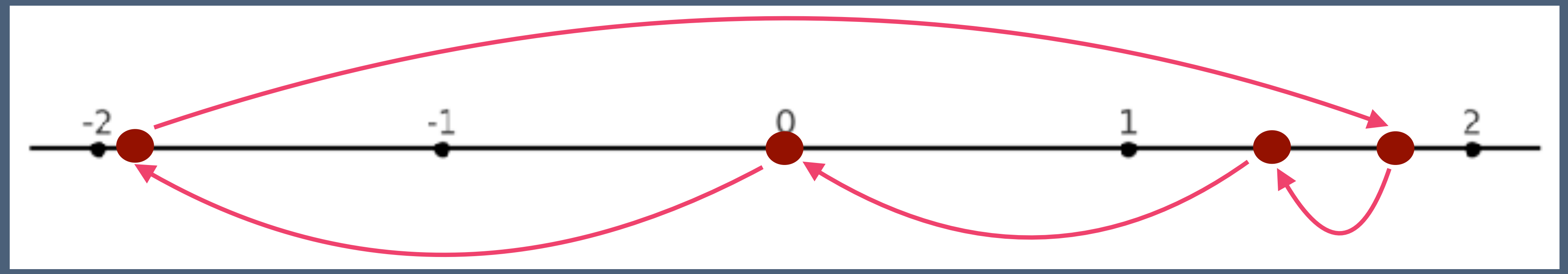

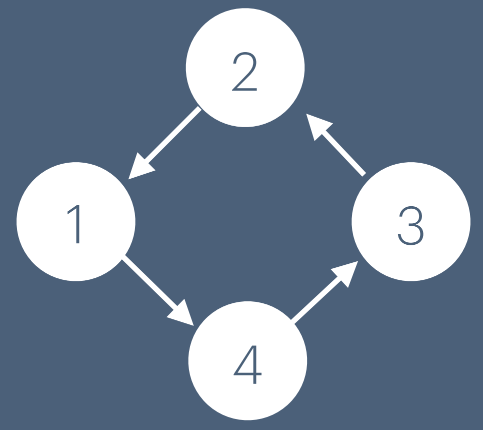

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4-cycle?

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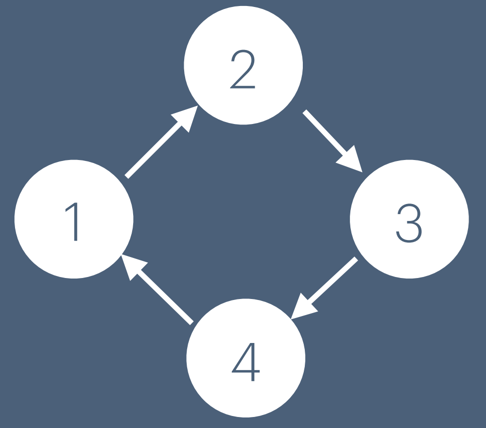

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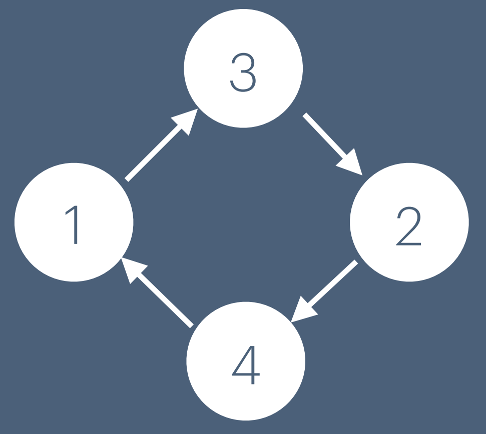

4321

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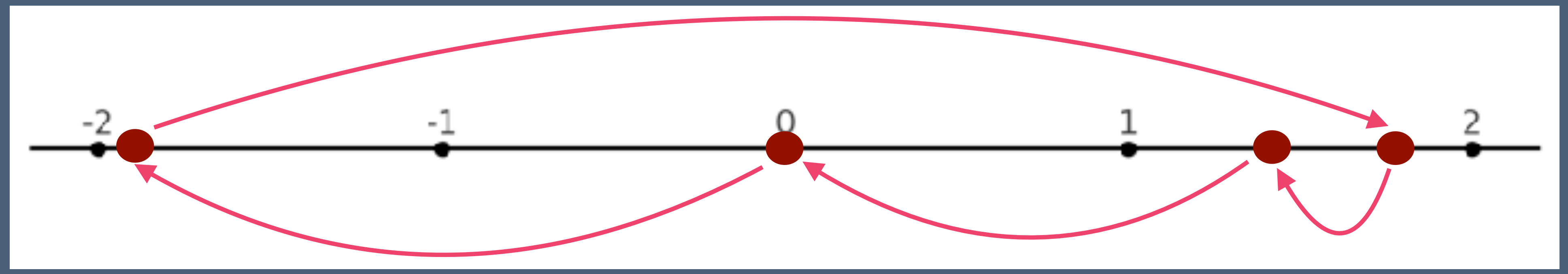

