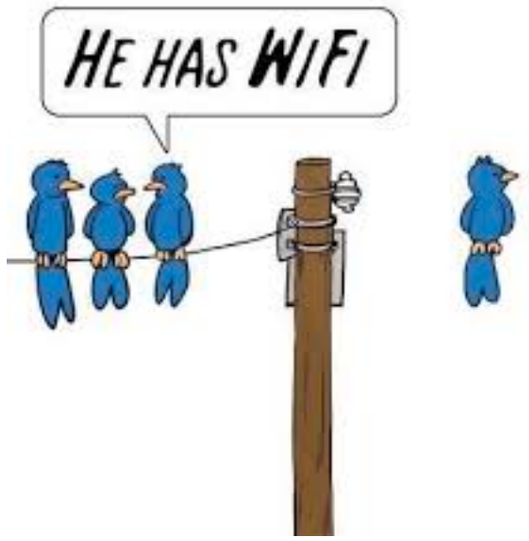
Graph Matchings and Wireless Communication



Rahul Vaze

In this talk

ZEE NEWS
पल पल आपके साथ
कॉल ड्रॉप से आज़ादी कब मिलेगी?
#CallDropFreeIndia
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In this talk

105 pictures 3 equations

In this talk

105 pictures 3 equations

color blind friendly

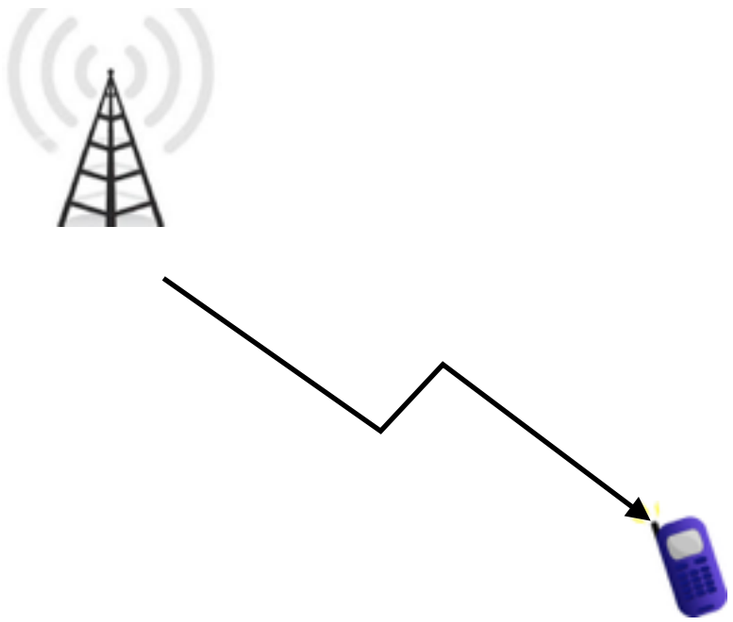
In this talk

105 pictures 3 equations

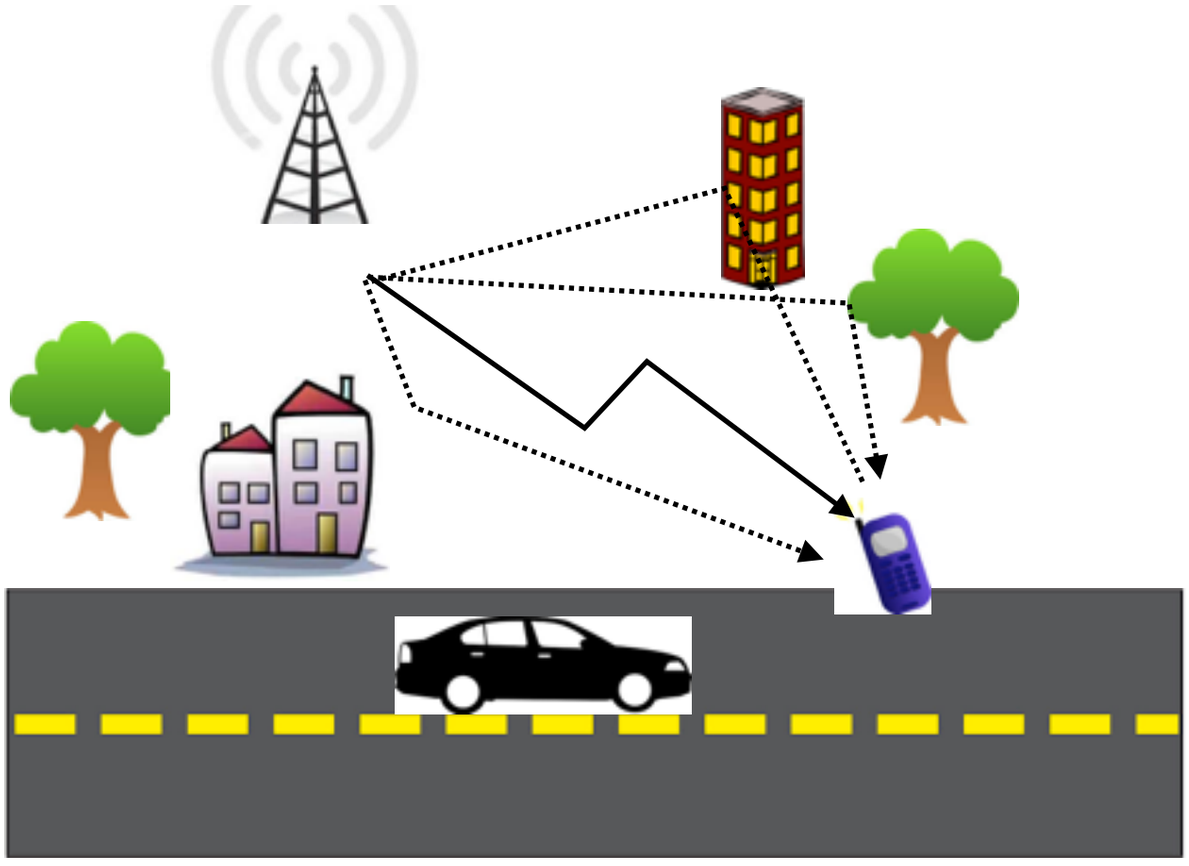
color blind friendly

NOT in this talk

Wireless Channel



Wireless Channel



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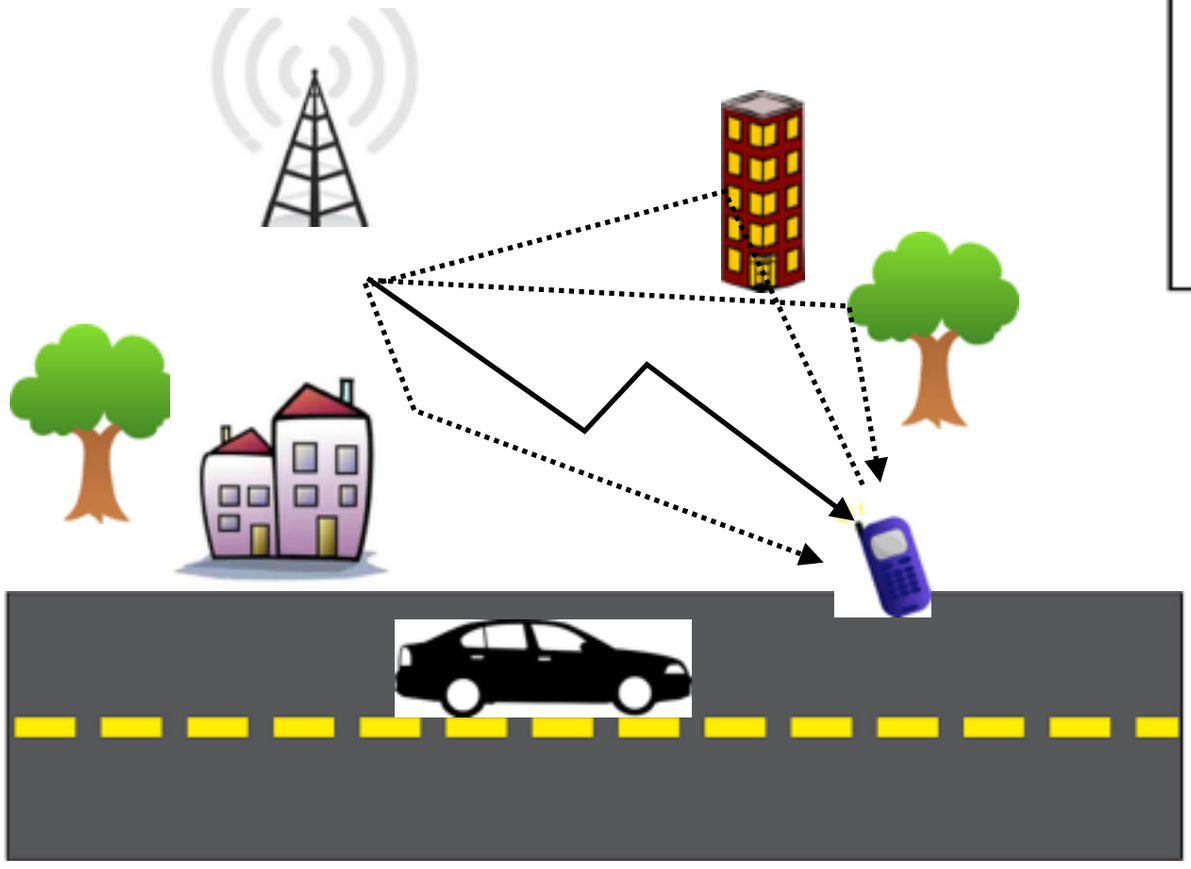

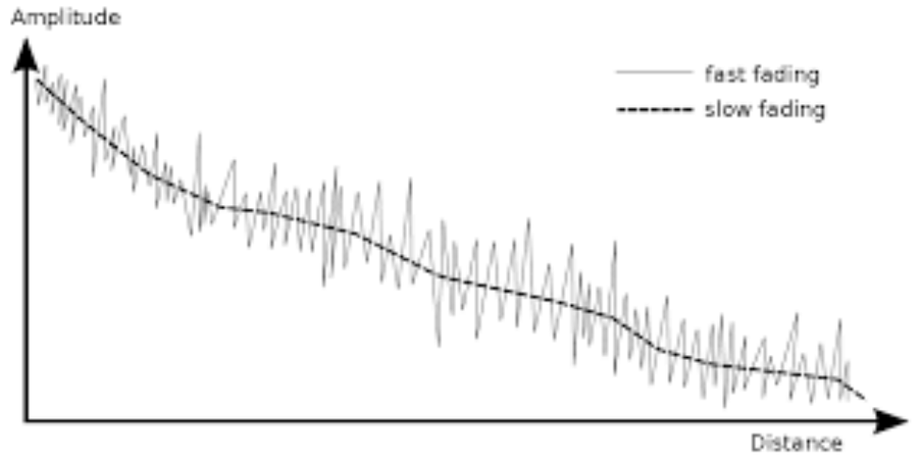

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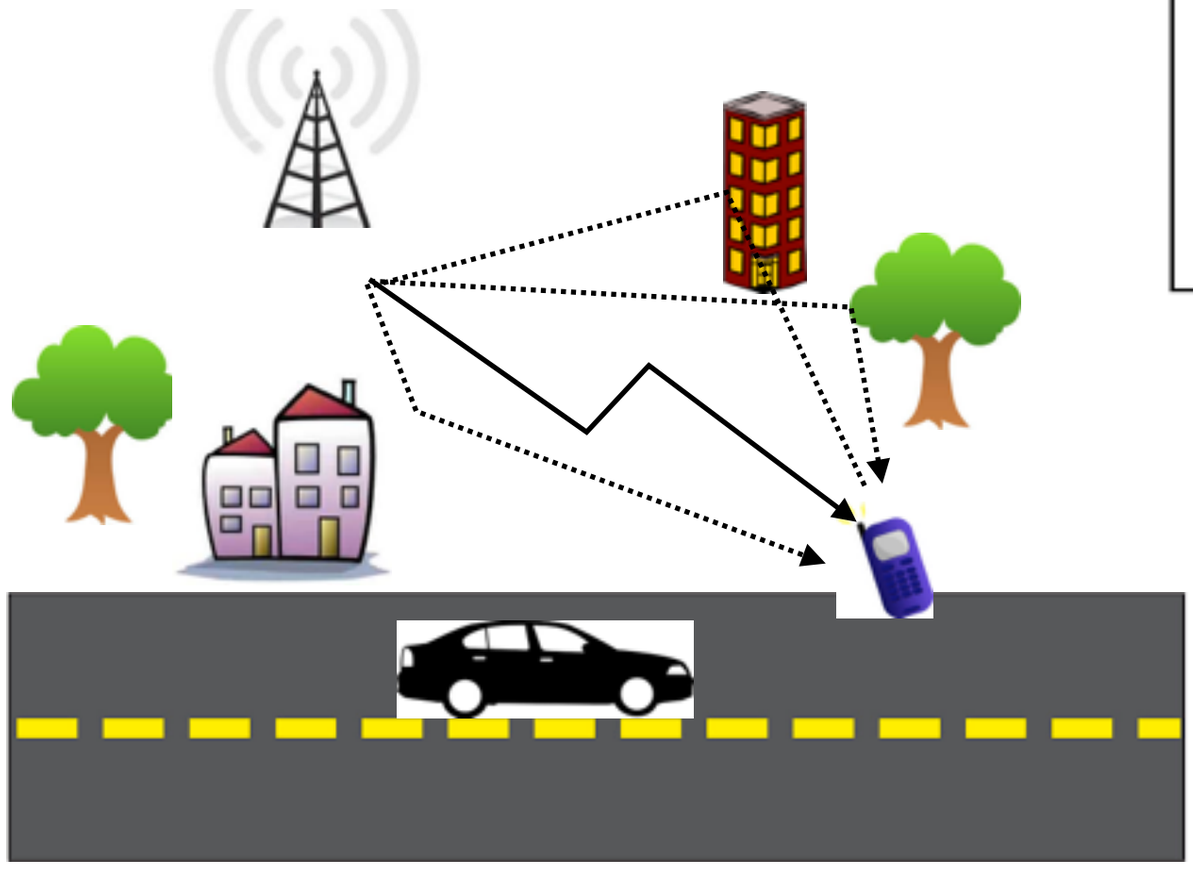
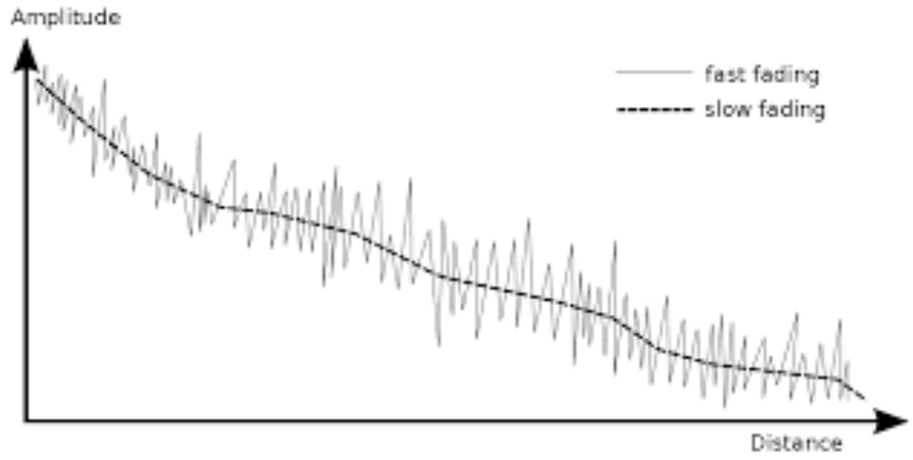
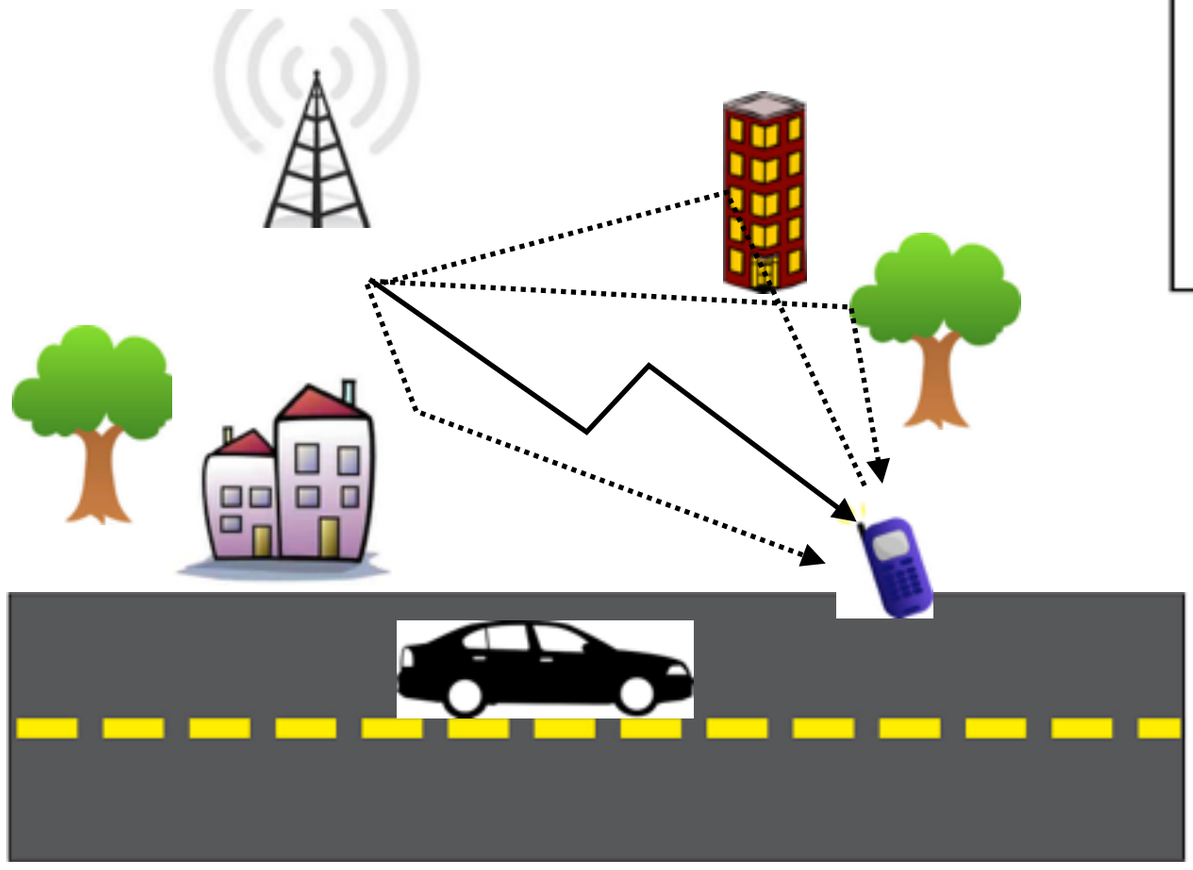
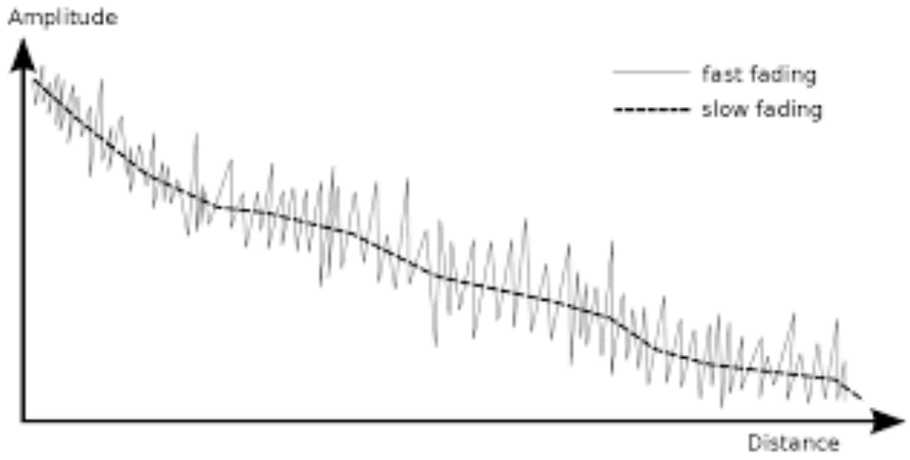




$$\text{Rate} = \log_2\left(1 + \frac{|h|^2 P}{N}\right) bits/sec/Hz$$

Wireless Channel





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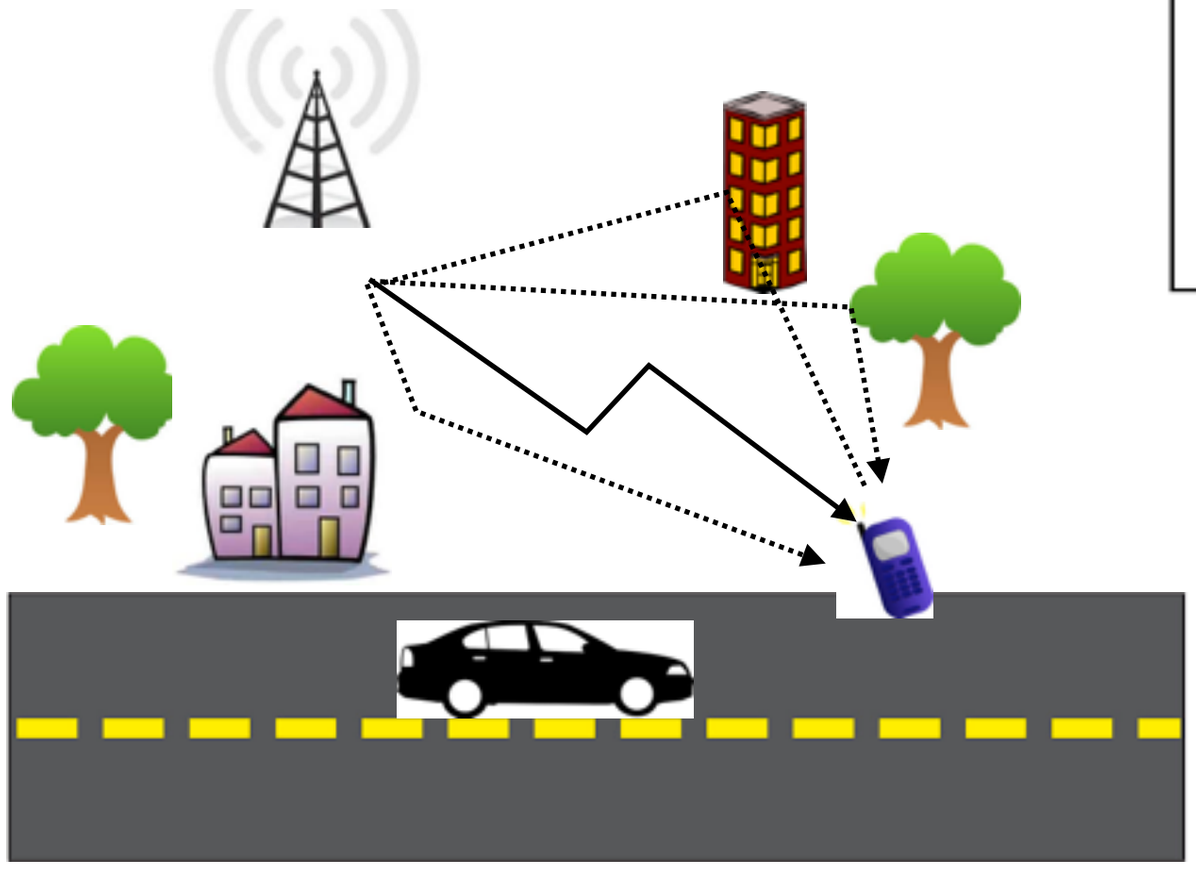
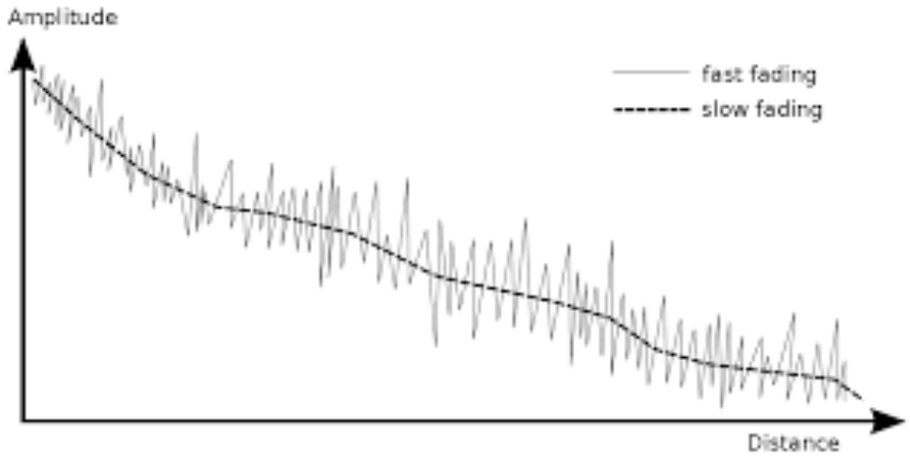
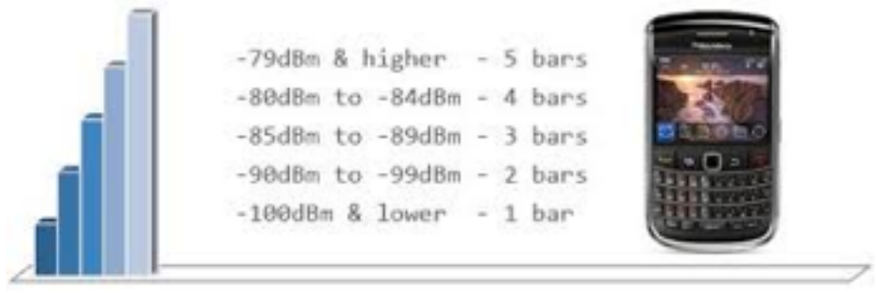
SNR

