Algorithmic Game Theory

Assignment 5: due December 12, 2022

Assignment policies:

1. While you may dicuss the problems with others, you must write up the solution by yourself, in your own words.
2. Please write in your submission the people with whom you discussed the problems, as well as any references you used.
3. Late submissions will not be accepted, unless agreed to by me **prior** to the last date for submission.
4. Please write clearly and legibly, and include how you arrived at the solution!

Question 1 [10]: Consider the sponsored search auction we discussed earlier, in the Bayesian setting: there are k advertisement slots, and each slot j has a (public) click-through rate (CTR) α_j. Each bidder's valuation is drawn iid from a known regular distribution F. Bidder i gets utility $v_i\alpha_j$ minus payments if it is allocated slot j. Describe the revenue-optimal auction, including payments, for this setting.

Question 2 [10]: Show that, in the Bayesian single-item single-parameter case, the revenue of Vickrey's second-price auction with n bidders is at least $(n-1)/n$ times that of the optimal auction.

Question 3 [10]: Consider the Bayesian single-parameter *public projects* environment, where the feasible allocation set $X = \{0^n, 1^n\}$, that is, either every bidder is allocated, or no bidder is (e.g., consider a public project as a park which everyone enjoys if it is built). Describe the revenue-optimal mechanism when bidder valuations are drawn iid from $U[0, 1]$.

Question 4 [10]: Consider the following variant of the house allocation problem: There are n agents and n houses, which are unassigned initially. Each agent must be allocated a house, and has a totally ordered preference list over the houses. An allocation A of houses to agents is said to be *Pareto-optimal* if every allocation A' that improves the allocation to an agent, also makes some agent worse off. Design a mechanism that is DSIC and produces an allocation that is Pareto-optimal.

Question 5 [10]: Consider the extension to the multi-unit auction studied in class where we now have two items, 1 and 2, and multiple copies m_1 and m_2 of these items. Agent i has value $v_i(x_1, x_2)$ if it is allocated x_1 units of item 1 and x_2 units of item 2. Agents continue to have free disposal, thus $v(x_1, x_2) \geq v(x_1', x_2')$ if $x_1 \geq x_1'$ and $x_2 \geq x_2'$. As before, we want to design a polynomial-time MIR DSIC mechanism that approximately maximizes social welfare. Design and analyse a 1/4-approximate mechanism for this. Describe the algorithm you analyse in detail.

Question 6 [10]: Recall the local search algorithm for stable matching we initially considered in class: start with an arbitrary matching $\pi : M \to W$. If there exists a blocking pair (m, w), remove edges $(m, \pi(m))$ and $(\pi(w), w)$ and replace these with the edges (m, w) and $(\pi(m), \pi(w))$.

Does this algorithm terminate? Either show a bound on the number of iterations taken by this algorithm to converge to a stable matching, or give an example where this algorithm cycles.

Hint: it is sufficient to consider stable matching instances where $|M| = |W| = 3$.

Question 7 [10]: Given an instance of fair division of additive goods, consider two allocations: allocation A, that maximizes the Nash social welfare, and B, that maximizes the utilitarian welfare (or the sum of agent utilities). Show that the utilitarian welfare of allocation A can be *arbitrarily* worse than the utilitarian welfare of allocation B, i.e., the ratio $\mathrm{UW}(B)/\mathrm{UW}(A)$ is not bounded by any function of m and n, where m and n are the number of items and agents respectively.