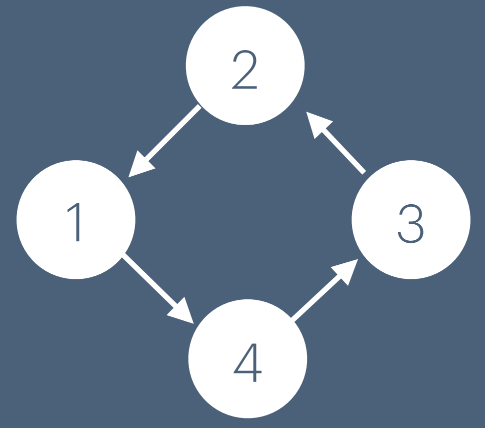

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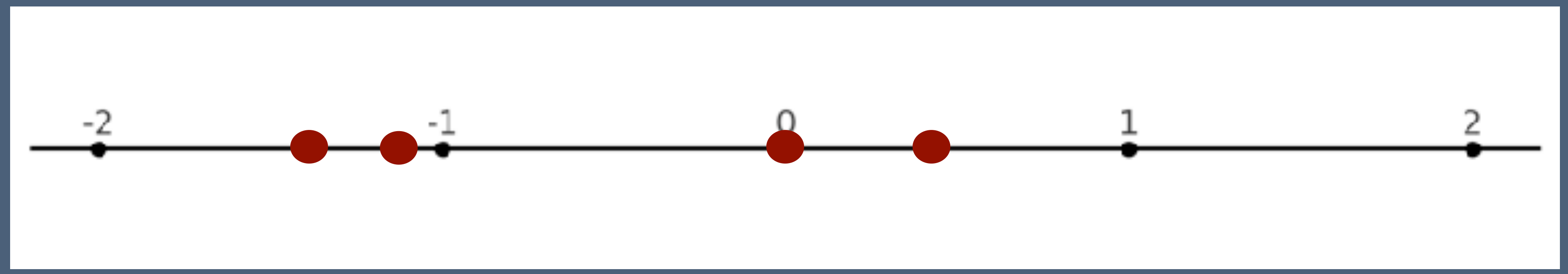

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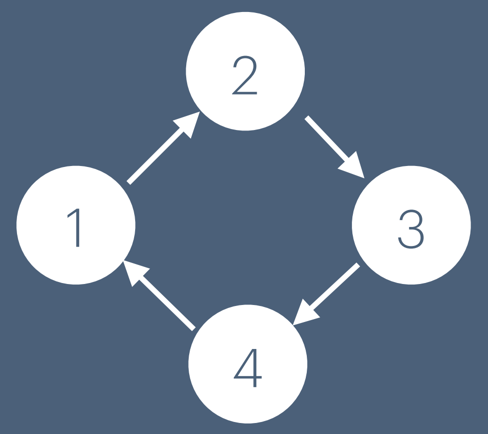
2341 ✓ 

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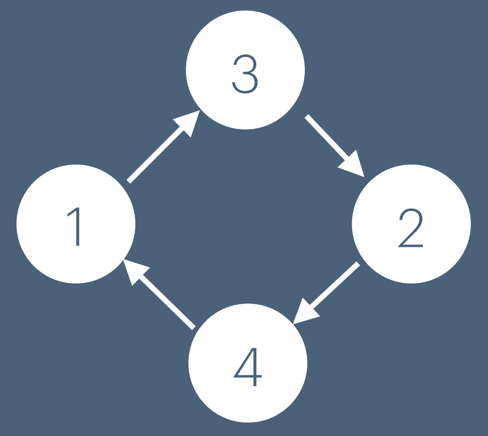
3421 ✓ 

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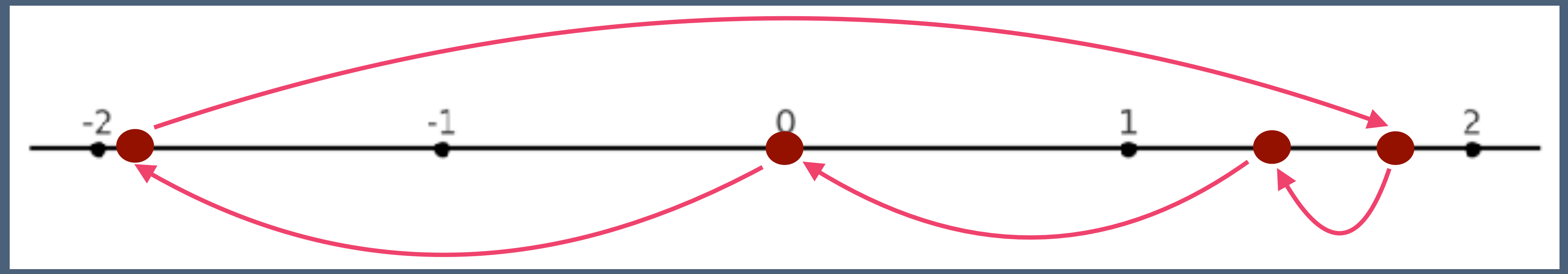