Wireless Channel



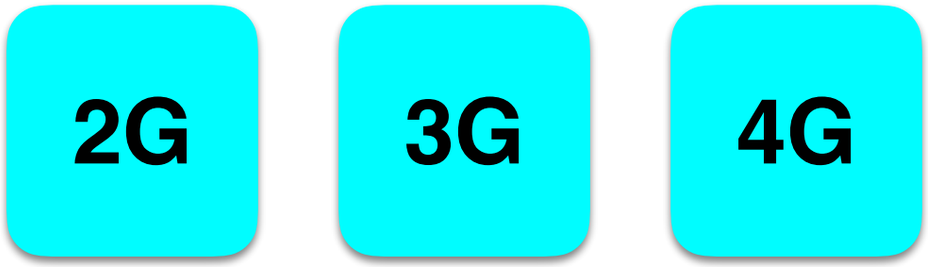




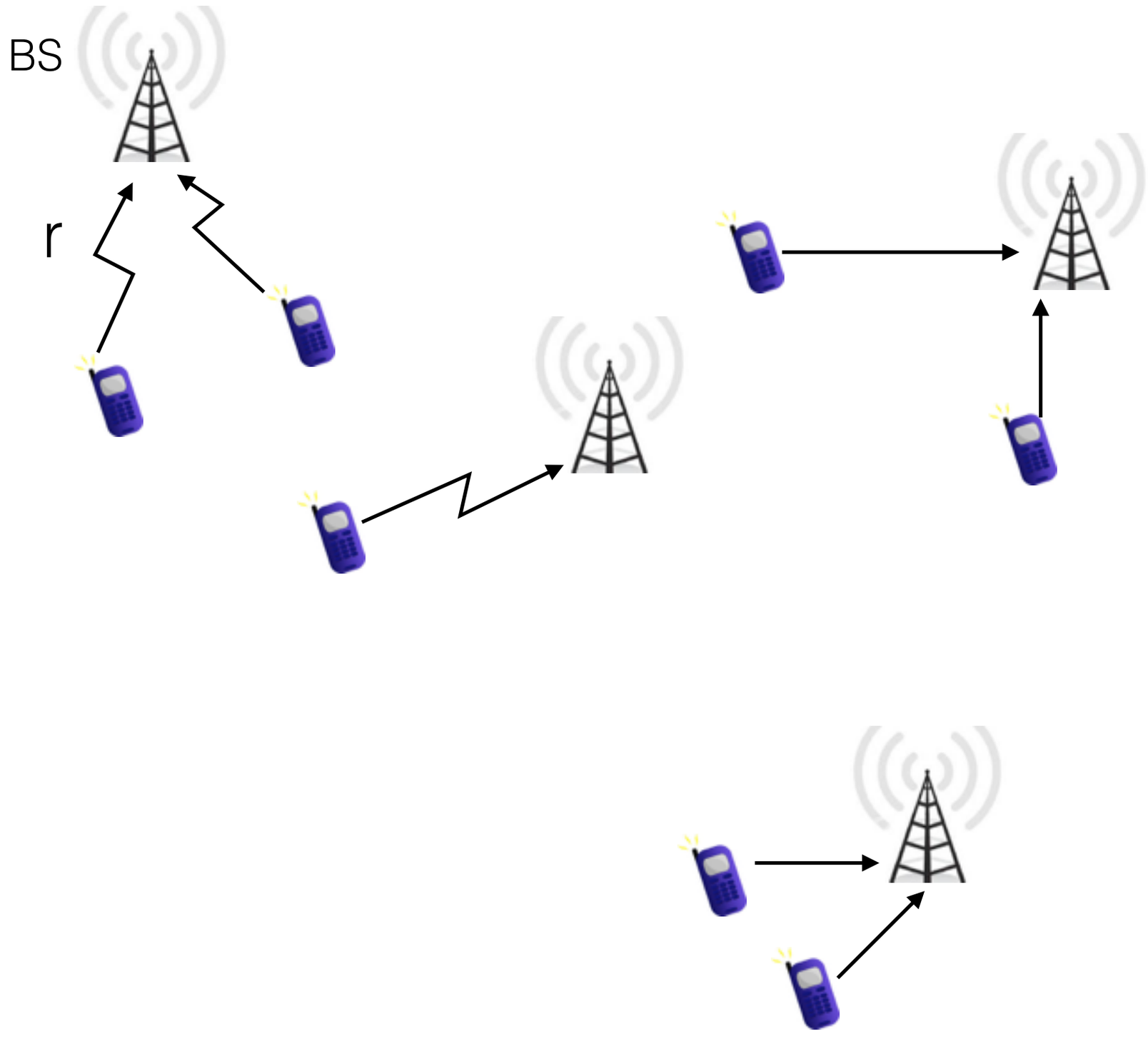
$$\text{Rate} = \log_2\left(1 + \frac{|h|^2 P}{N}\right) bits/sec/Hz$$