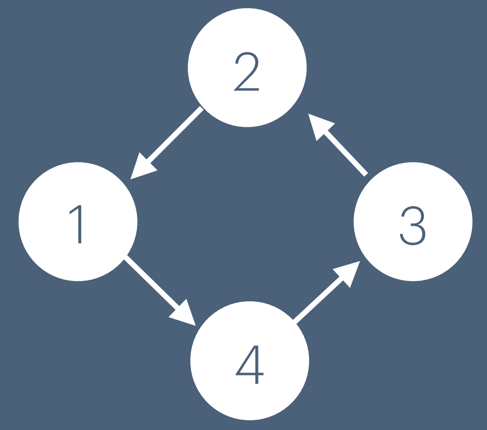

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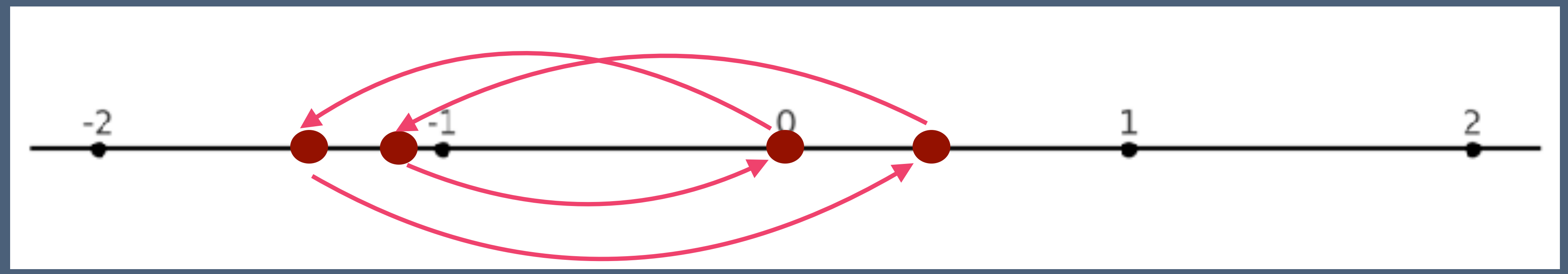

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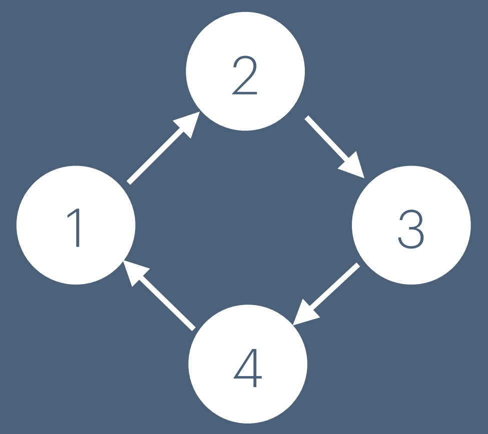

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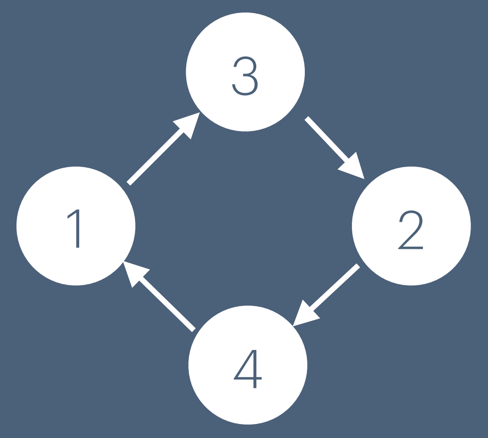

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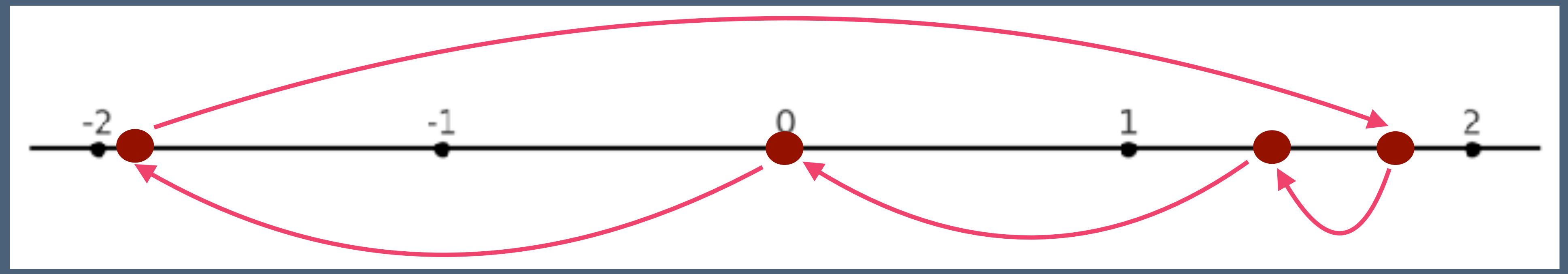

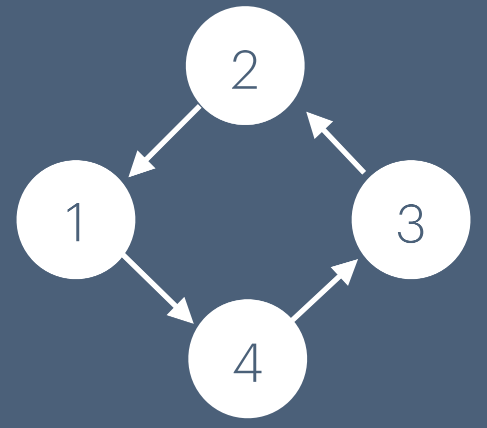

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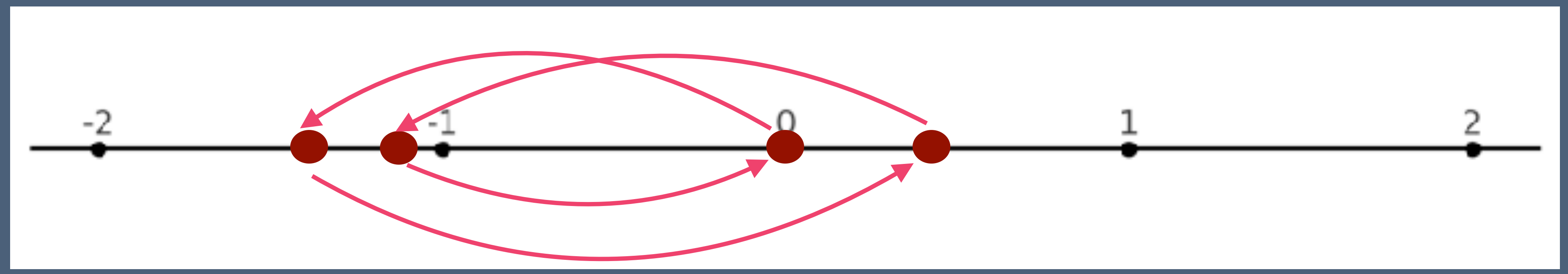

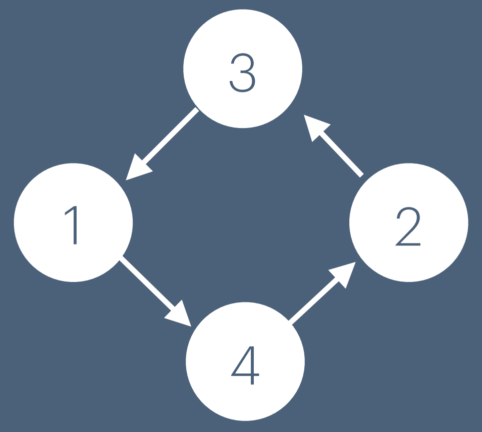

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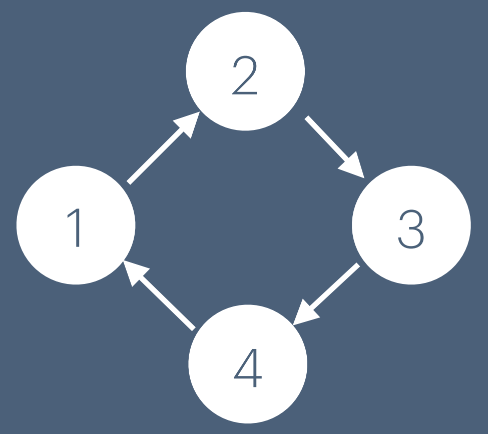

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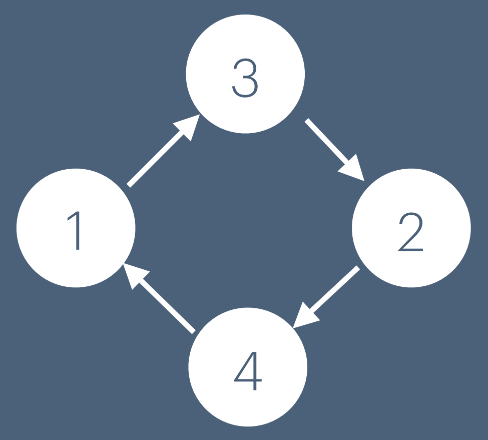

4321

Theorem [Milnor-Thurston]:

Cyclic Gilbreath permutations of size n
are in one-to-one correspondence with
real periodic Mandelbrot sequences with period n.

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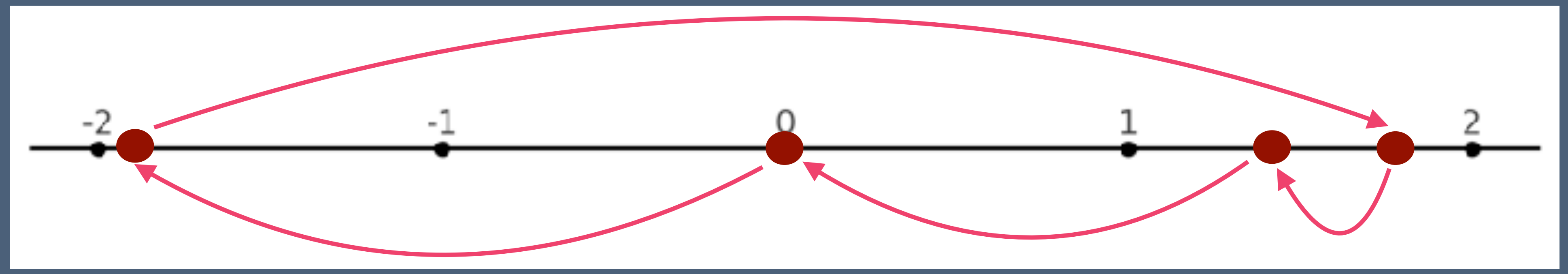