SNR

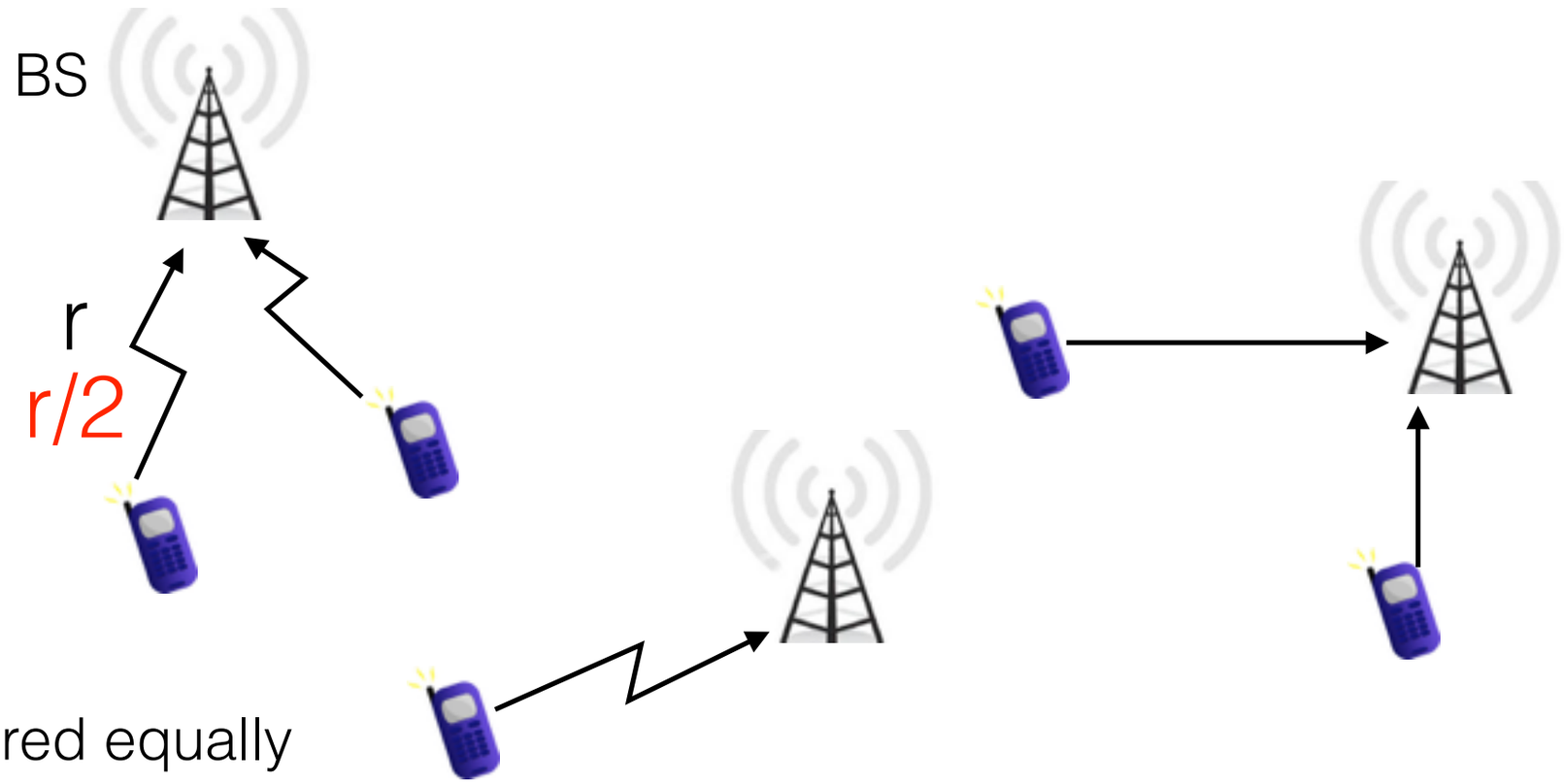
Legacy Problem



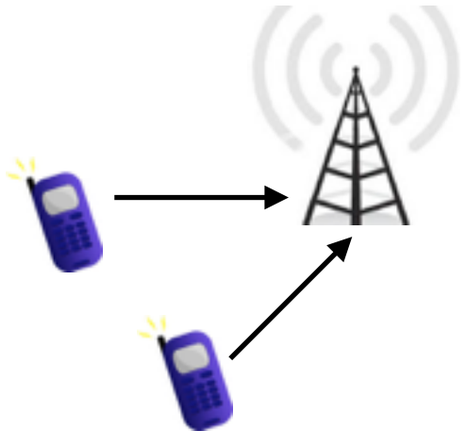
Legacy Problem -Wireless Communication



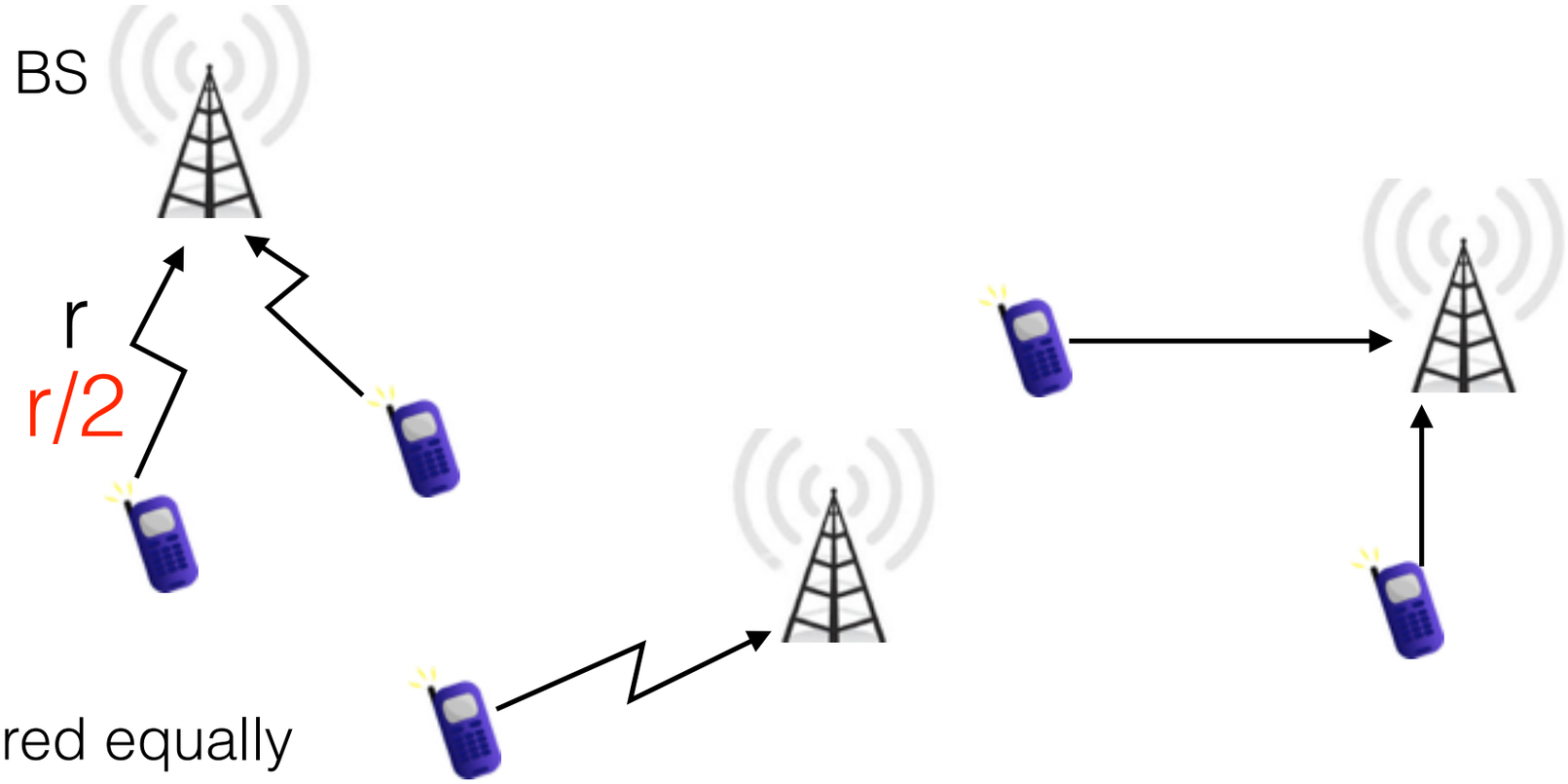
Legacy Problem -Wireless Communication



Resources shared equally

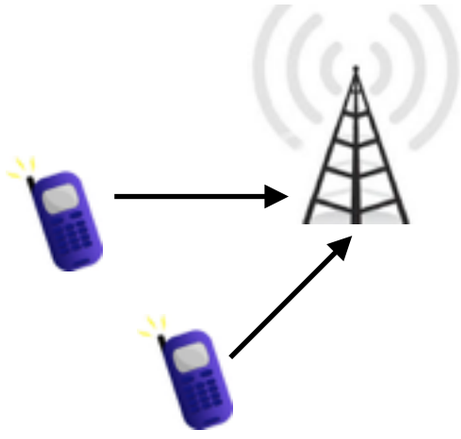


Legacy Problem -Wireless Communication

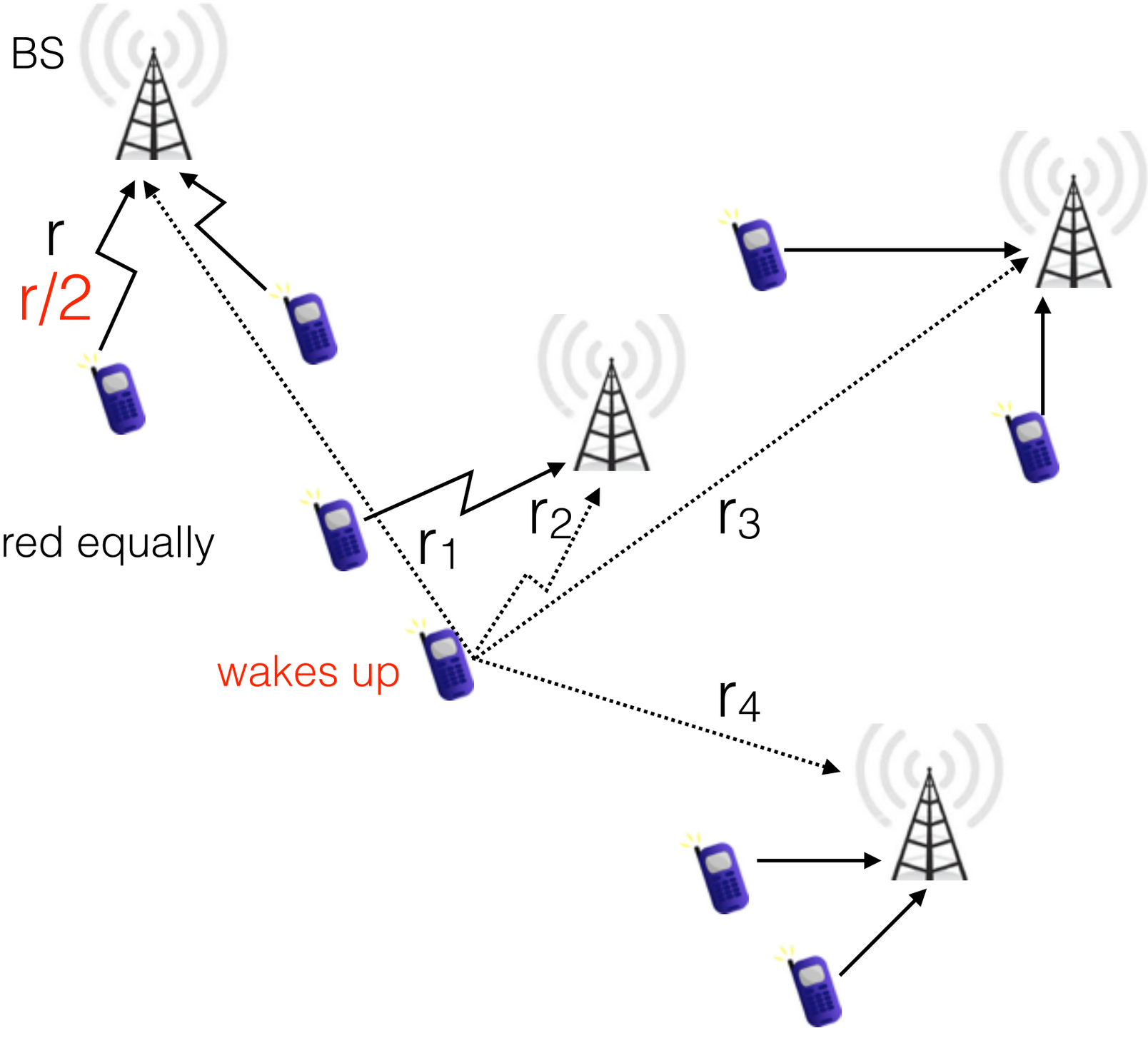


Resources shared equally

wakes up

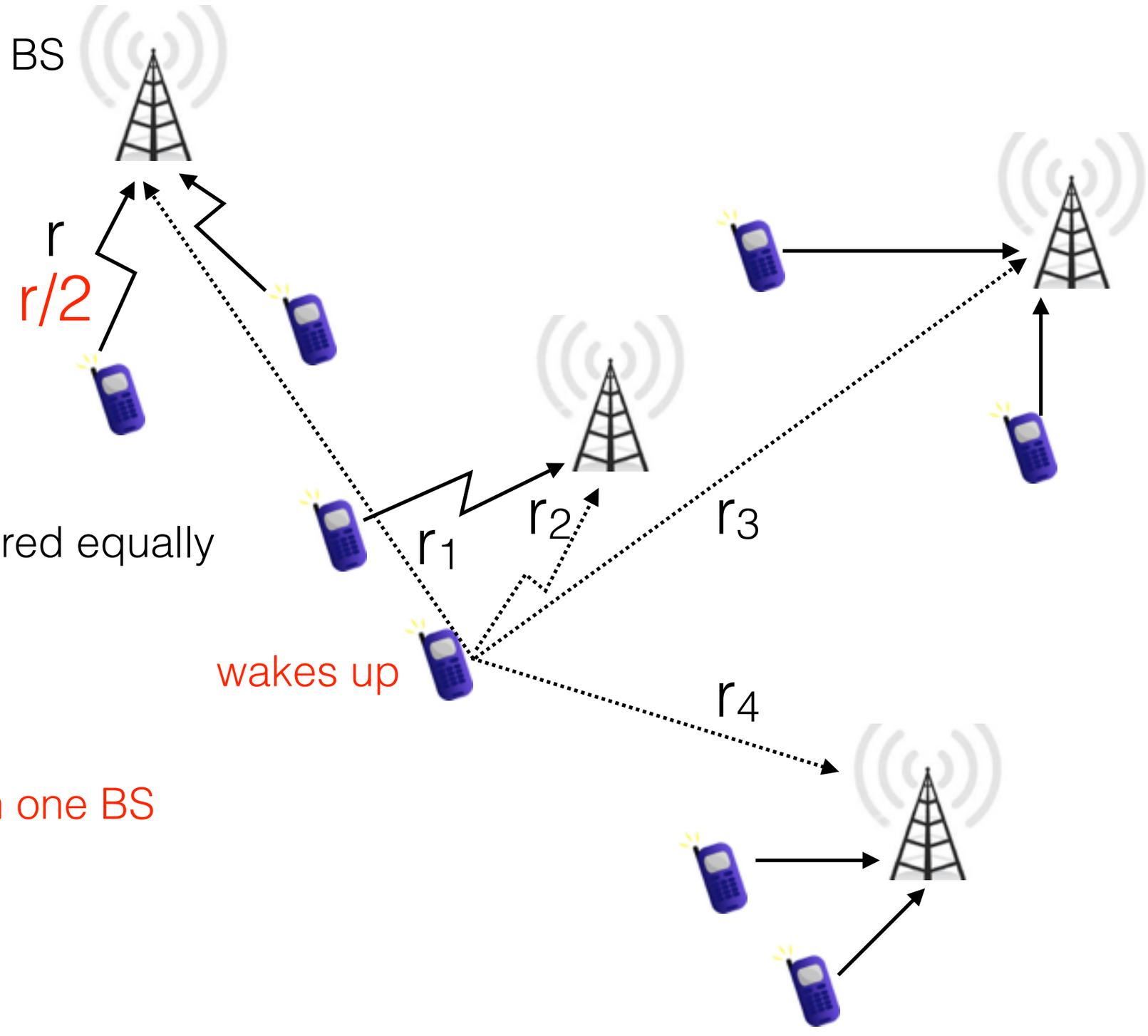


Legacy Problem -Wireless Communication



Resources shared equally

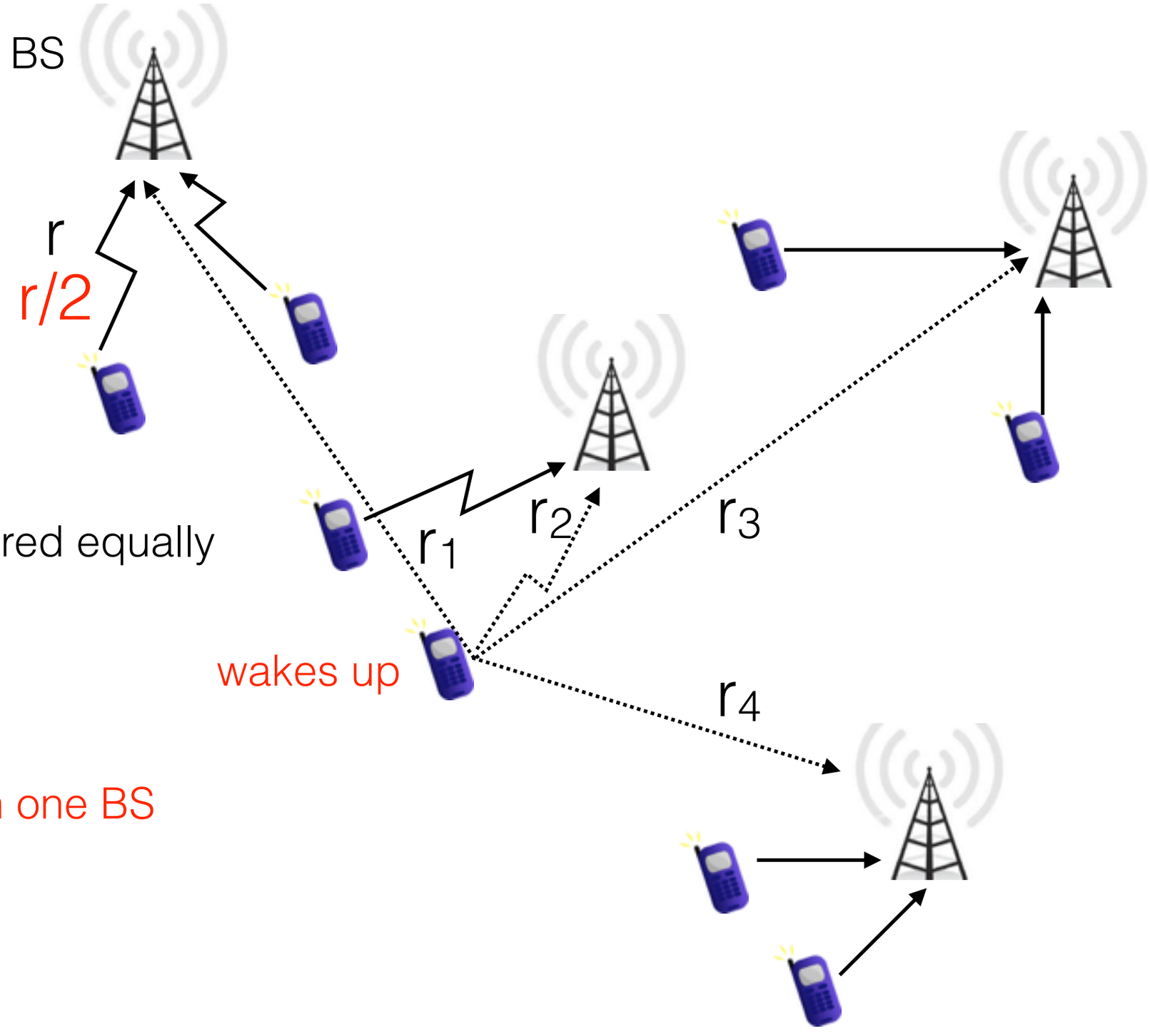
Legacy Problem -Wireless Communication



Resources shared equally

Associate with one BS

Legacy Problem -Wireless Communication



Resources shared equally

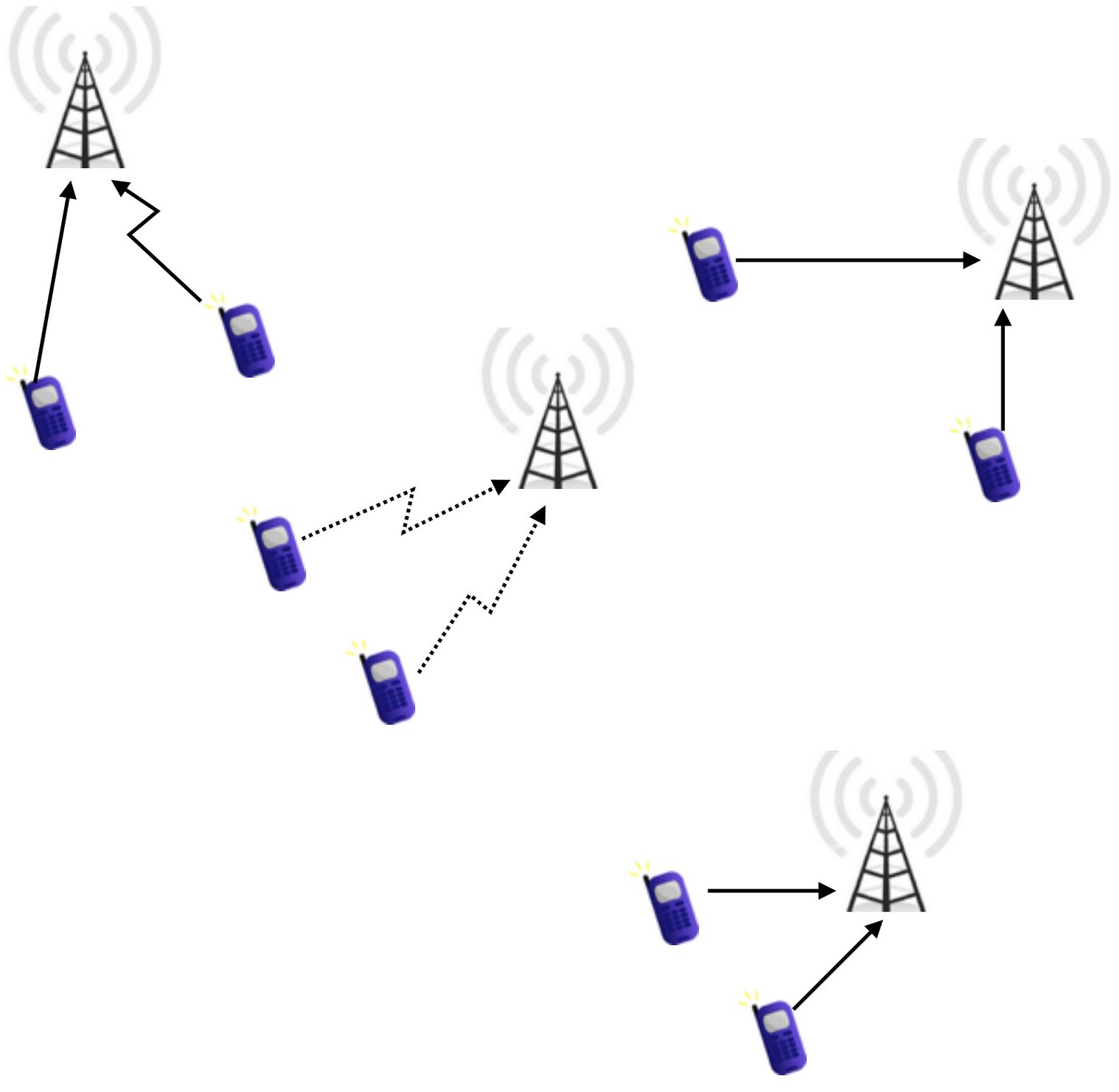
Associate with one BS

Find optimal BS allocation to maximize sum-rate

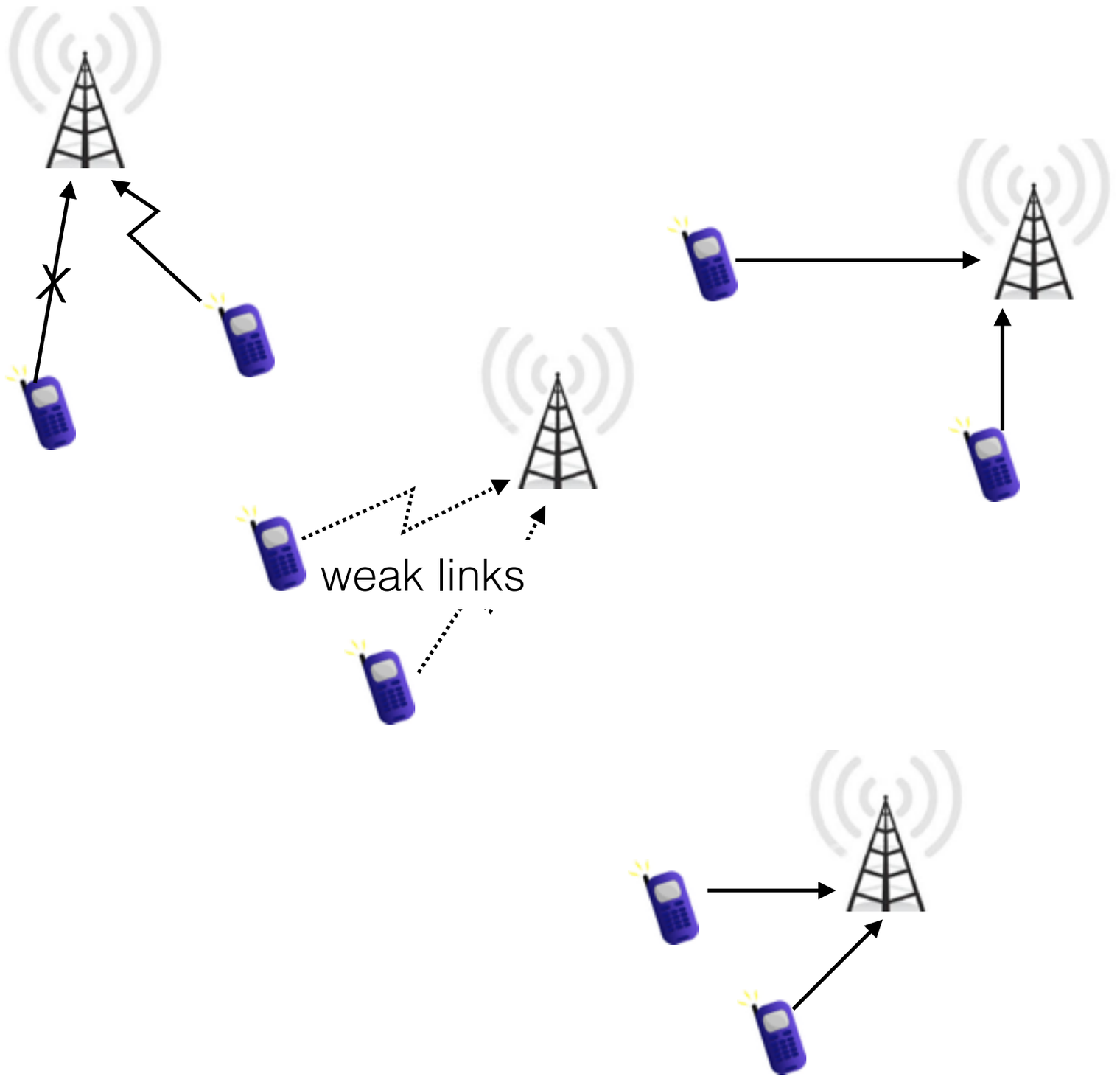
Modern Problem

5G

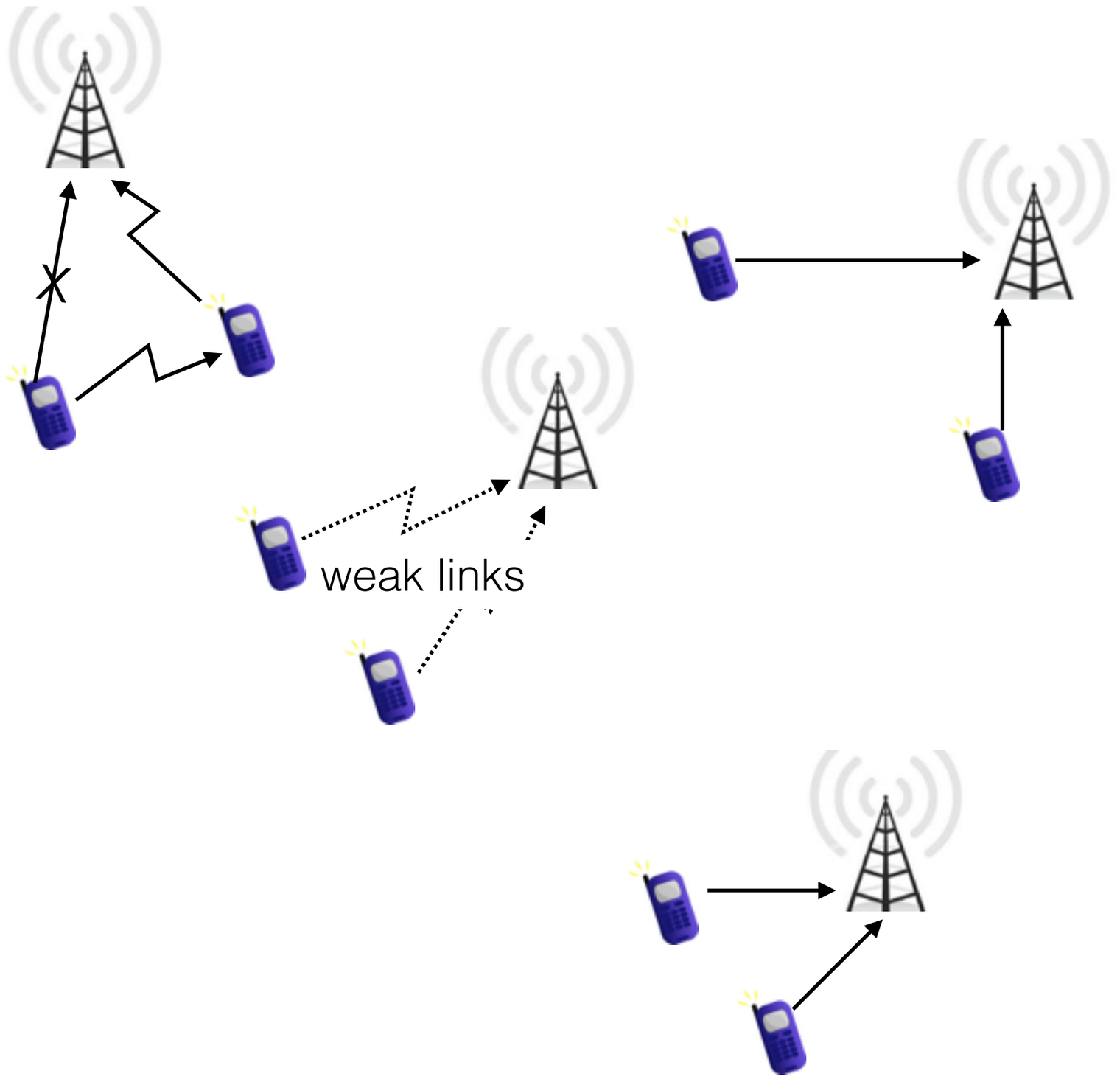
Modern Problem Device-2-Device Communication



Modern Problem Device-2-Device Communication



Modern Problem Device-2-Device Communication



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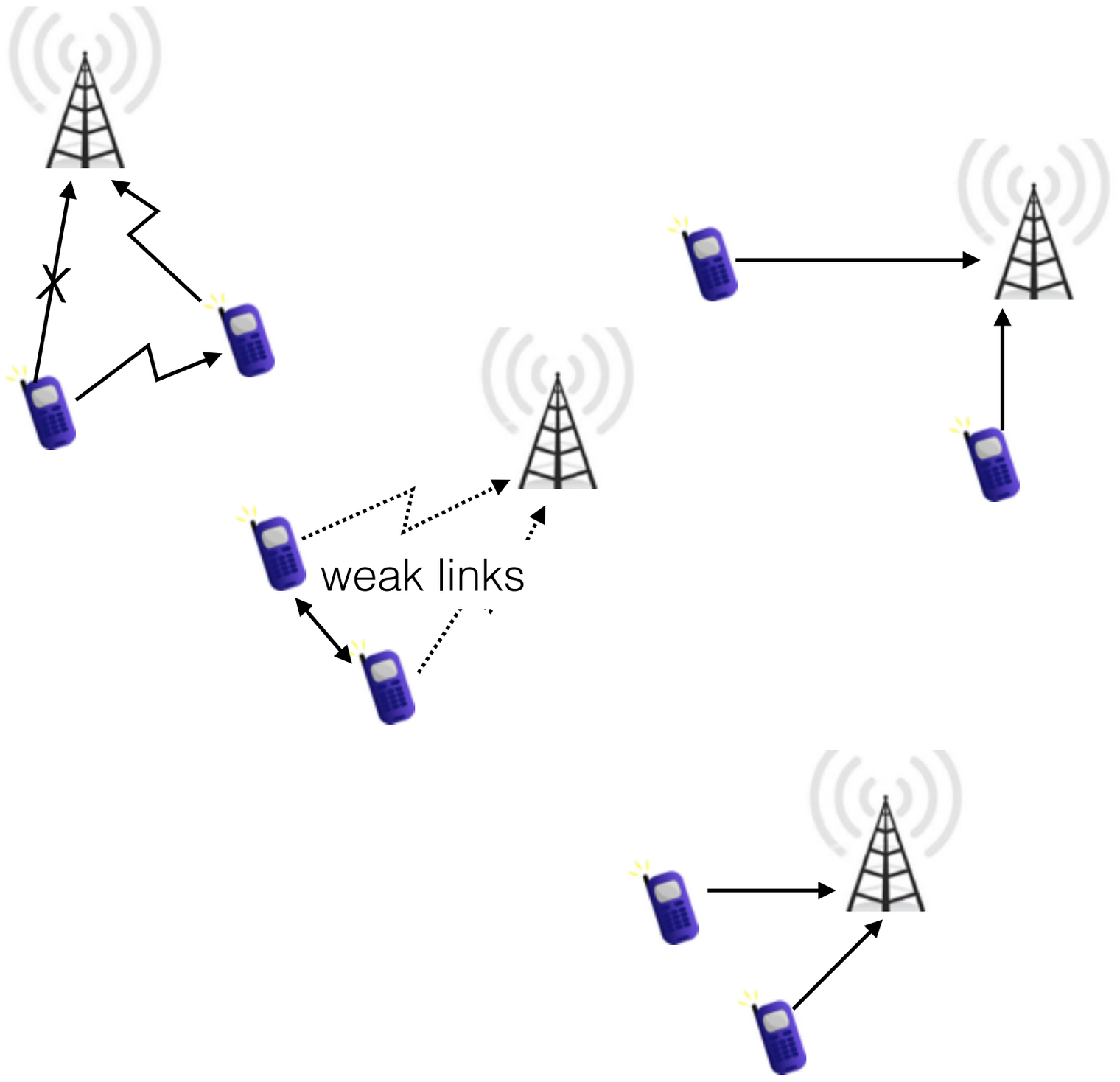

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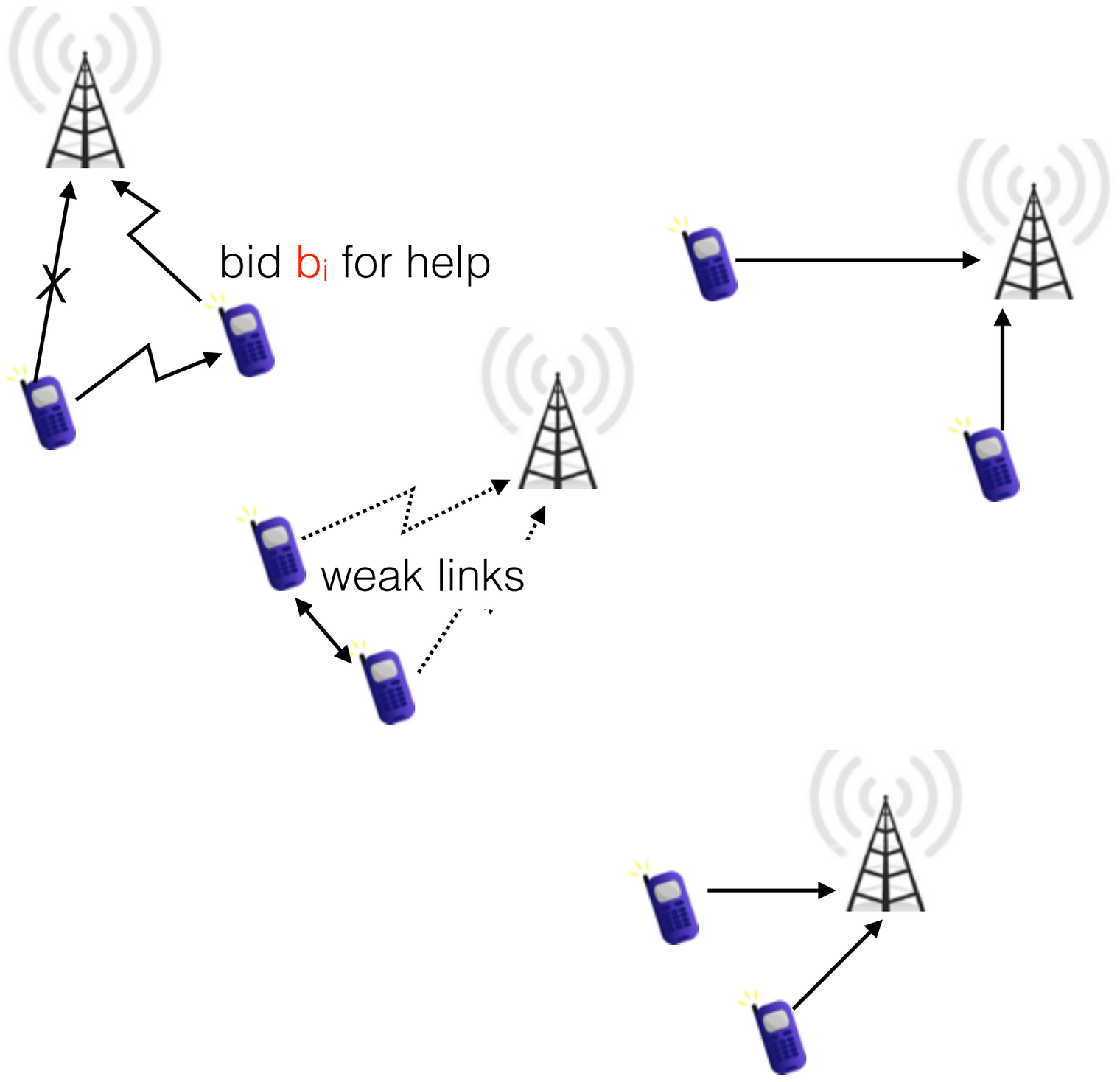

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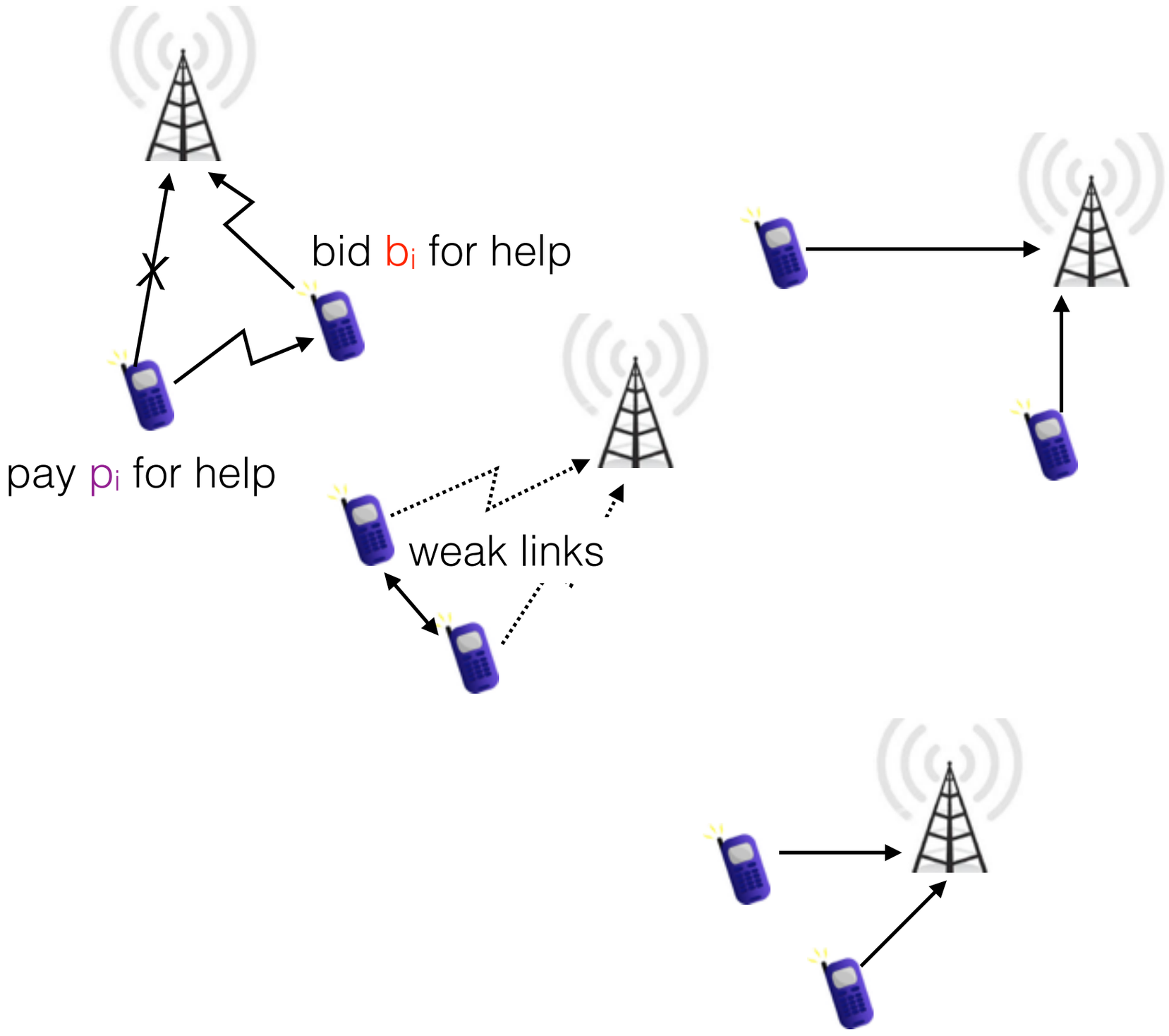
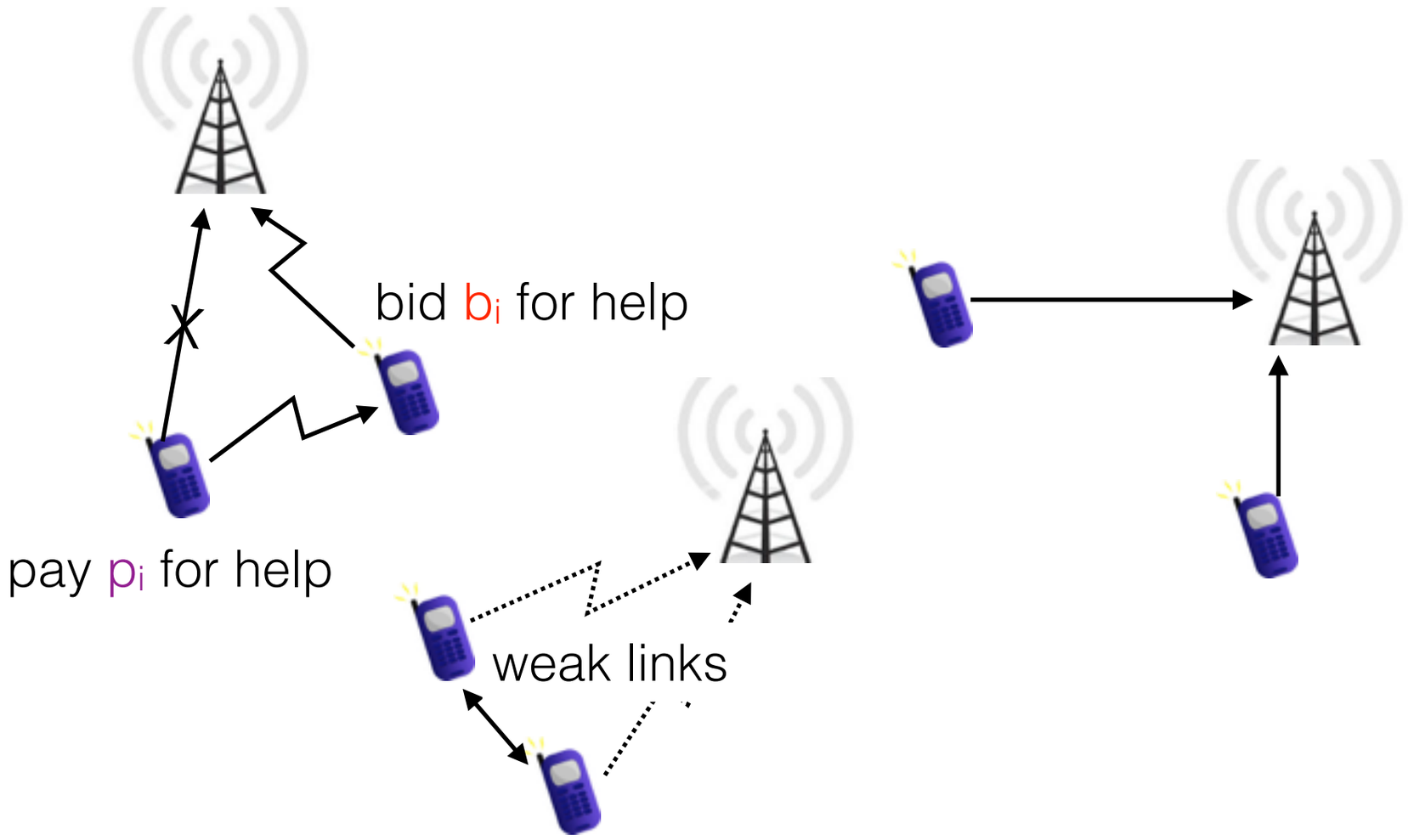
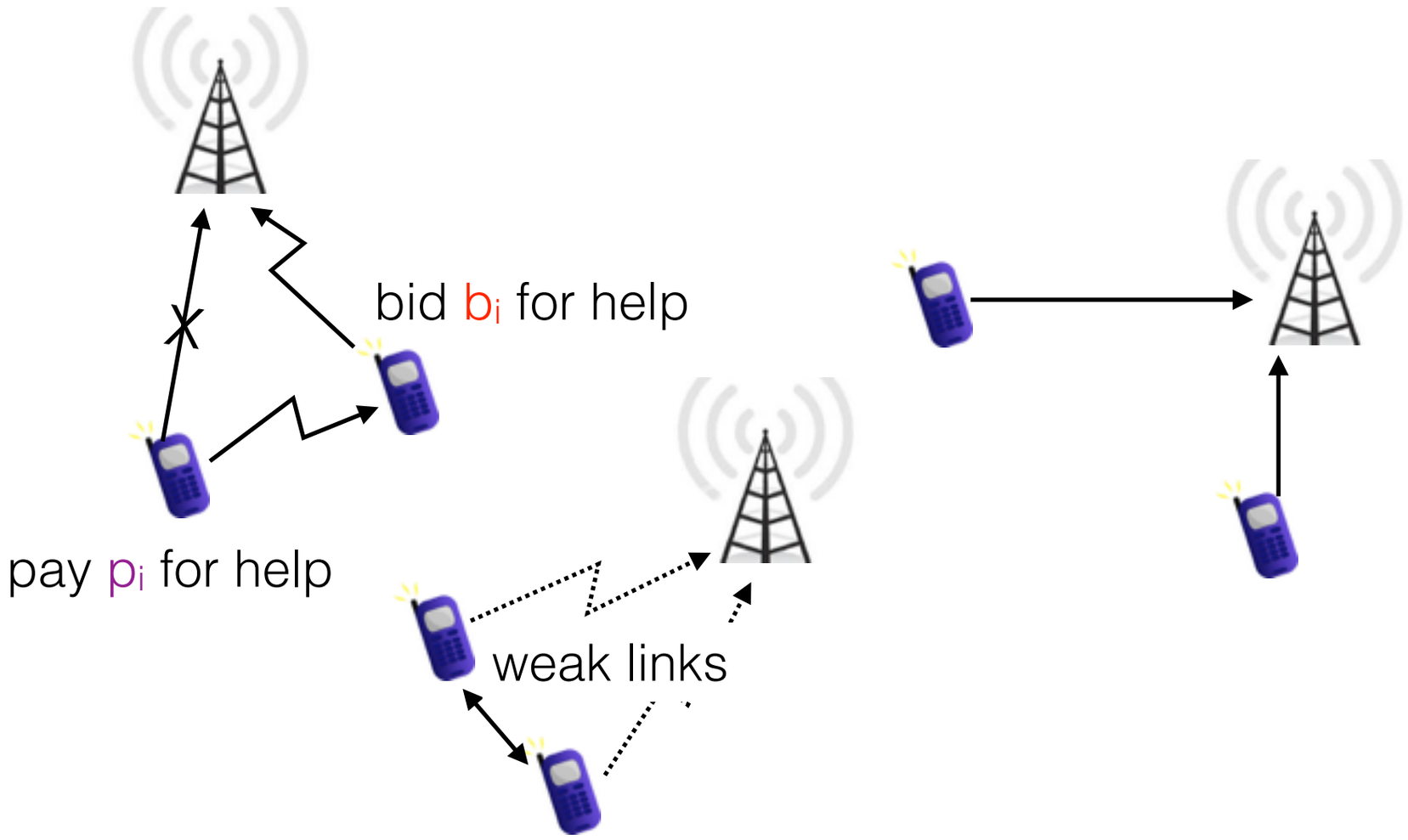


Modern Problem Device-2-Device Communication



mechanism to avoid cheating

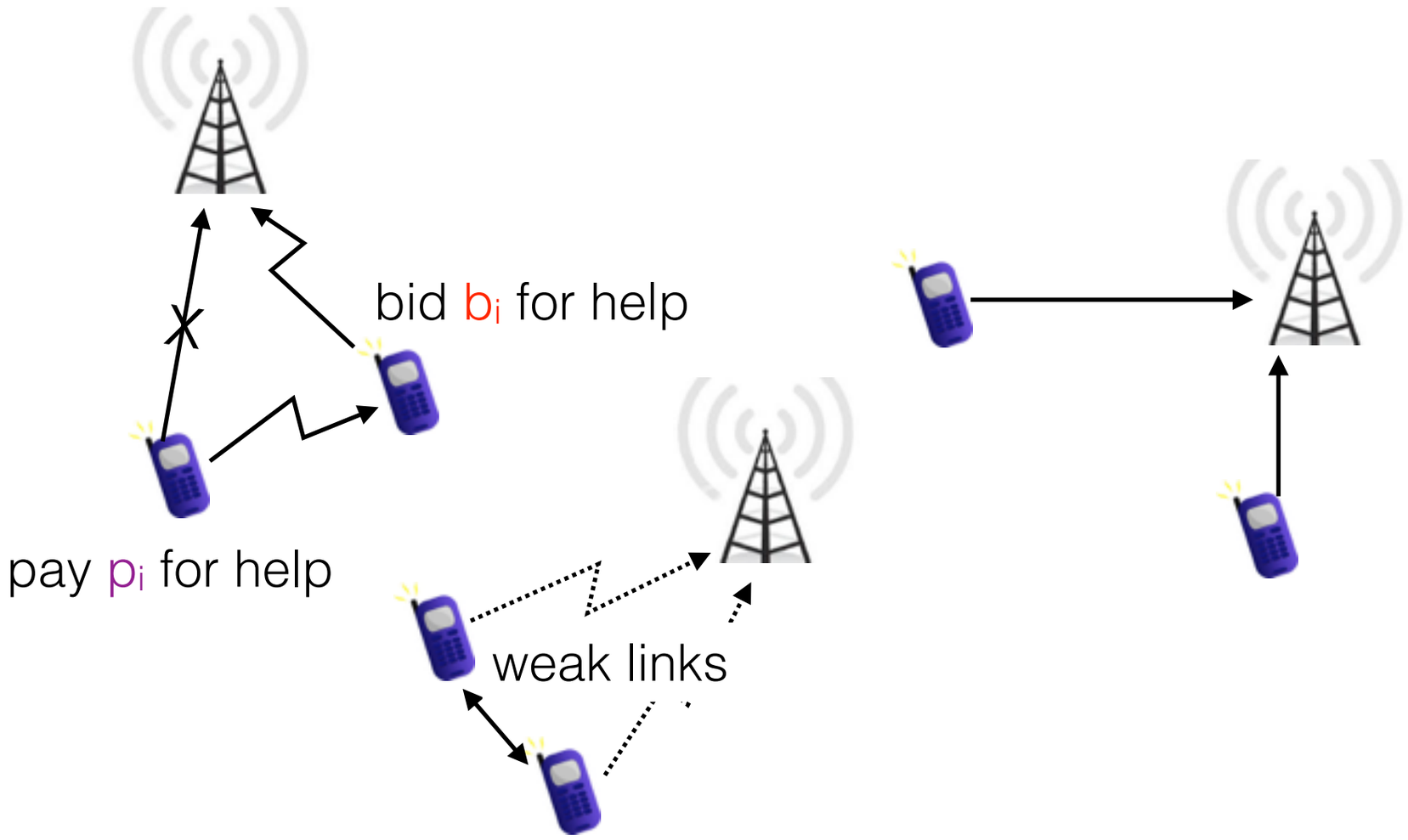
Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

Find optimal helper association and incentive rule that is truthful

Why care about truthfulness ?

Why care about truthfulness ?

8.28 Crores - 2011

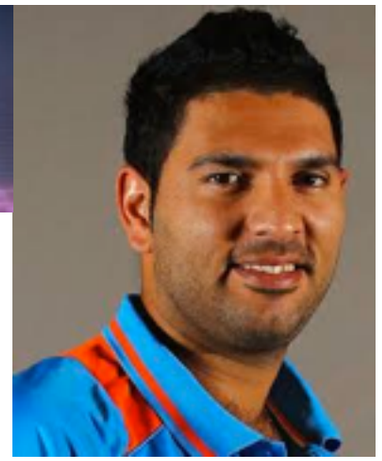
Why care about truthfulness ?

8.28 Crores - 2011

14 Crores

PUNE WARRIORS INDIA

Why care about truthfulness ?



8.28 Crores - 2011

14 Crores

16 Crores

Why care about truthfulness ?

8.28 Crores - 2011

14 Crores

16 Crores

7 Crores

W. Vickerey

E. Clarke

T. Groves

Truthful Auction

Truthful Auction

Truthful Auction

Winner: Largest bid

Price: Second-Largest bid

Truthful Auction

Winner: Largest bid

Price: Second-Largest bid

No incentive to bid more than private utility/price

small detour

at the height of their popularity.

The mathematics behind a perfect match

It's a problem that has had many names – the secretary problem, the Sultan's dowry problem and the optimal stopping problem, but it's also one that can be used to decide when to stop dating and settle down. "Settle down early and you might forgo the chance of a more perfect match later on. Wait too long to commit, and all the good ones might be gone. You don't want to marry the first person you meet, but you also don't want to wait too long," writes Ann Swanson, as she describes the mathematics behind finding the best spouse.

The spouse problem is an old one, and has a solution – a simple mathematical rule. "In the scenario, you're choosing from a set number of options. For example, let's say there is a total of 10 potential mates who you could seriously date and settle down with in your lifetime. If you could only see them all together at the same time, you'll have no problem picking out the best. But this isn't how a lifetime of dating works, obviously," writes Swanson. The problem, of course, is that suitors don't appear together, but at random. And of course, once you've rejected someone, it can be embarrassing - if not impossible, to go back.

MAGIC NUMBER: Dump the first 37% of the (other) you may have ever your lifetime

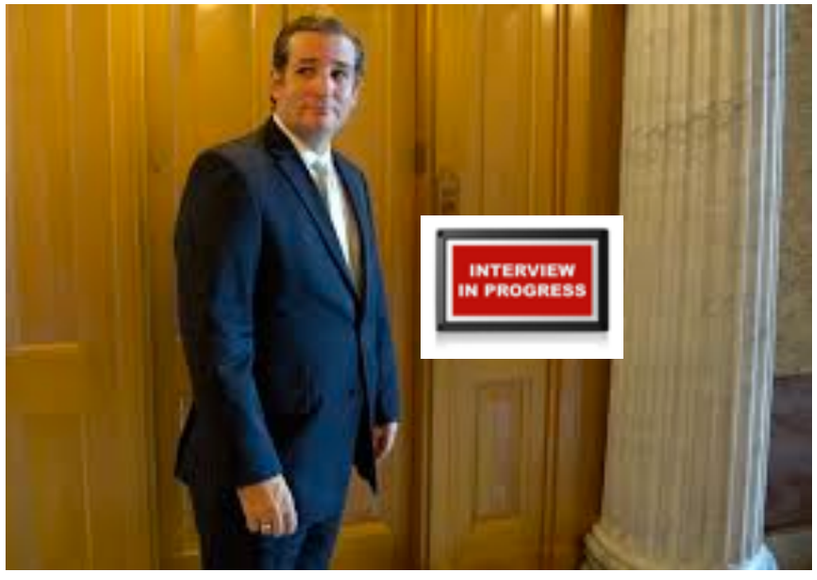
The answer lies in probability – and its mathematics. To find the perfect partner, you have to date and reject the first 37% of the group your total lifetime suitors. Then find the next person who is better than anyone you've ever dated before. Naturally, for this to work in real life, you need to know how many suitors you will have, but you can make a guess.

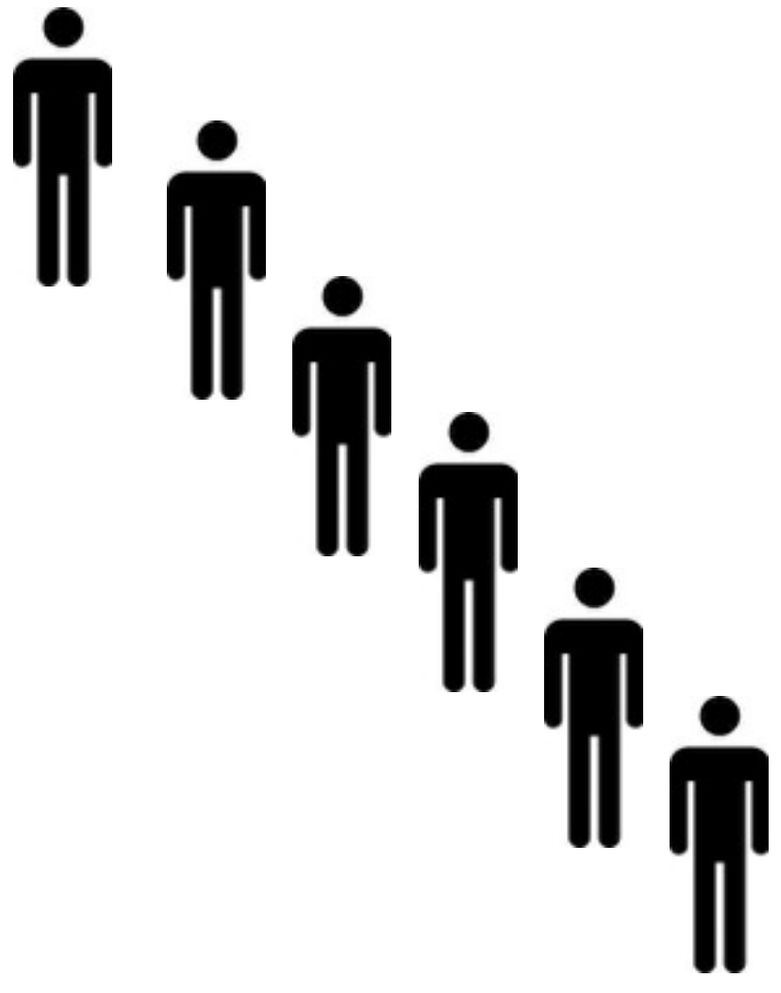
There are risks, of course. "There's the risk, for example, that the first person you date really is your perfect partner. If you follow the rule, you'll reject that person anyway. And as you continue to date other people, no one will ever measure up to your first love, and you'll end up rejecting everyone, and end up alone with your cats. Of course, some people may find only preferable to be friends or girlfriends anyway)," writes Swanson.

how many to date before decide to marry!

Hiring impatient staff

Hiring impatient staff





Hiring staff - not adversarial

Hiring staff - not adversarial

random order

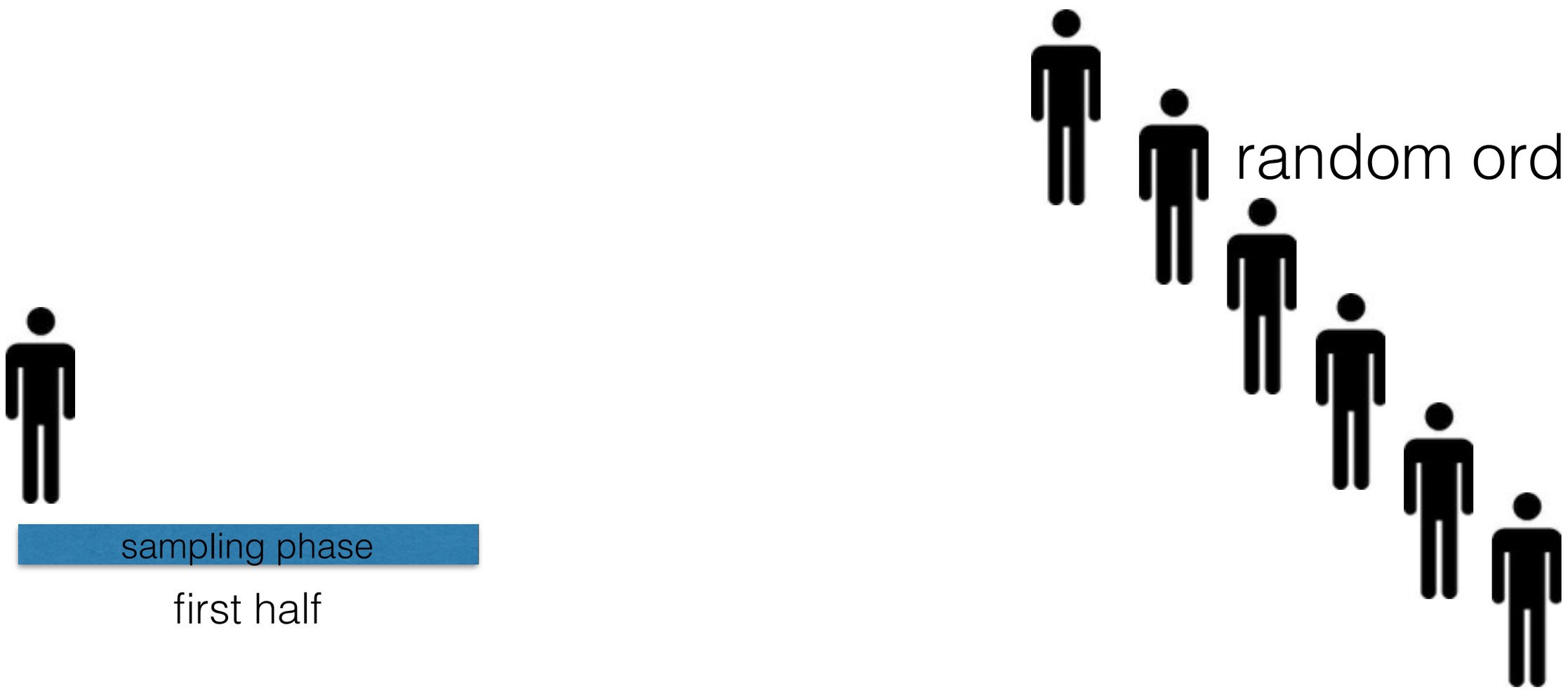
Hiring staff - not adversarial

sampling phase

first half

random order

Hiring staff - not adversarial



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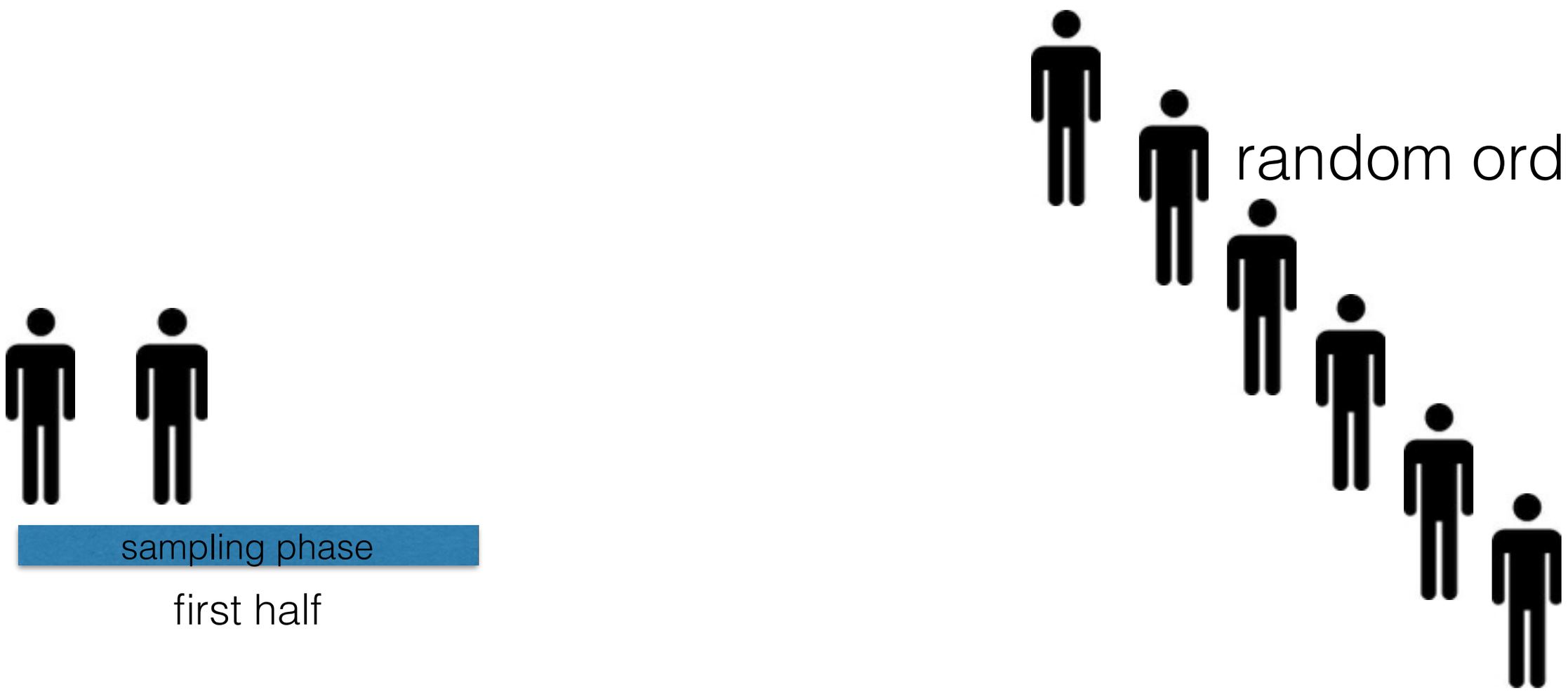

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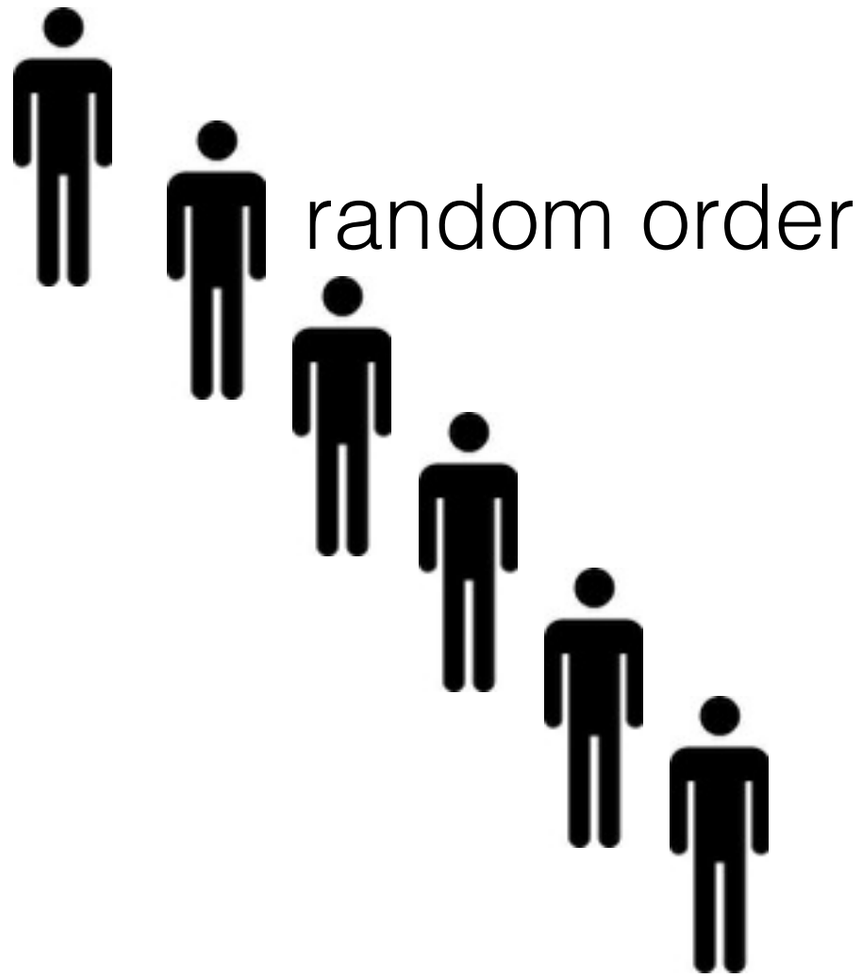

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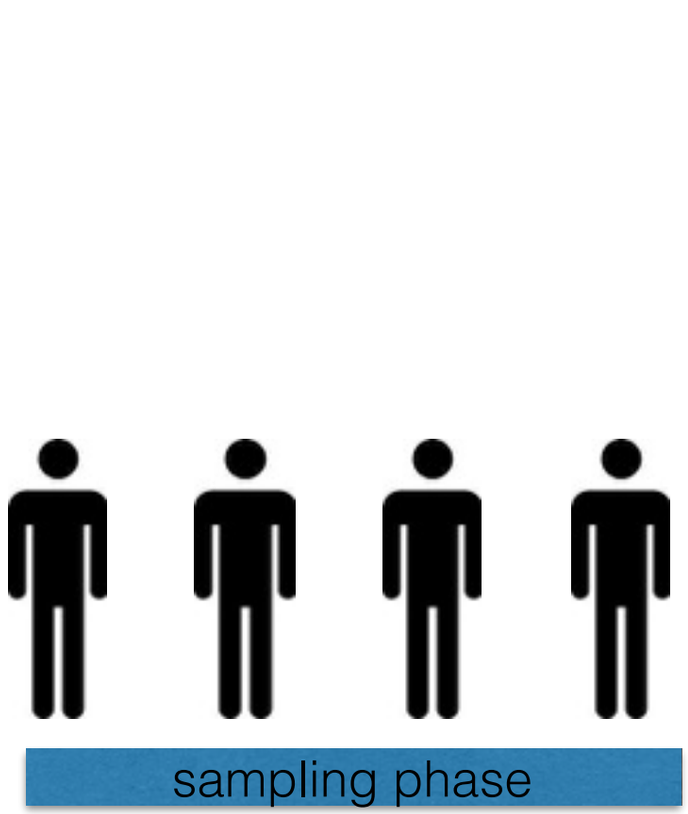

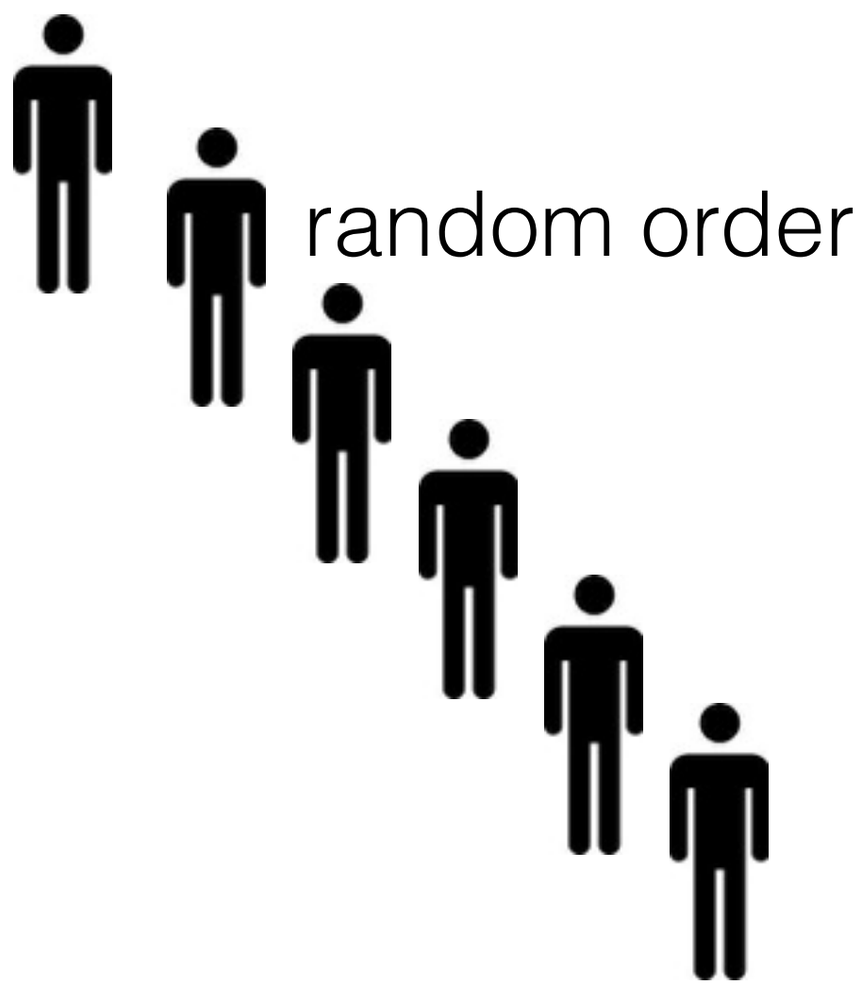

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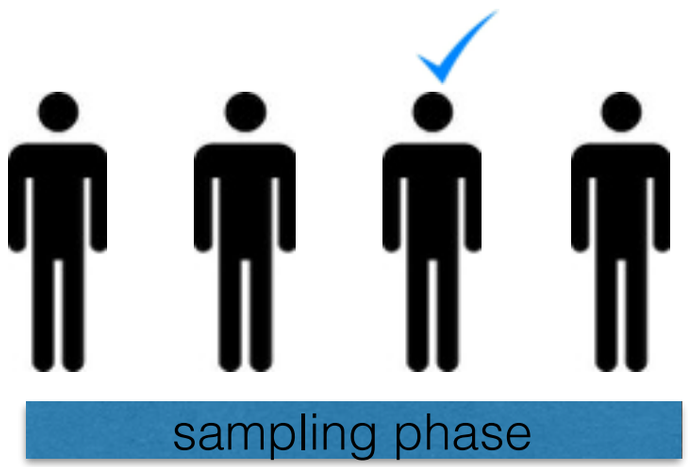