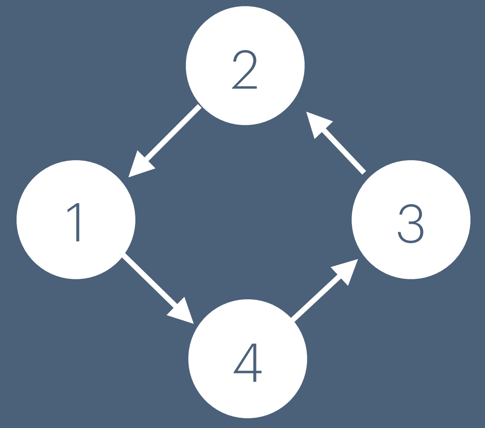

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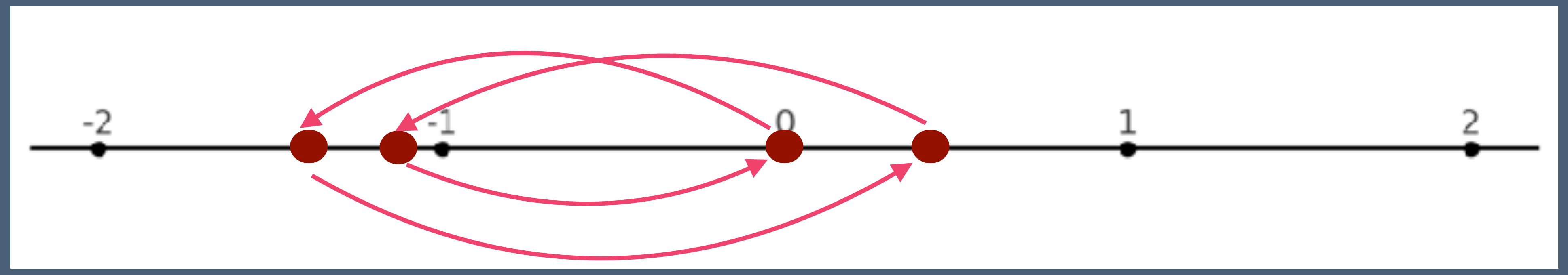

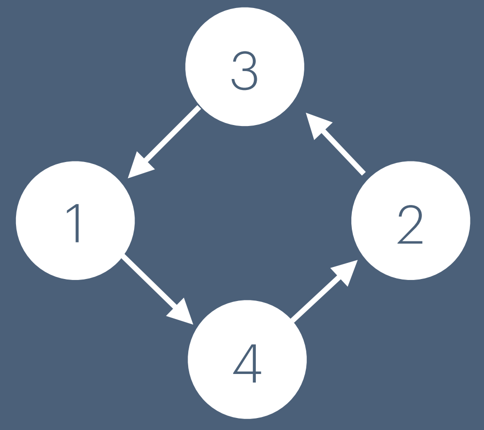

Going back to hobbies

“Serious” hobbies

(not a rigorous definition)