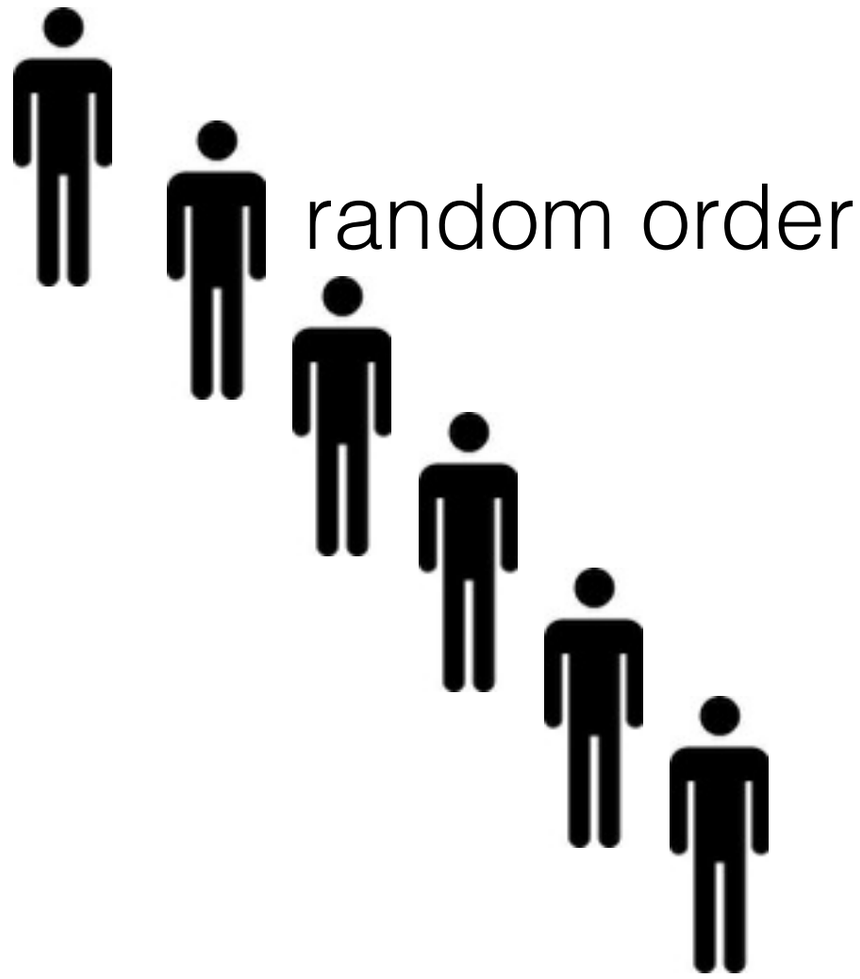

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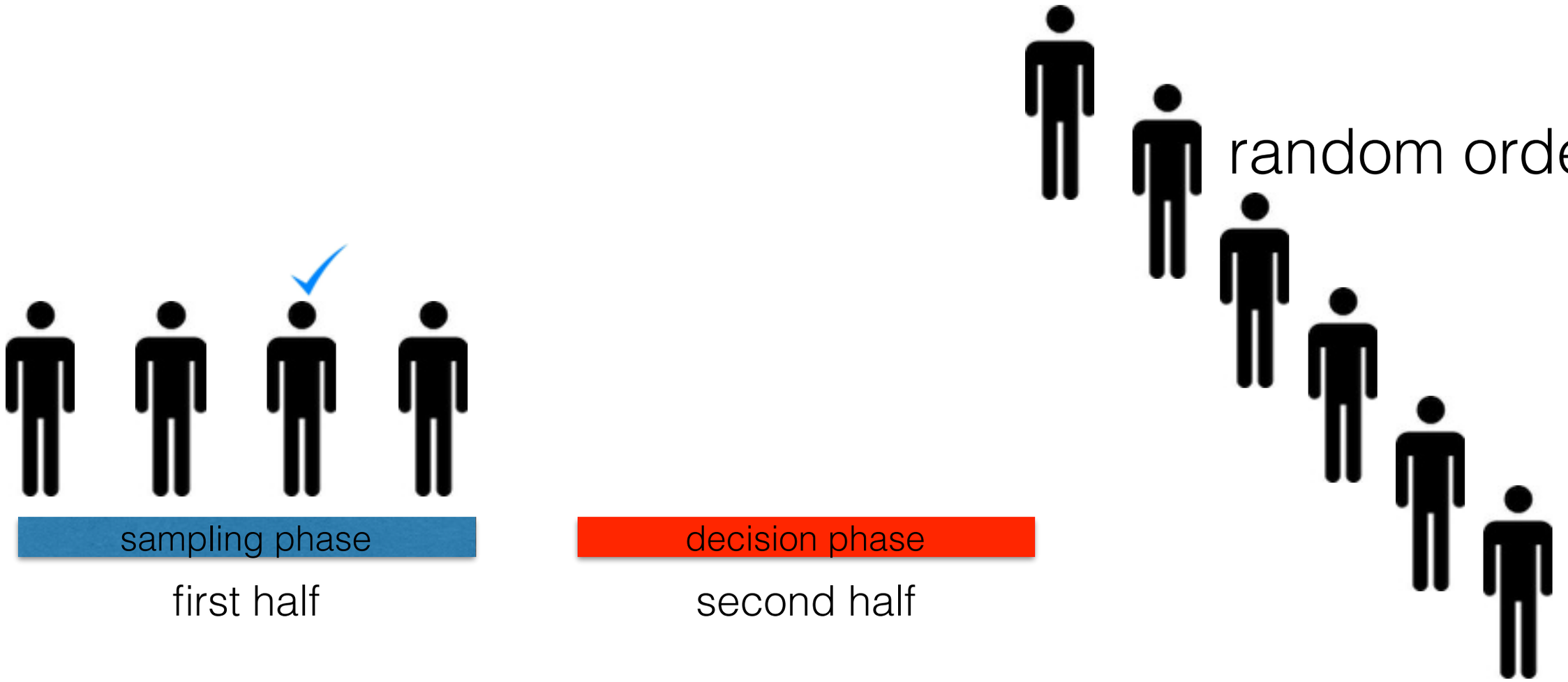

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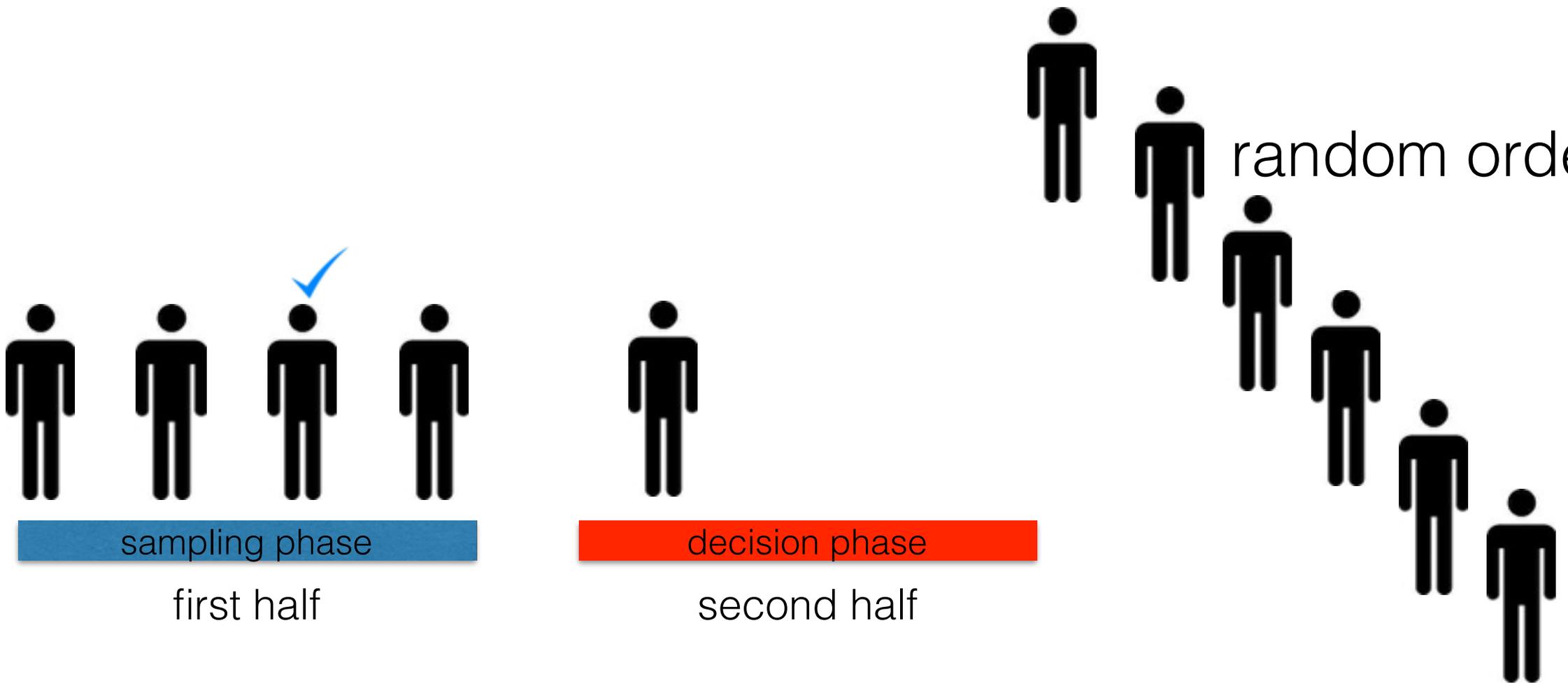

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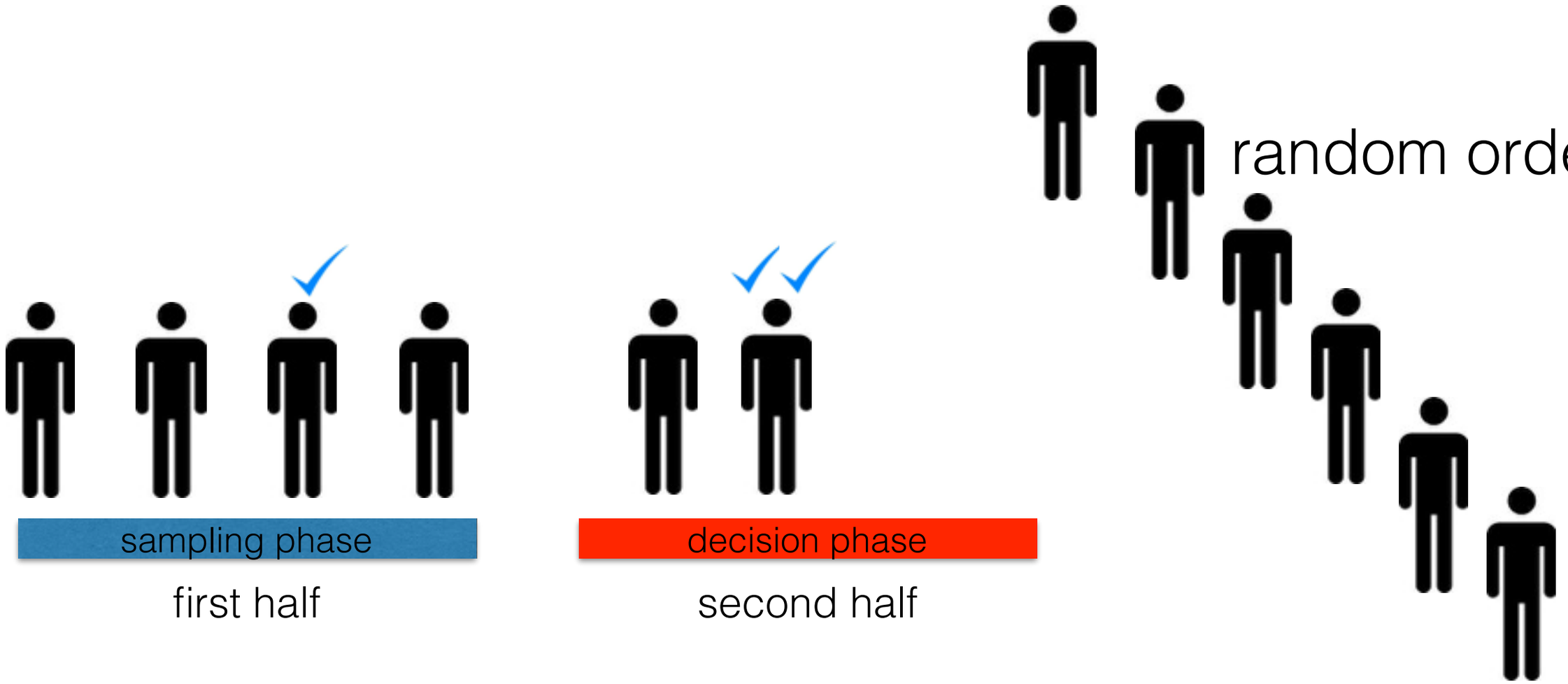
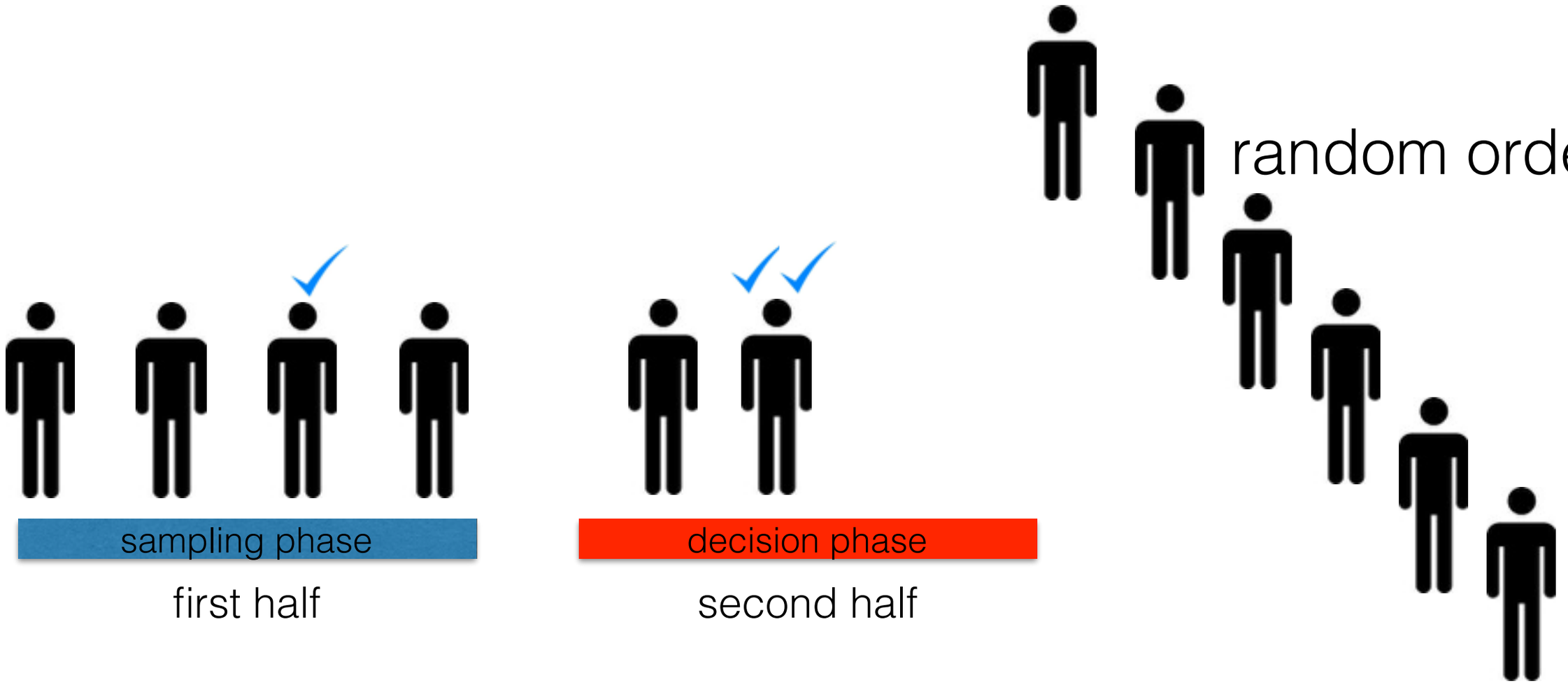


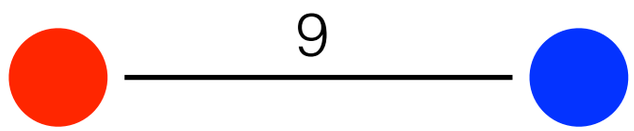
Hiring staff - not adversarial

Success with prob > 1/4



Actually Matching

Actually Matching



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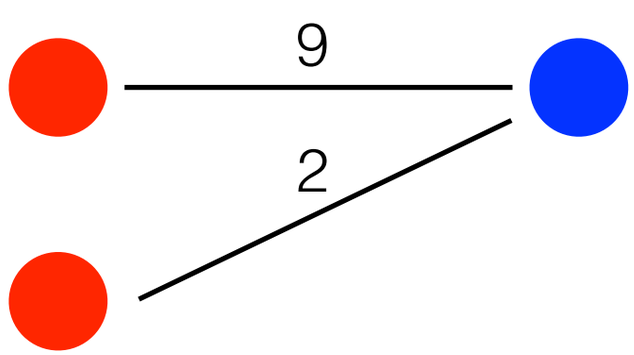

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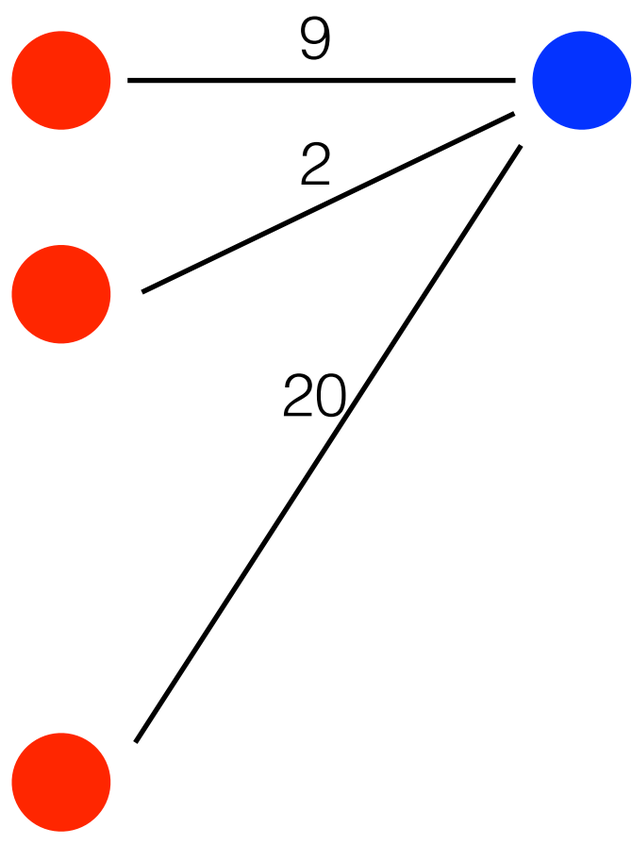
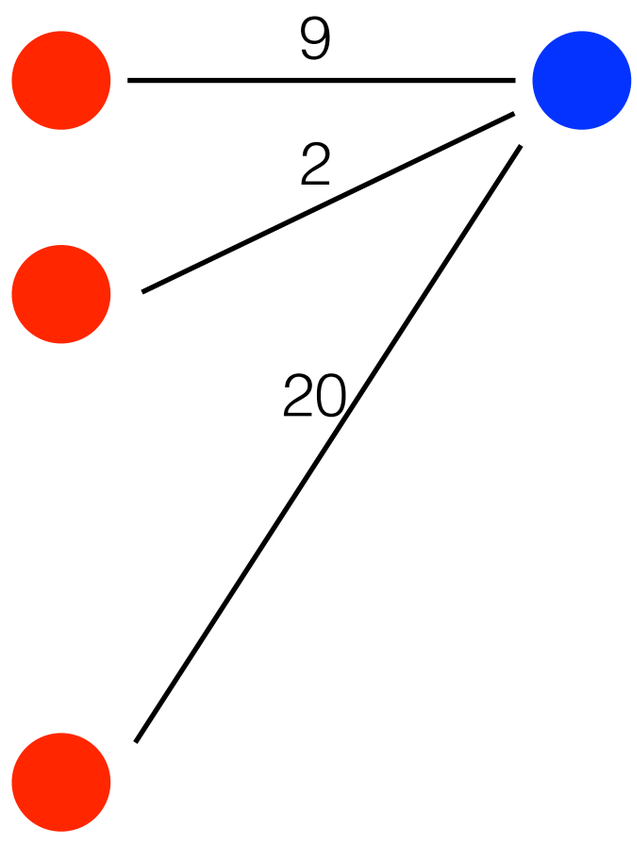
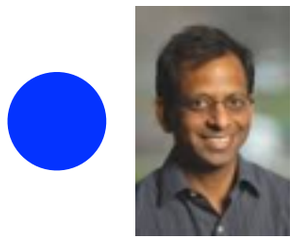