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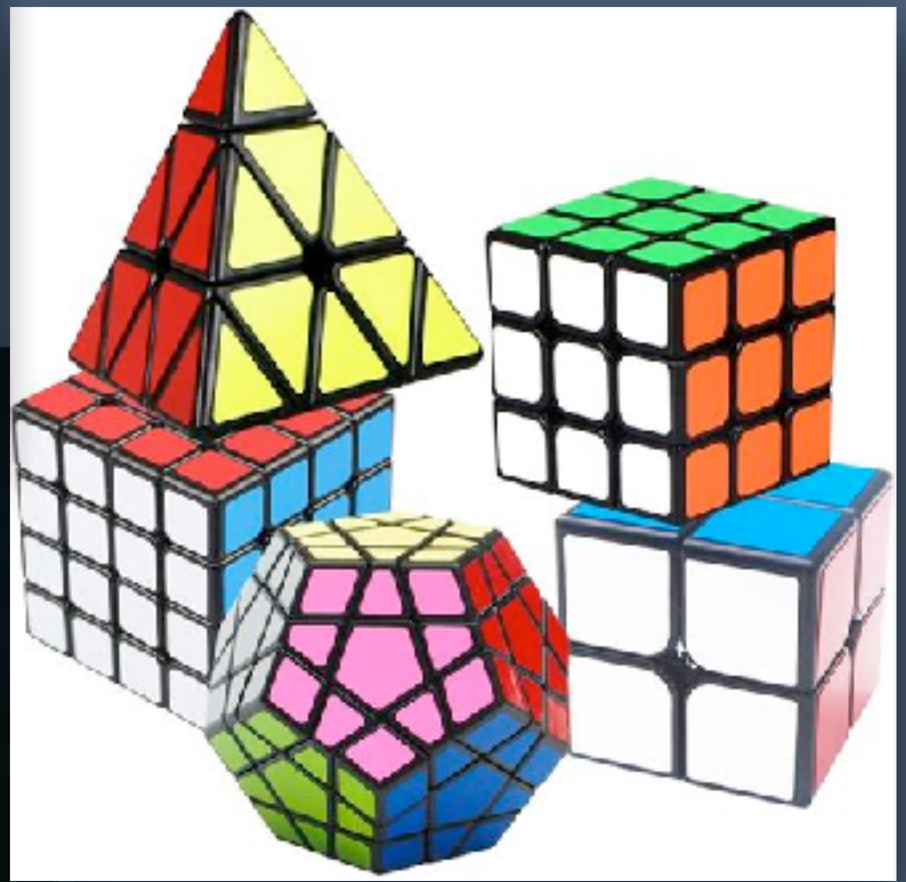
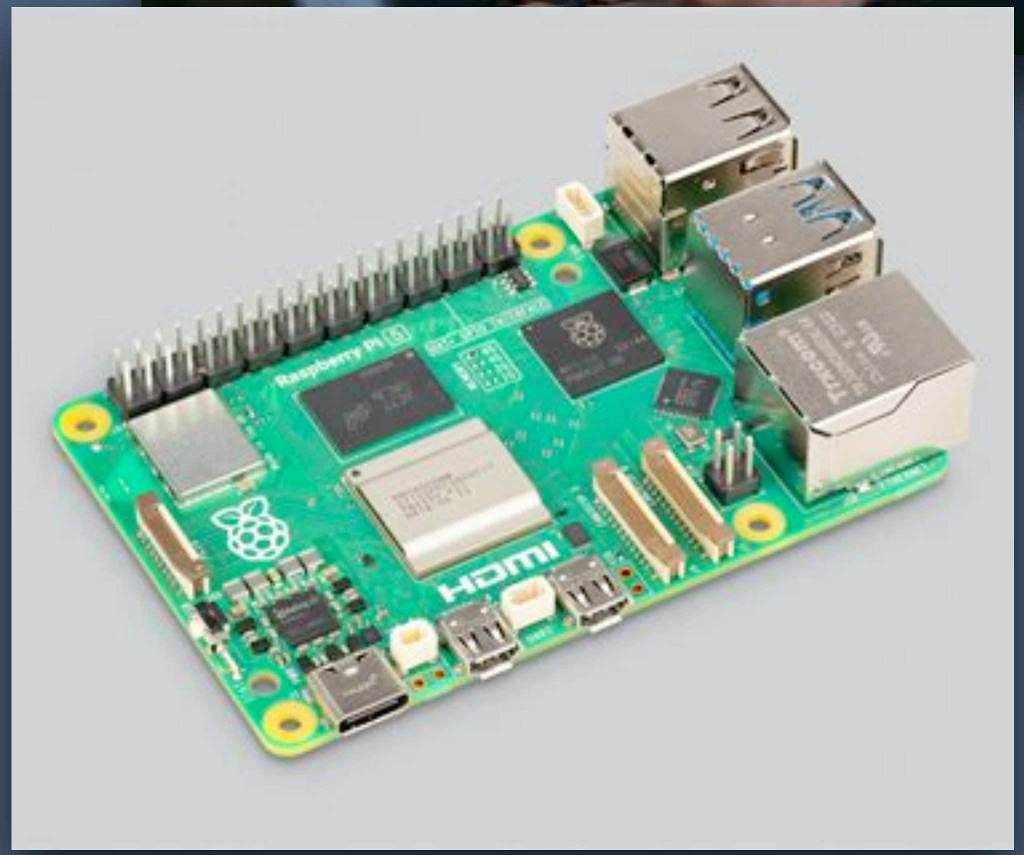
- Intrinsic motivation





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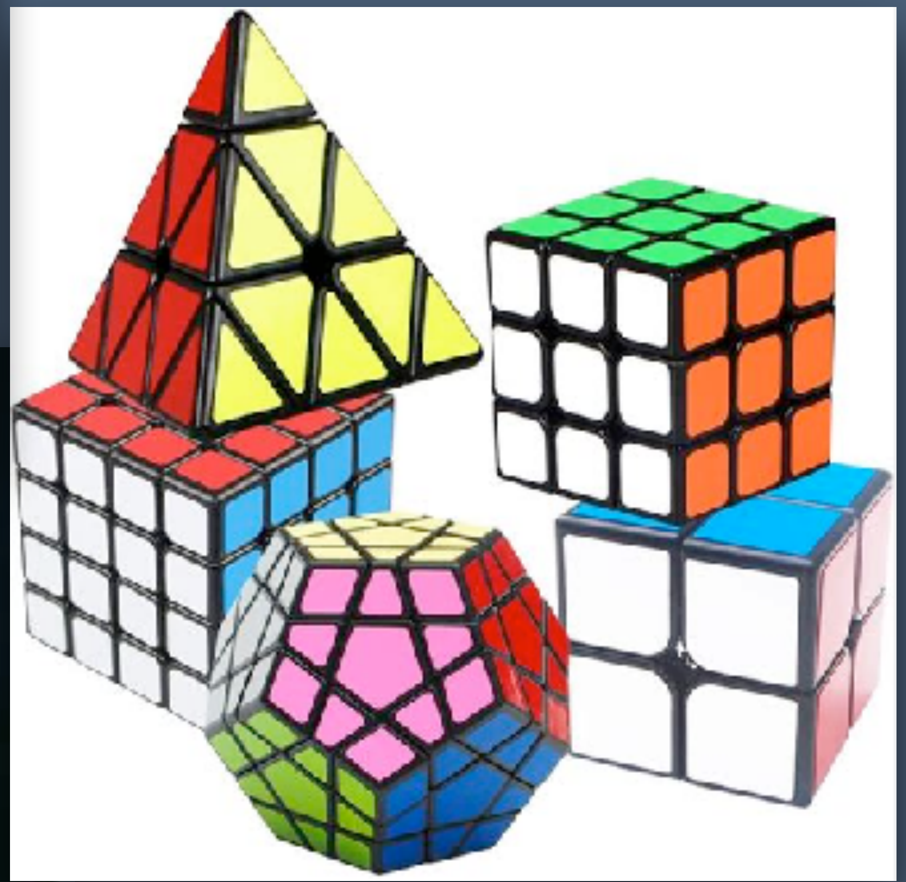

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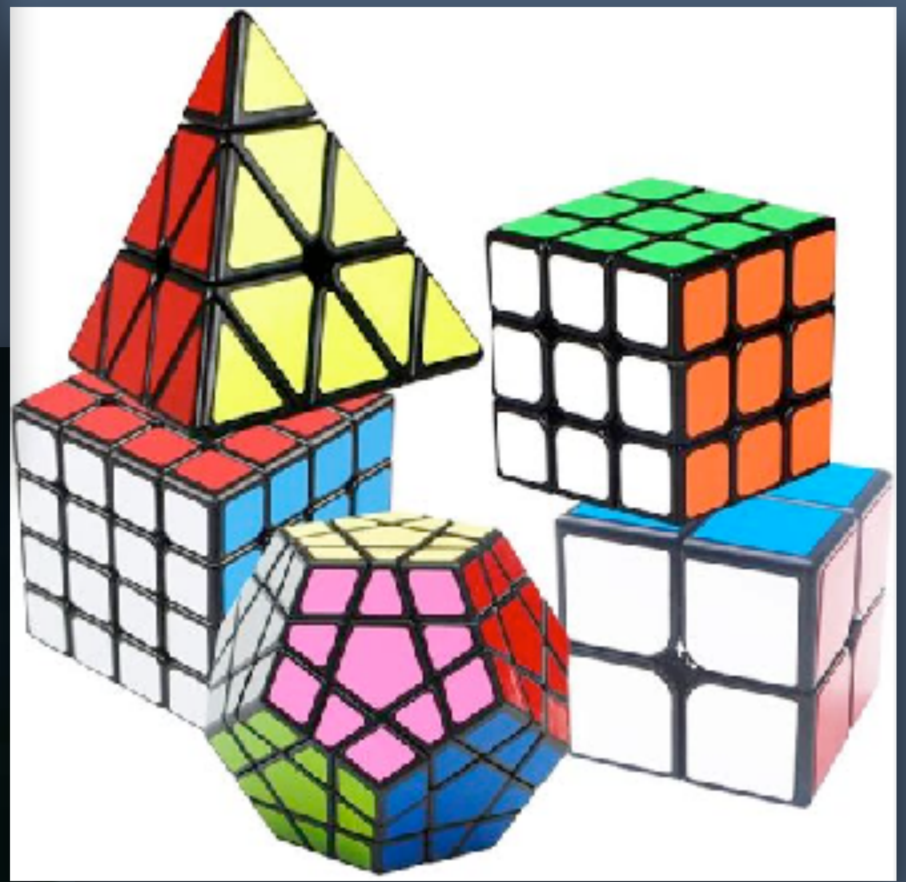