Actually Matching

accept the edge with the largest weight instantaneously



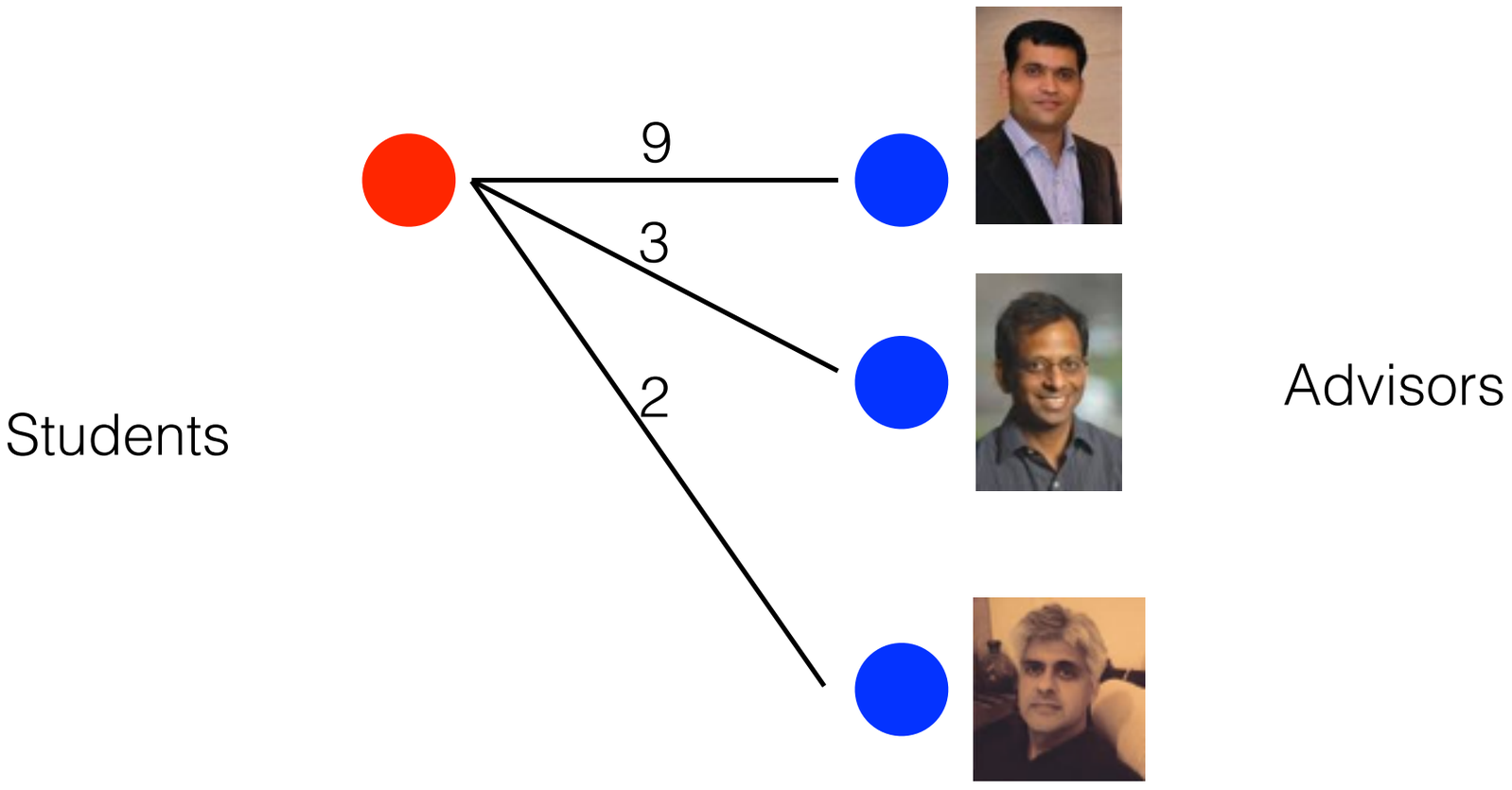
Natural Generalization



Students

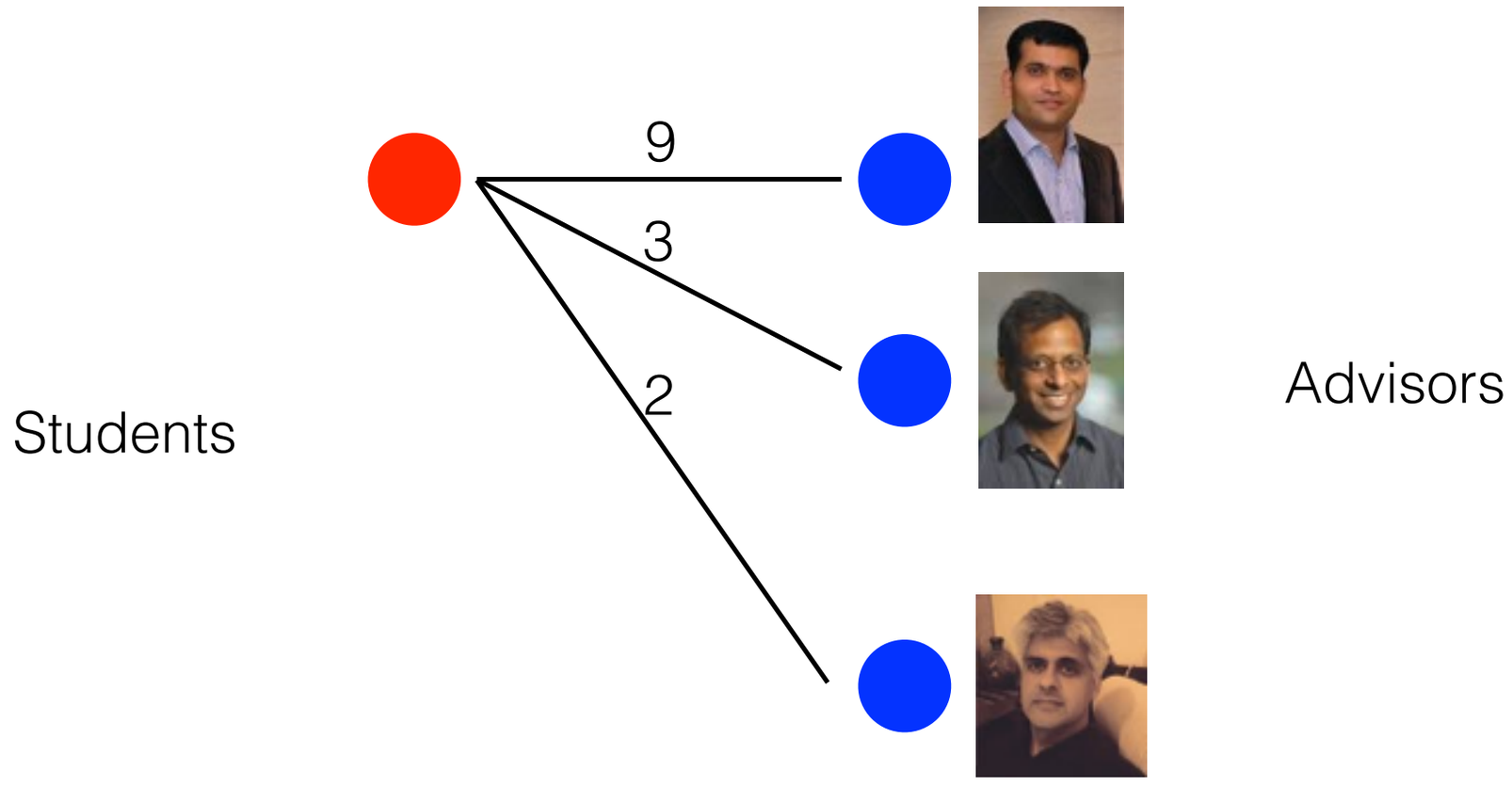
Advisors

Natural Generalization



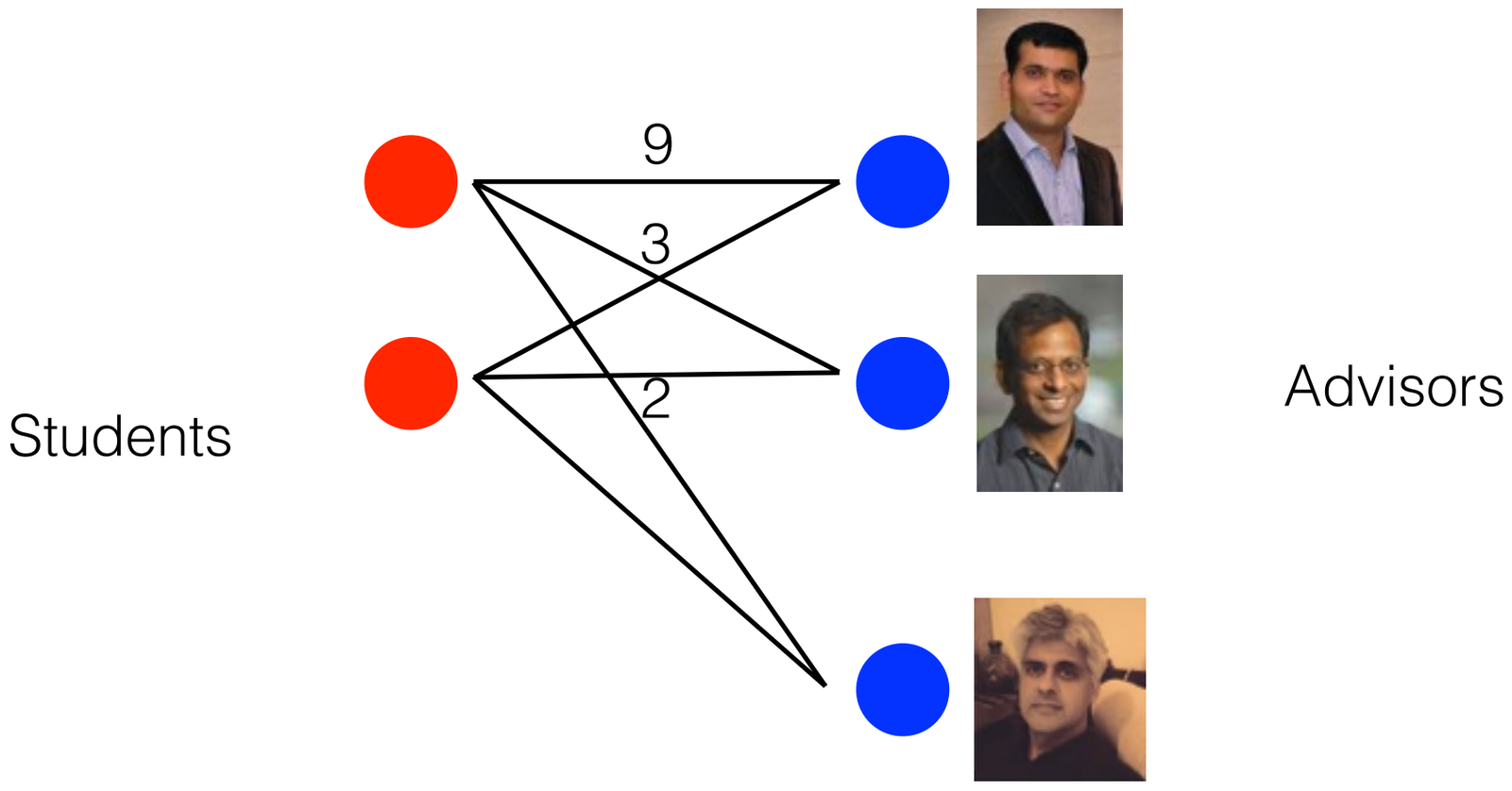
Natural Generalization

each advisor gets at most one student



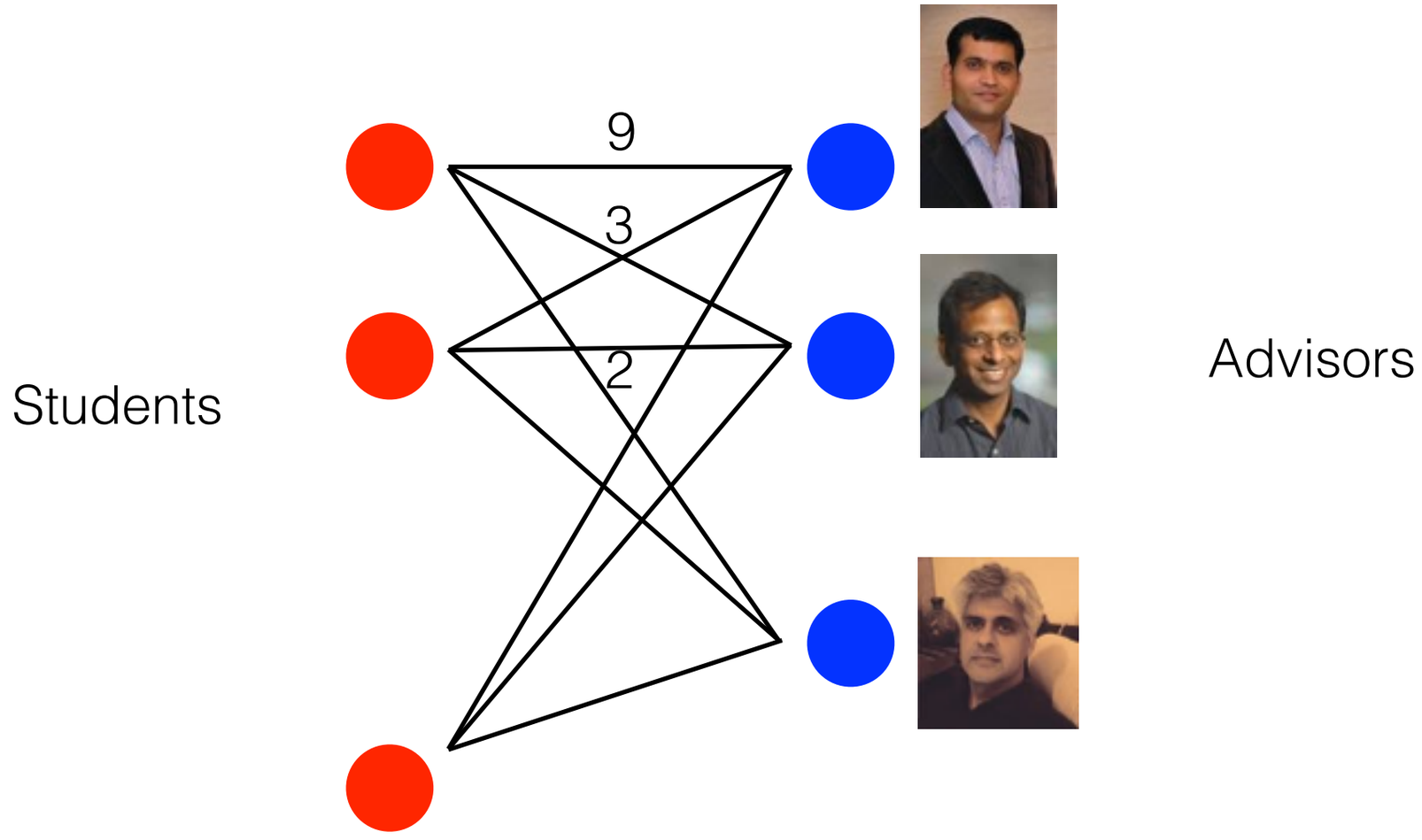
Natural Generalization

each advisor gets at most one student



Natural Generalization

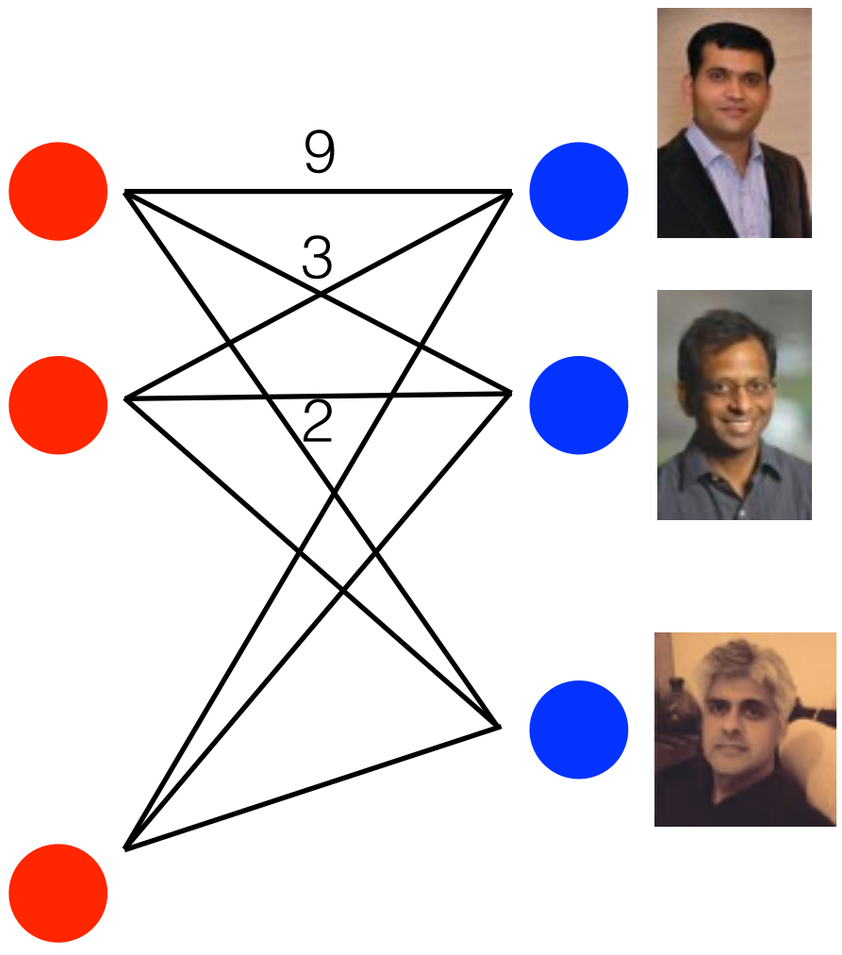
each advisor gets at most one student



Natural Generalization

each advisor gets at most one student - allocation made by

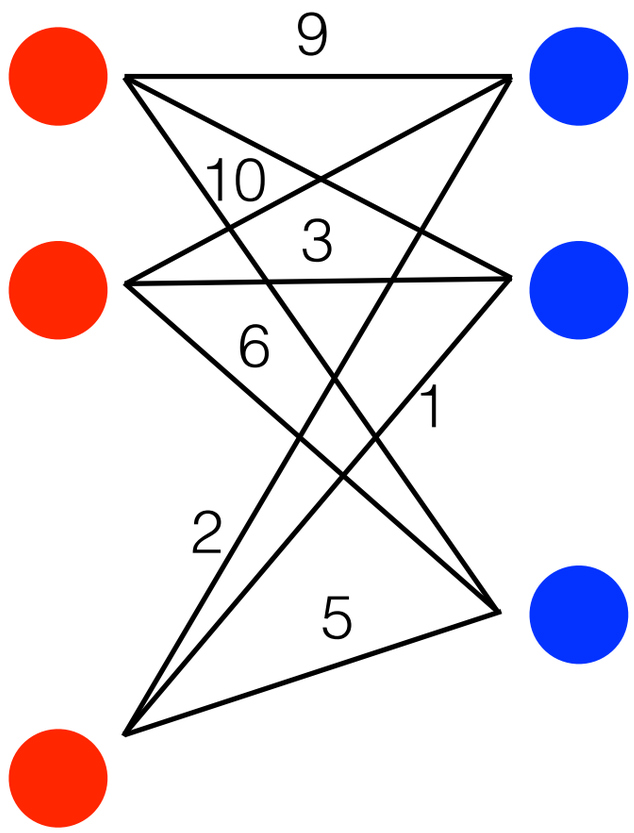
Students



Advisors

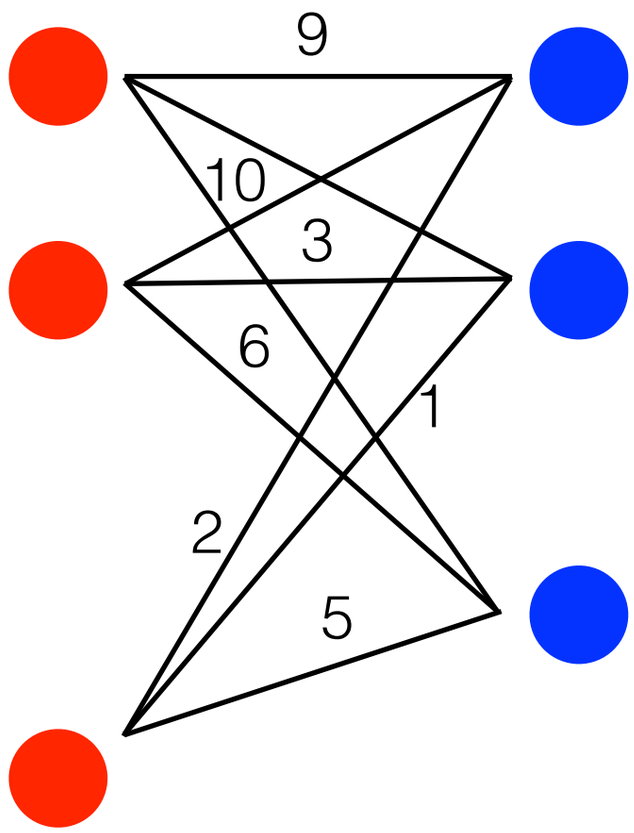
Objective: Matching with largest sum weight

Example



Example

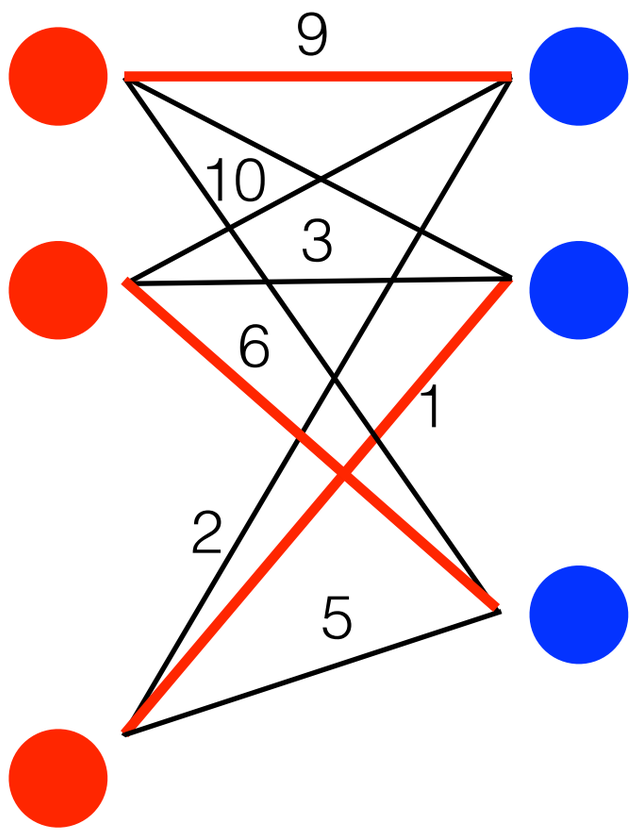
at most one accepted edge



Objective: Matching with largest sum weight

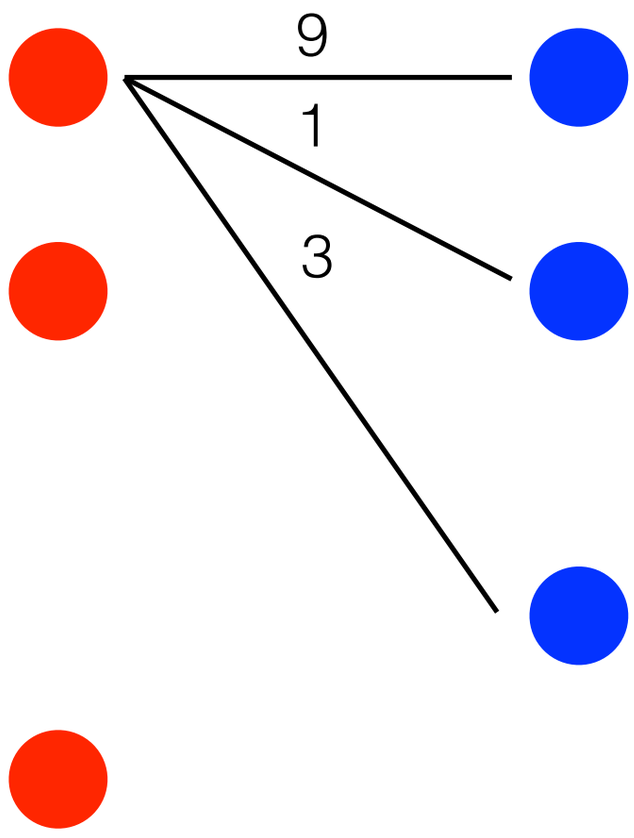
Example

at most one accepted edge

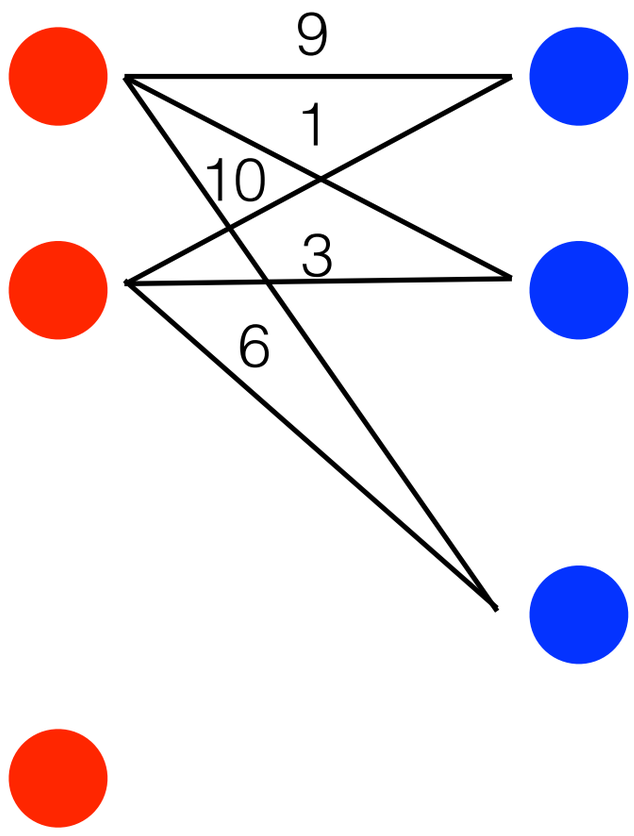


Objective: Matching with largest sum weight

How to solve this ONLINE

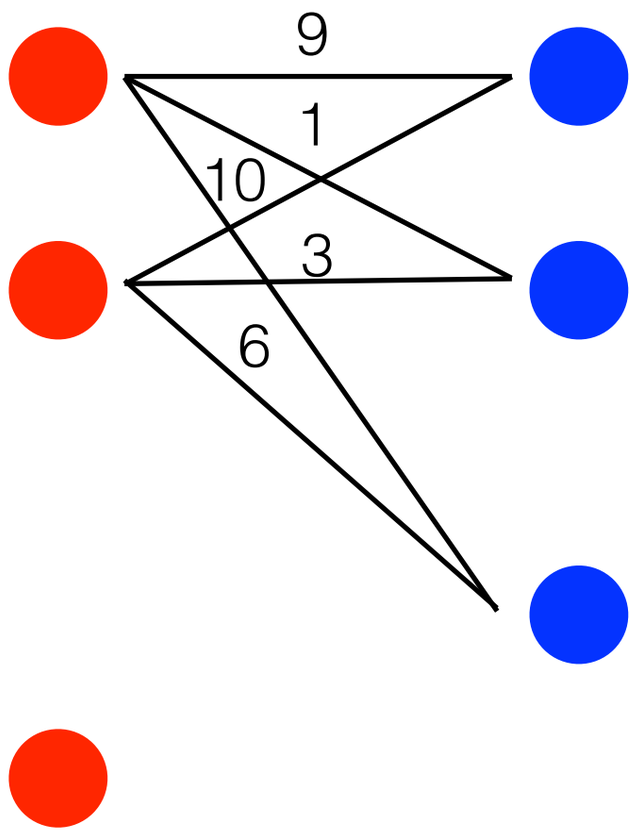


How to solve this ONLINE



How to solve this ONLINE

Sampling idea as before



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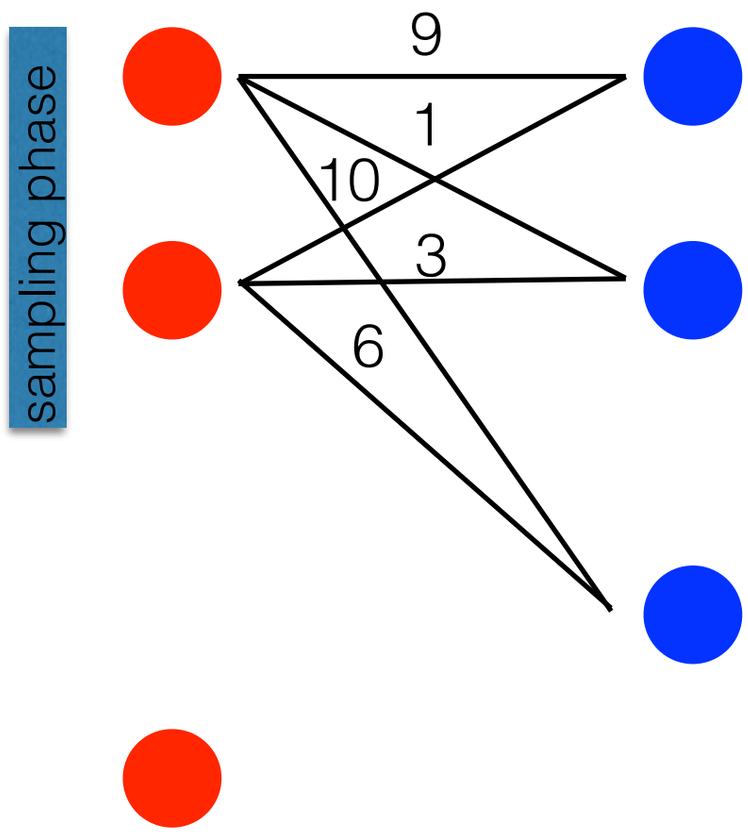
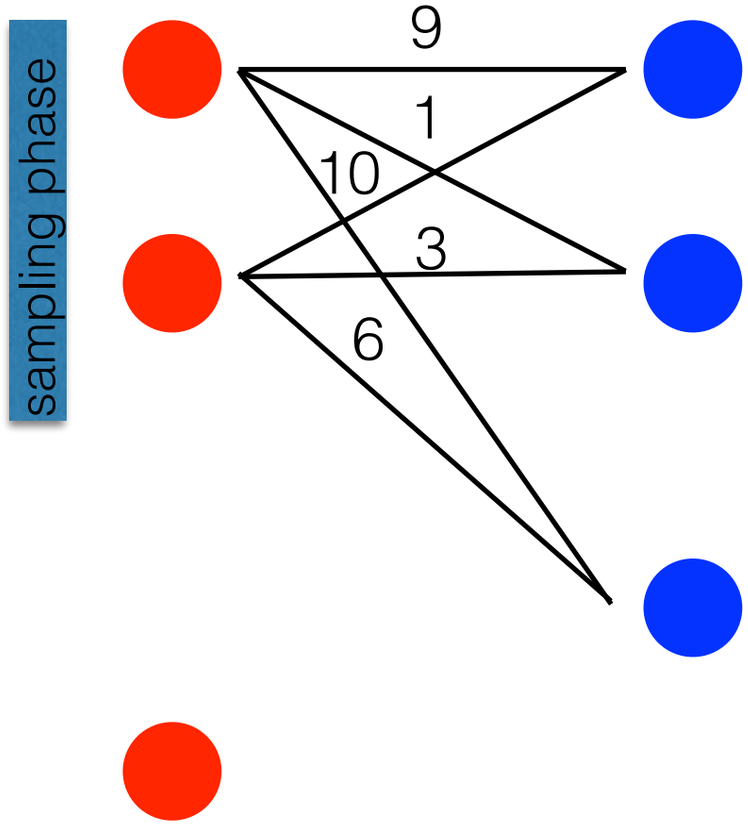


How to solve this ONLINE

Sampling idea as before

Find best matching



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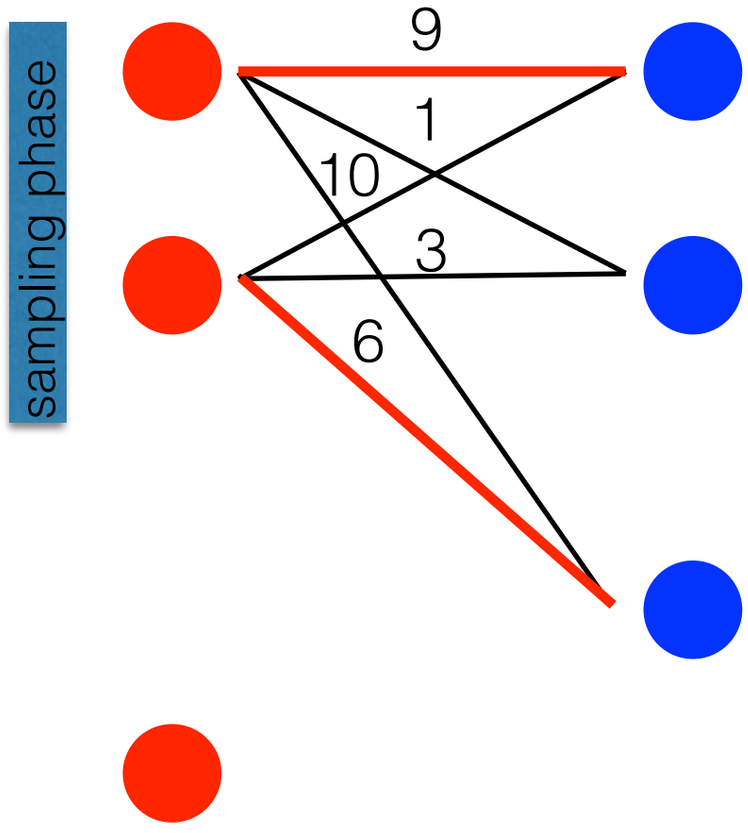
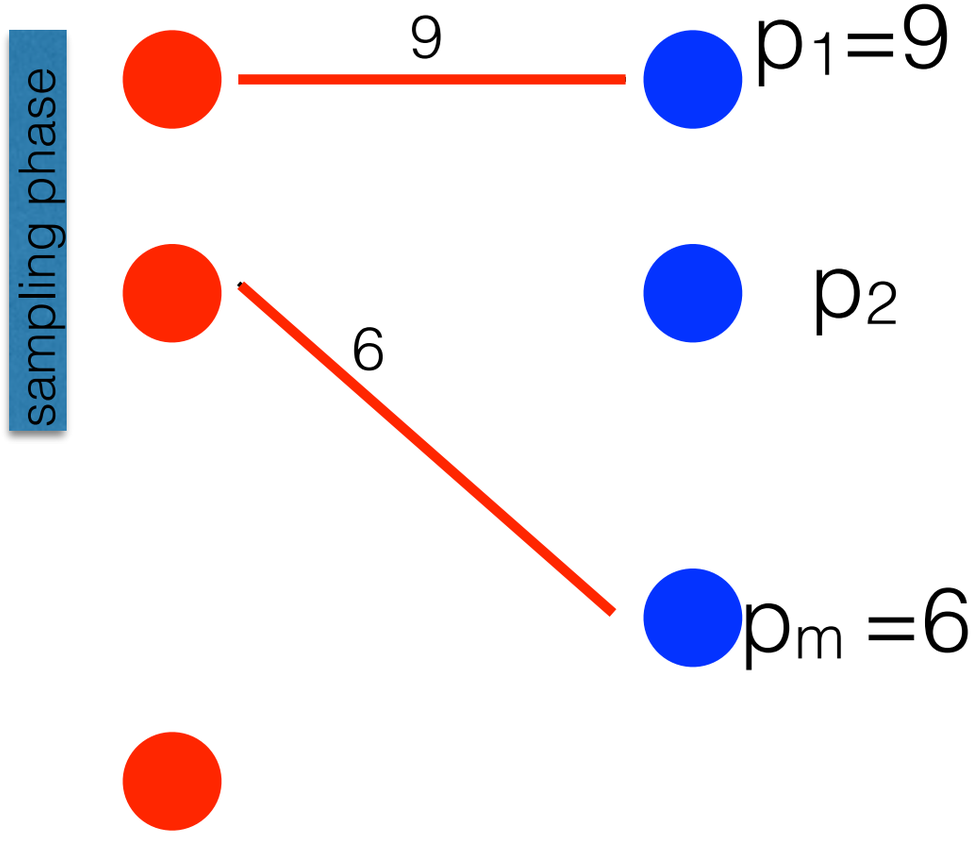


How to solve this ONLINE

Sampling idea as before

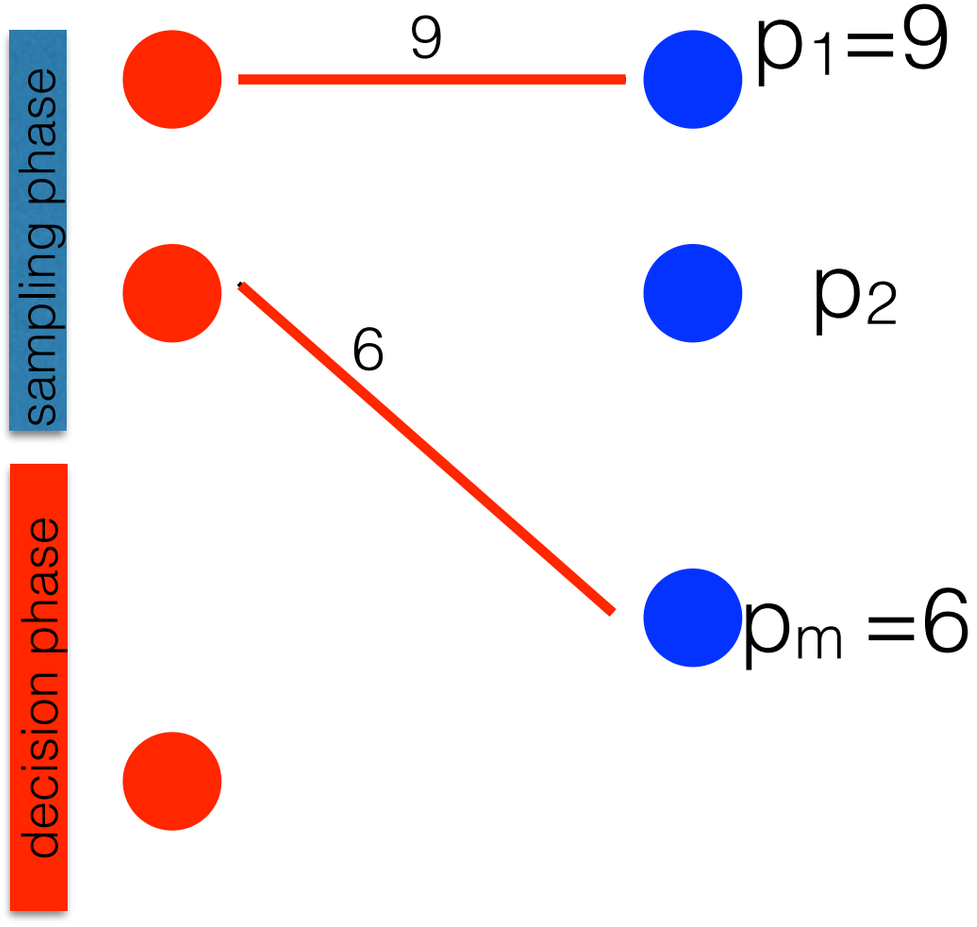
Find best matching



How to solve this ONLINE

Sampling idea as before

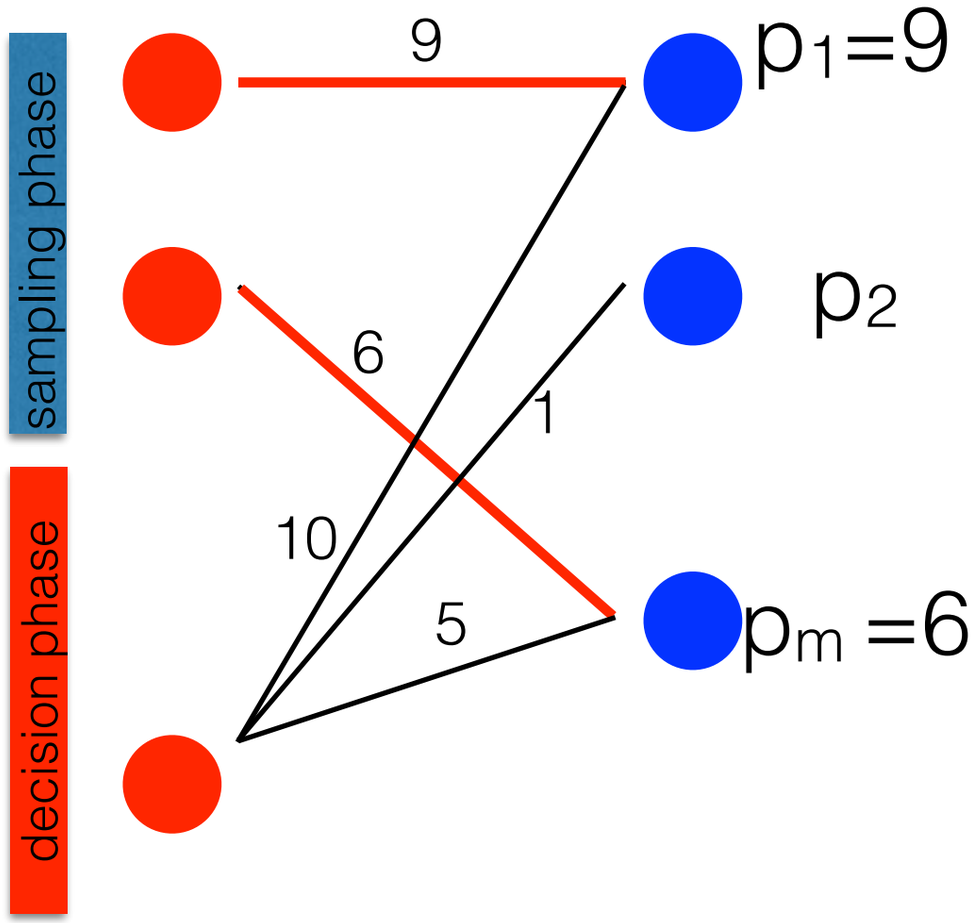
Find best matching



How to solve this ONLINE

Sampling idea as before

Find best matching

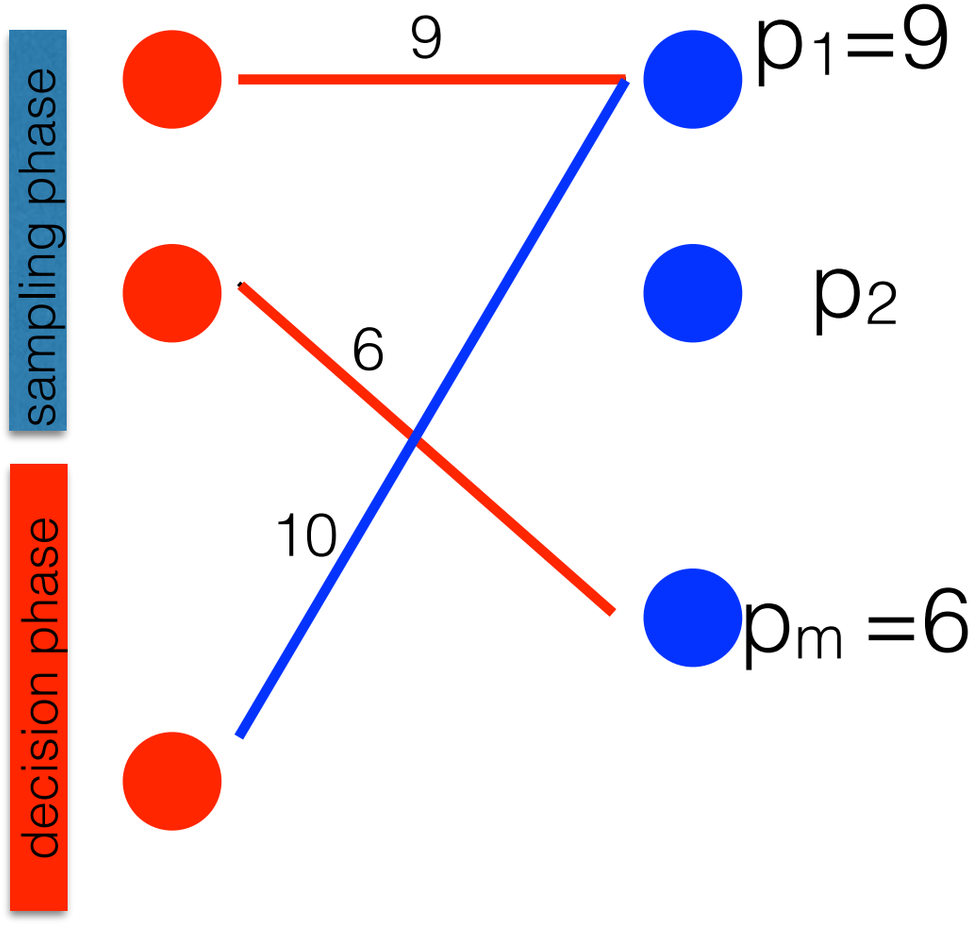


How to solve this ONLINE

Sampling idea as before

Find best matching

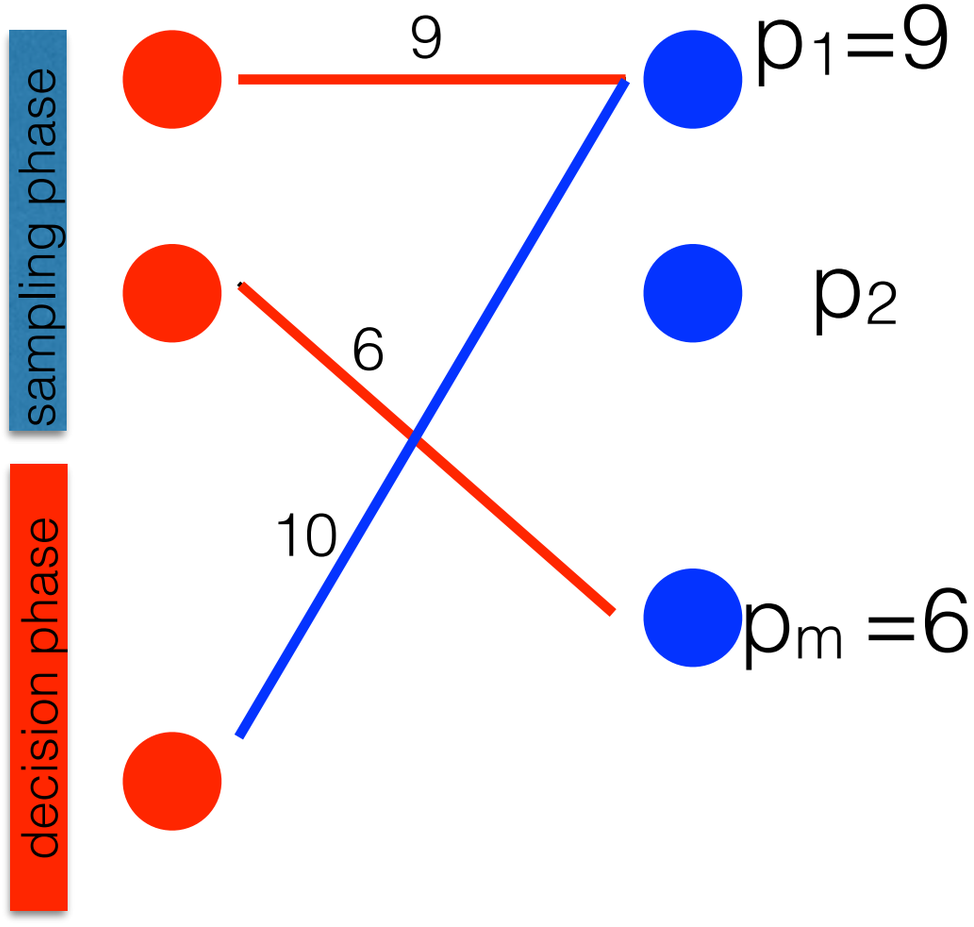
match the largest weight more than price



How to solve this ONLINE

Sampling idea as before

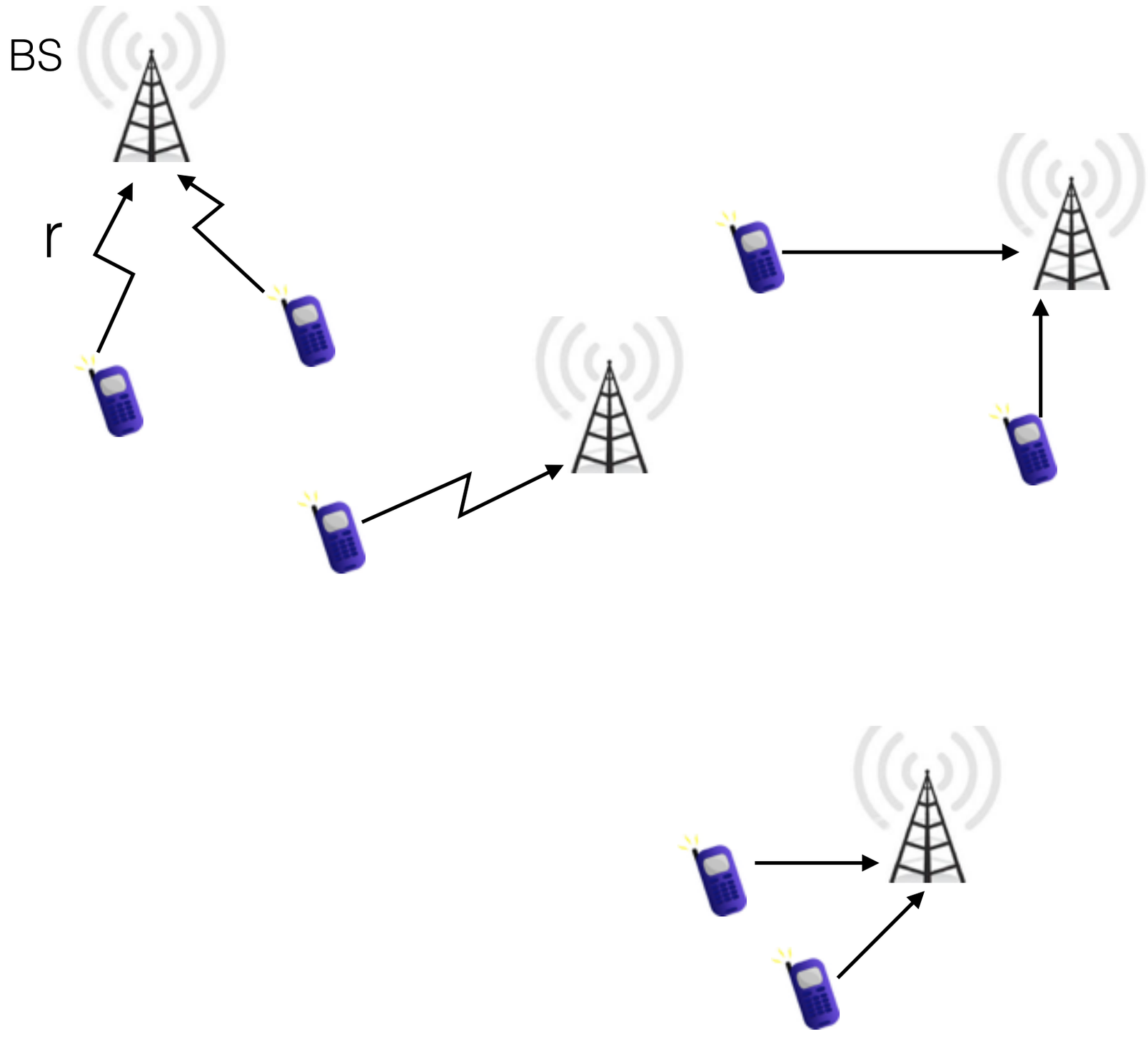
Find best matching



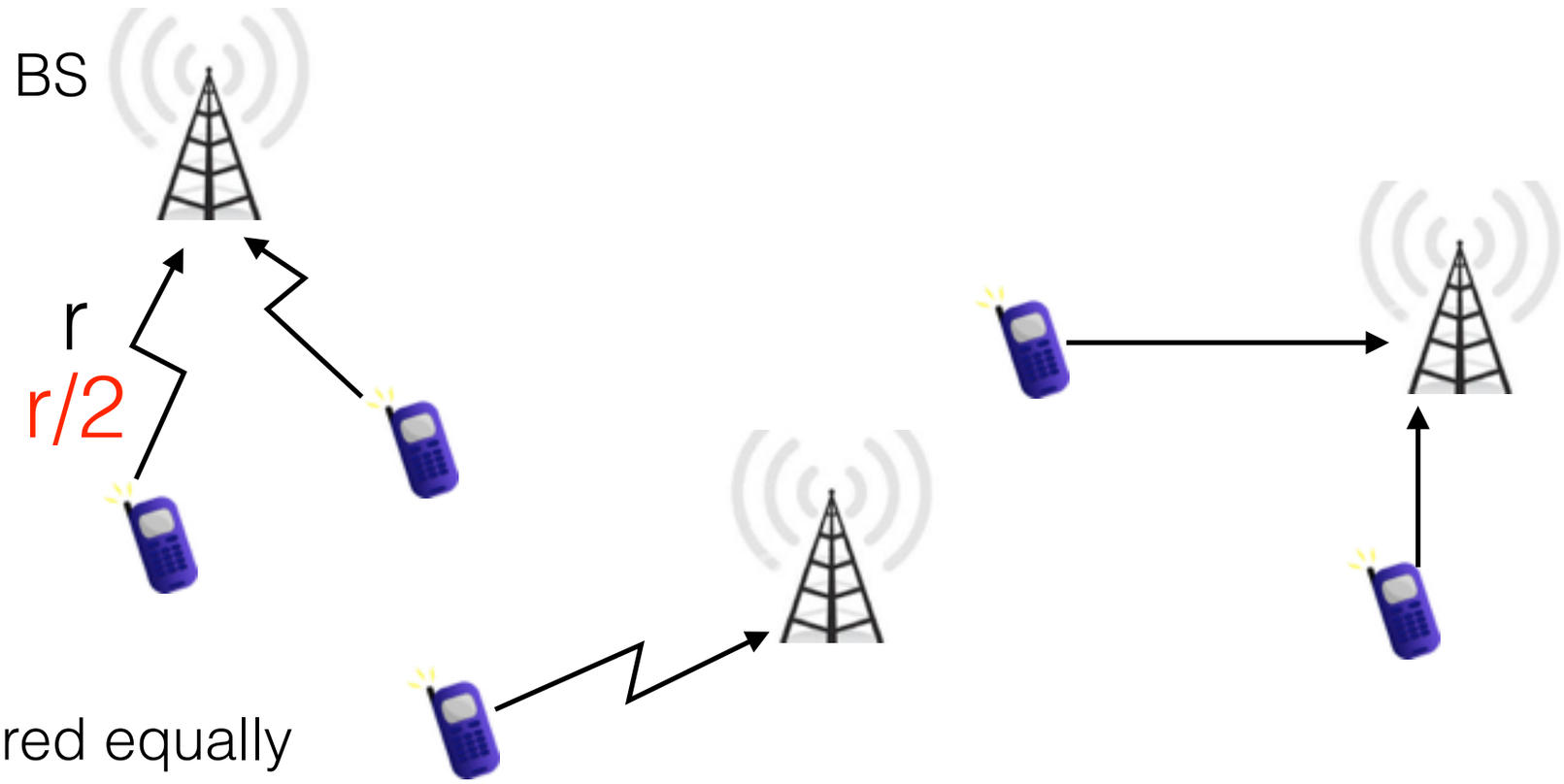
match the largest weight more than price