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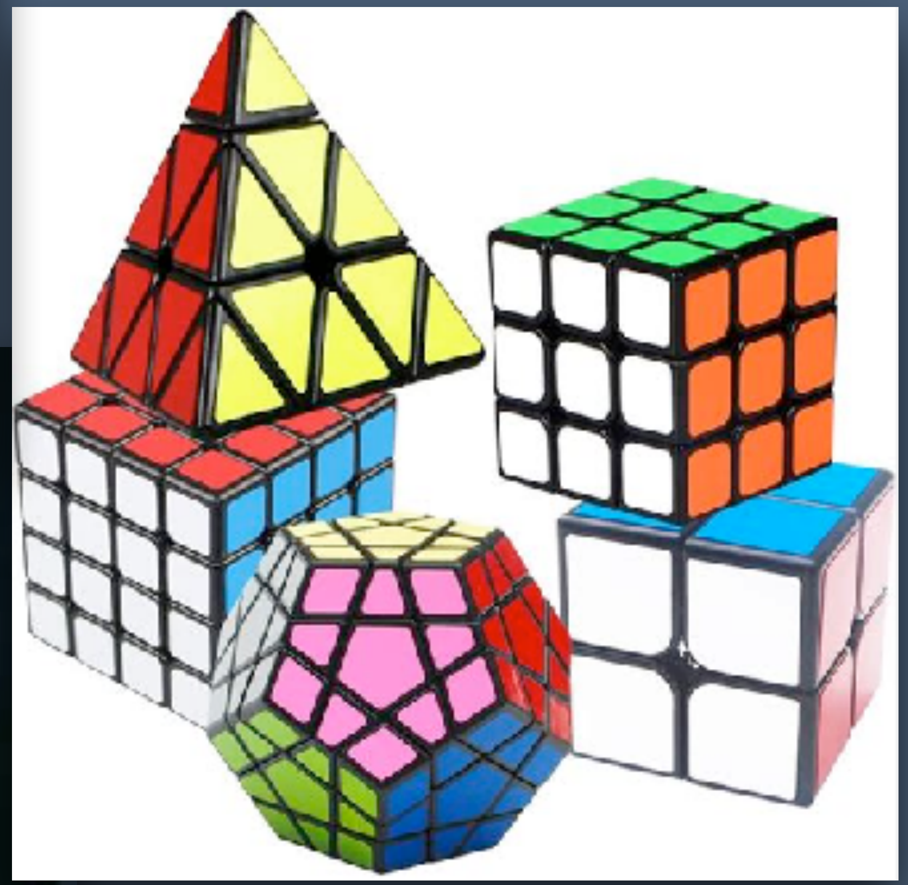
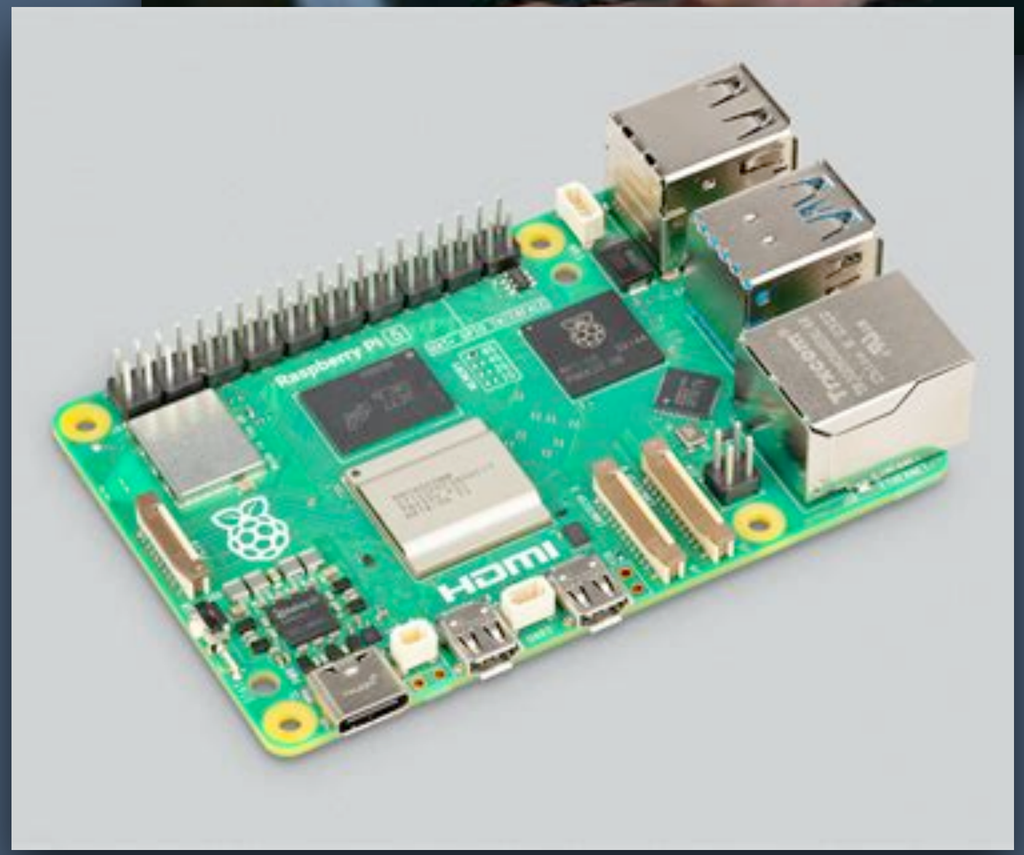
... research feels similar

(in some respects)




A beautiful area of mathematics

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Why the hell are these almost integers?

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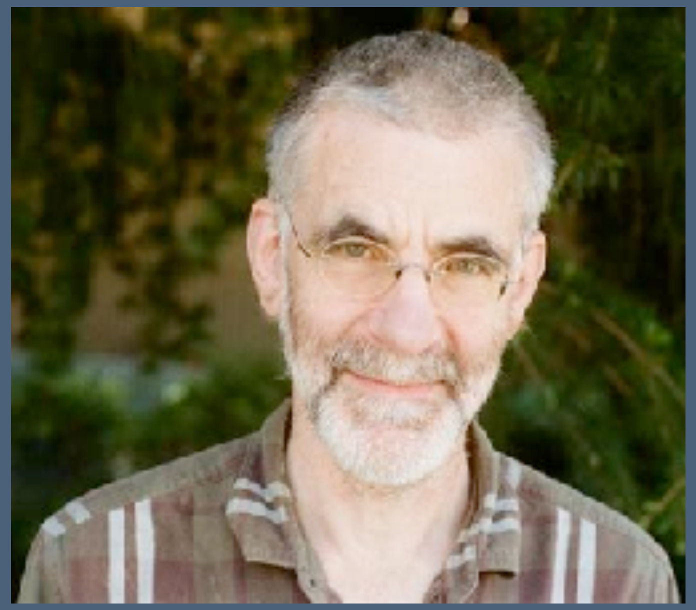

Richard Borcherds proved the 'Monstrous Moonshine Conjecture'

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A must-read for students

"Margins of my dissertation: Life lessons that my PhD taught me"
by Chhavi Yadav

Do I Recommend doing a PhD?

Only if you know it's going to break you—and rebuild you—in ways you didn't expect. If you come in with a romantic and rozy idea of research, it will shatter extremely quickly. But if you're curious, willing to unlearn, and open to being reshaped by failure and growth—it can be profound.

Just don't go in expecting only answers. You'll come out with better questions.

Link to the blogpost

What this talk was NOT about?

- Chaos theory and complex dynamics
- Mandelbrot set
- A theorem of Milnor and Thurston

What this talk was about?

Some mathematical card tricks!

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What to expect (and not expect) from a career in research

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Thank you for your time!