Result: 8-competive/optimal [*Korula, Pal' 08***]**

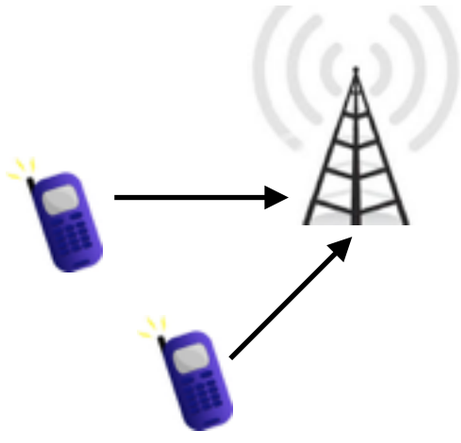
Legacy Problem -Wireless Communication



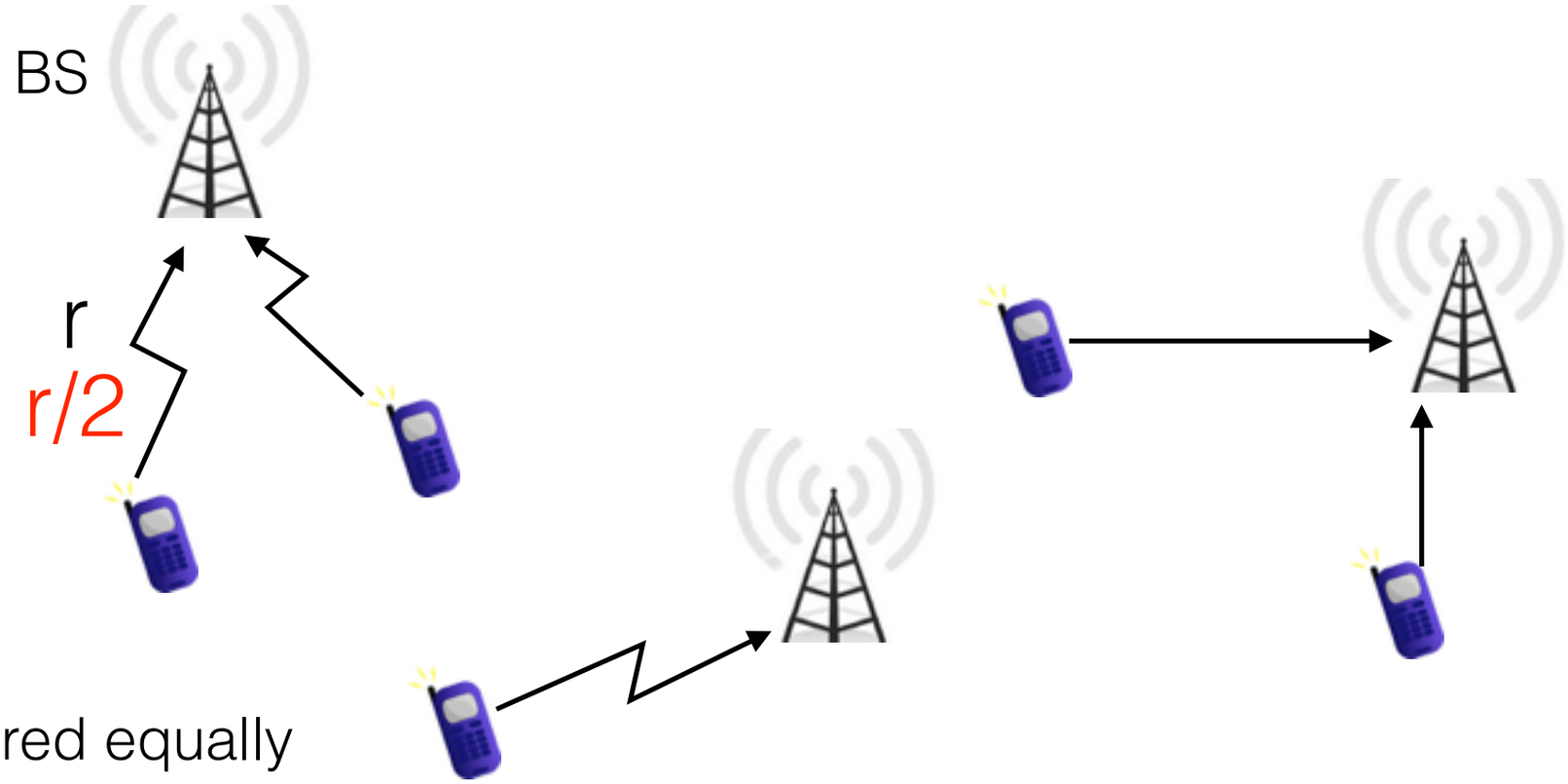
Legacy Problem -Wireless Communication



Resources shared equally

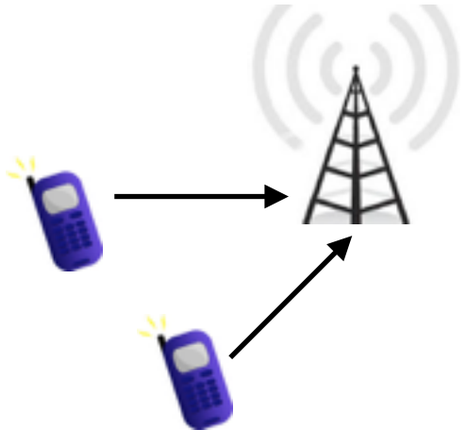


Legacy Problem -Wireless Communication

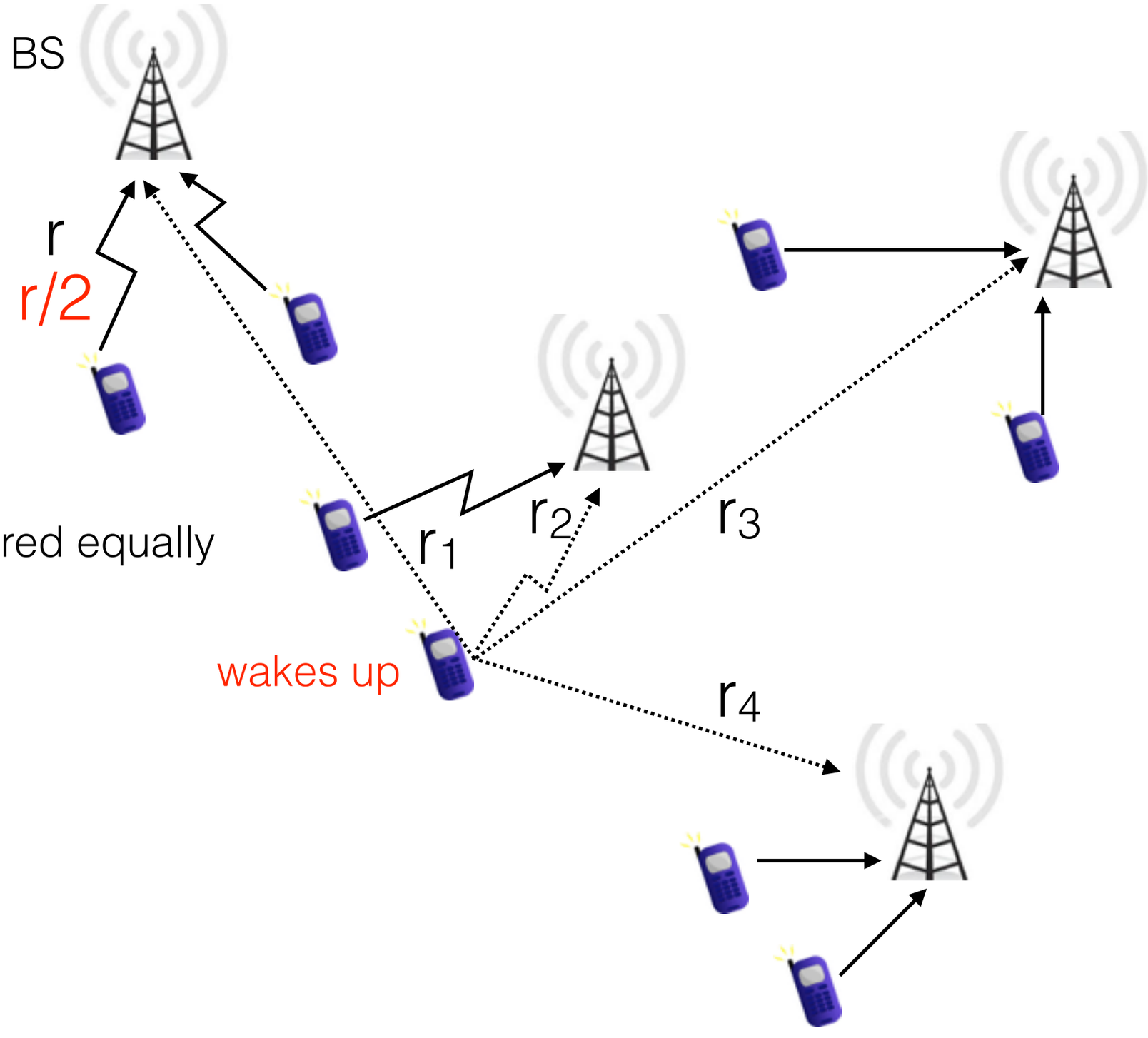


Resources shared equally

wakes up

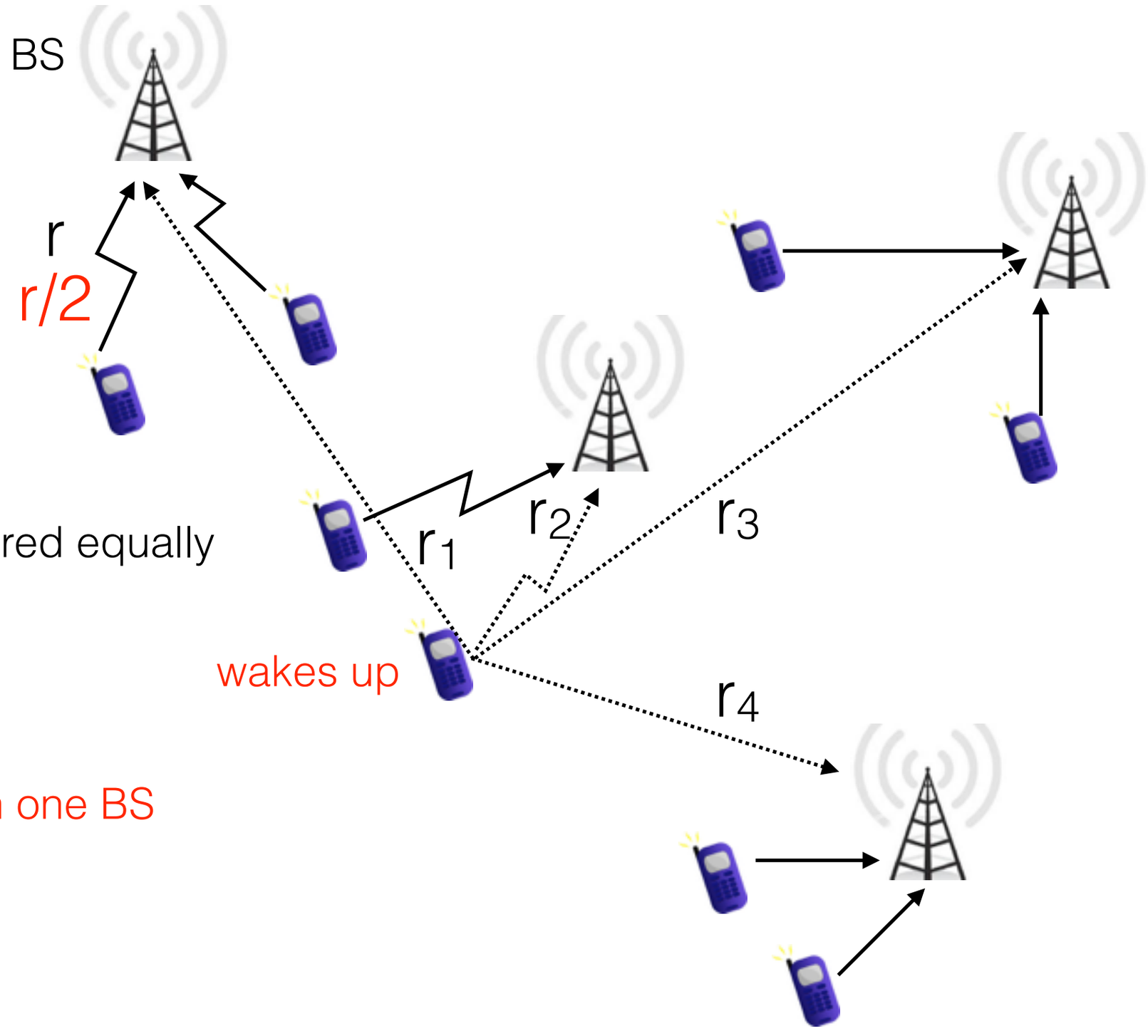


Legacy Problem -Wireless Communication



Resources shared equally

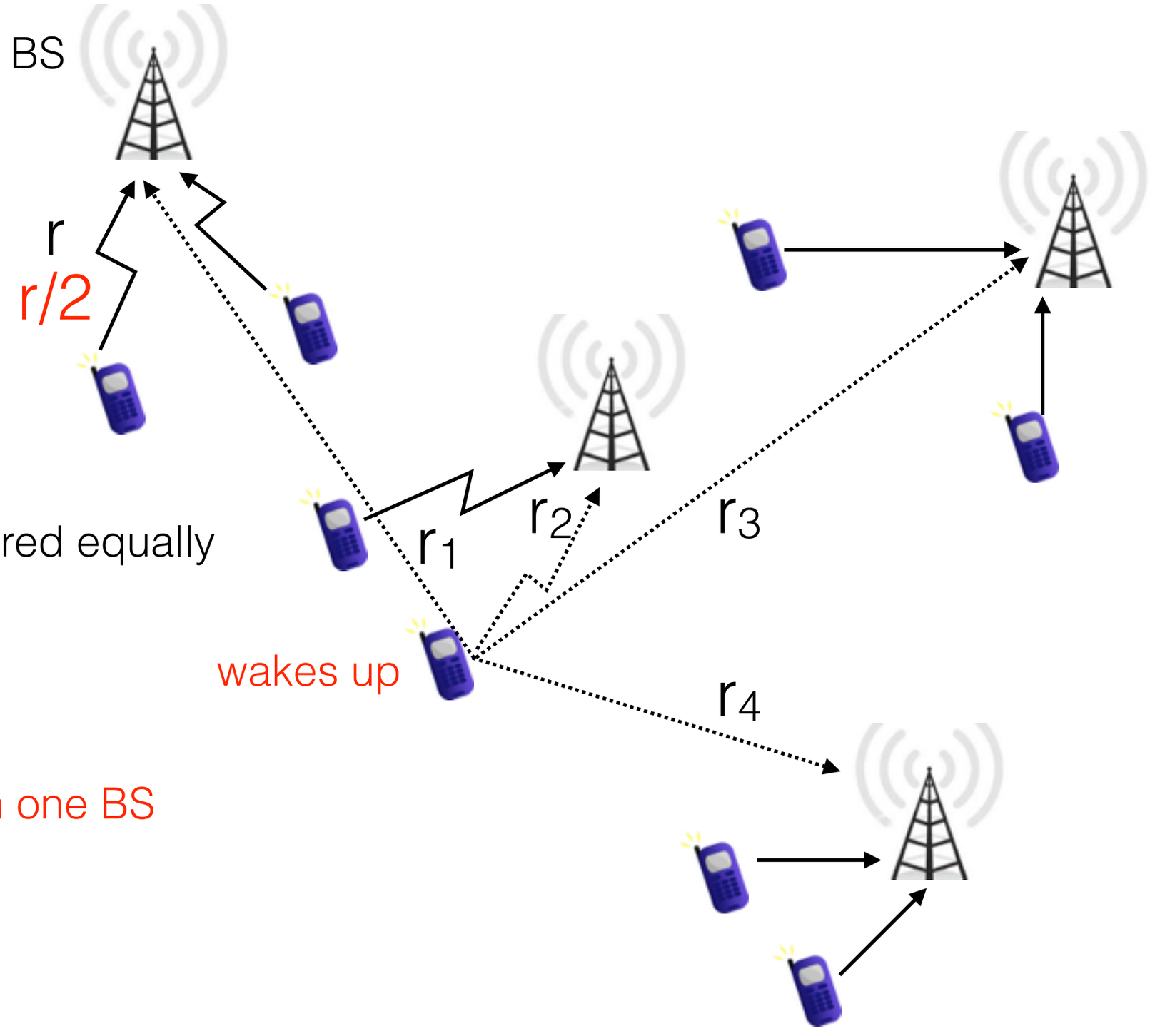
Legacy Problem -Wireless Communication



Resources shared equally

Associate with one BS

Legacy Problem -Wireless Communication

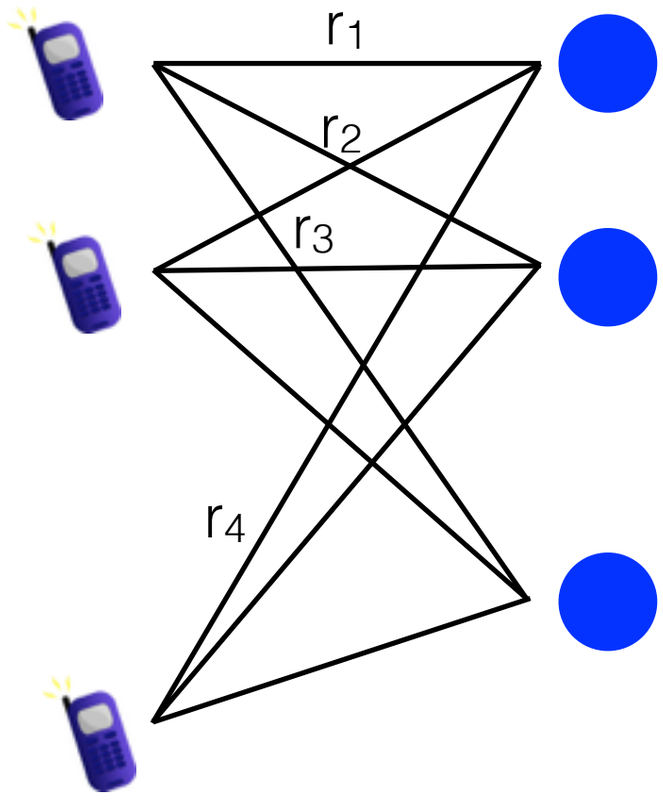


Resources shared equally

Associate with one BS

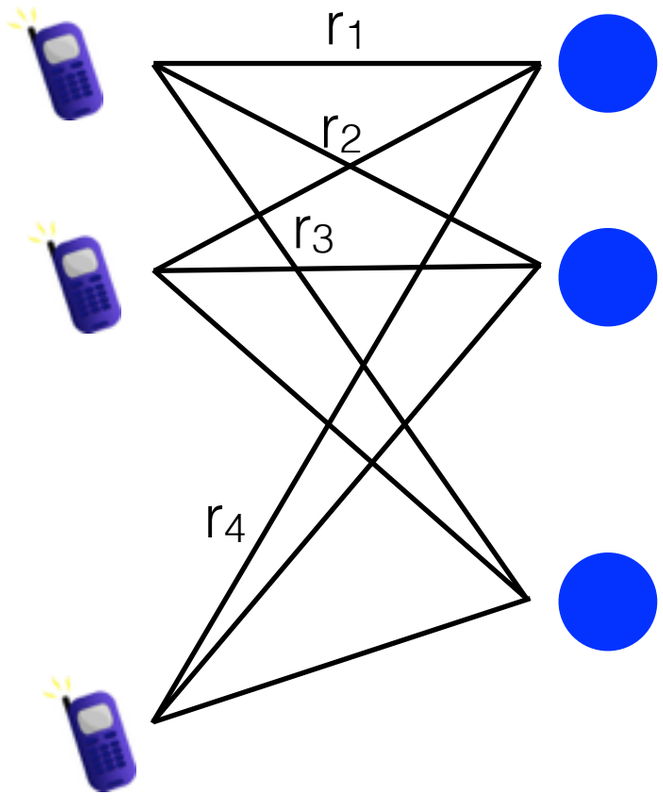
Find optimal BS allocation to maximize sum-rate

Example



Example

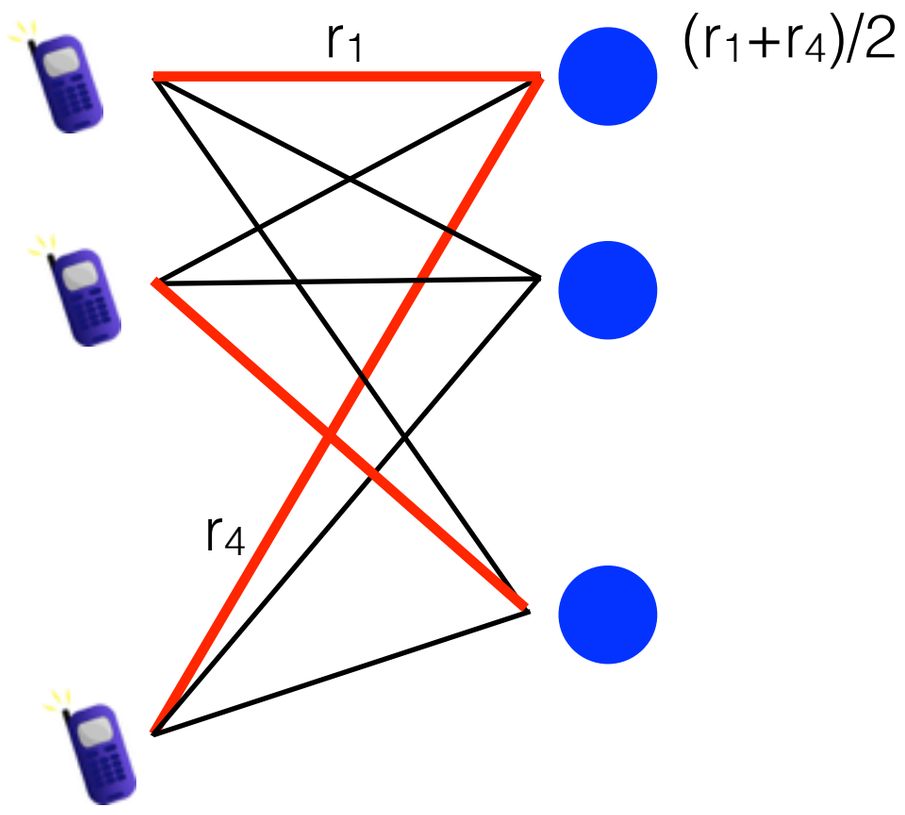
Still Interested in largest sum-weight but **No** longer MATCHING



Objective: Association with largest sum weight

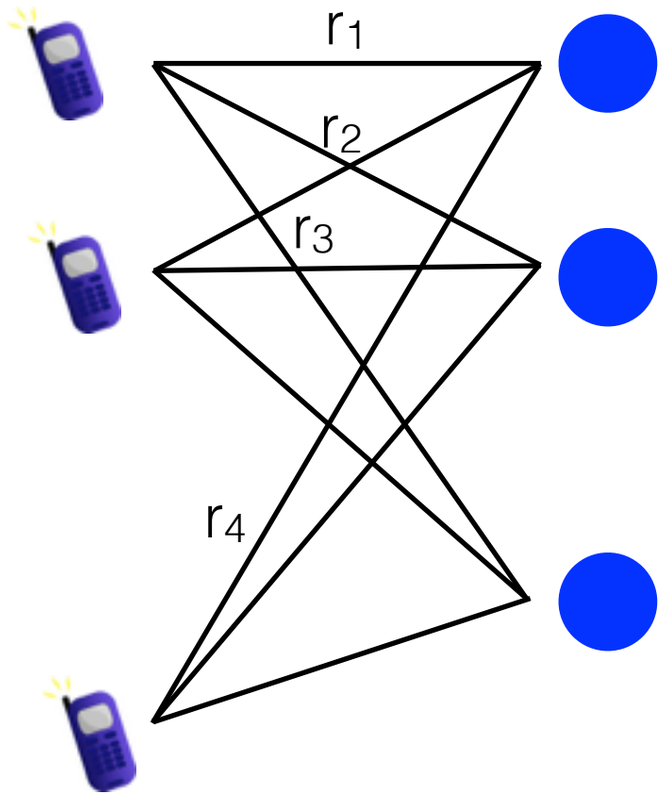
Example

Still Interested in largest sum-weight but **No** longer MATCHING



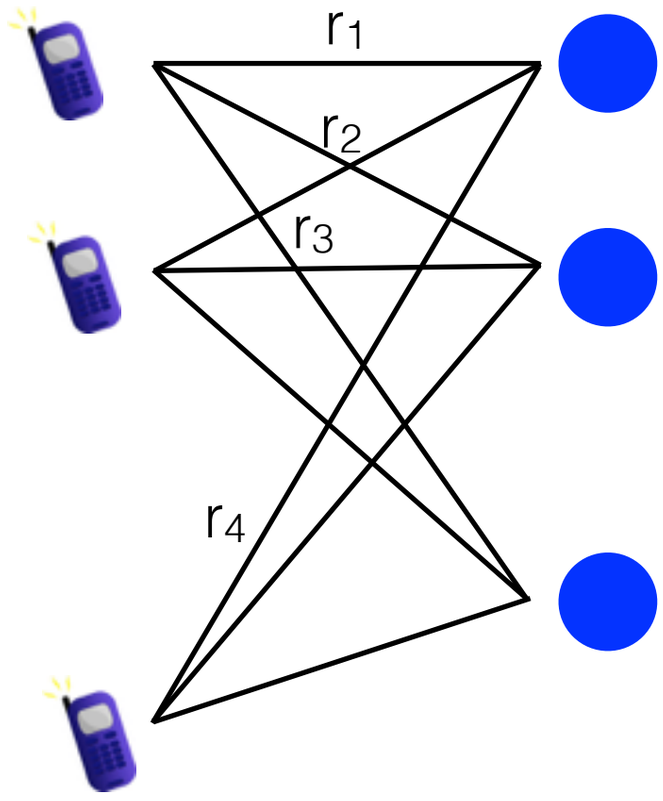
Objective: Association with largest sum weight

Important Observation



Important Observation

Note that sum-weight is still dominated by Max-Weight with MATCHING



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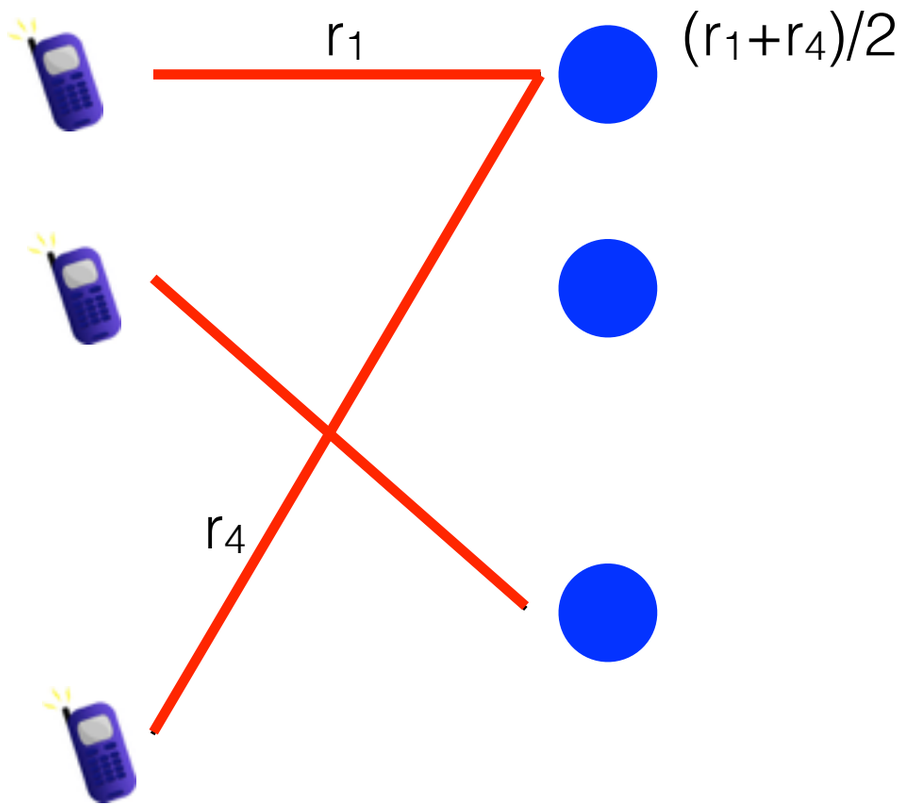

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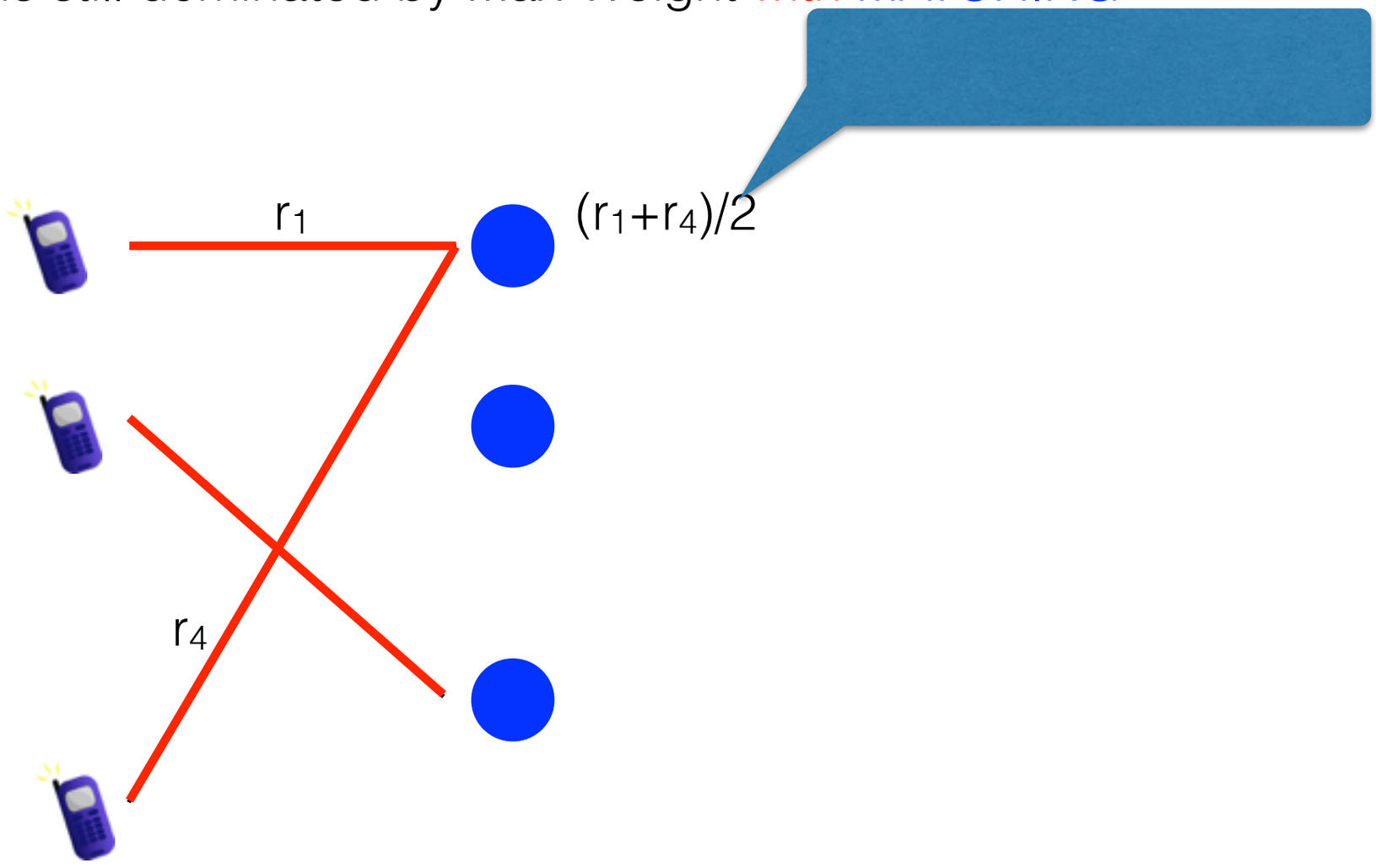

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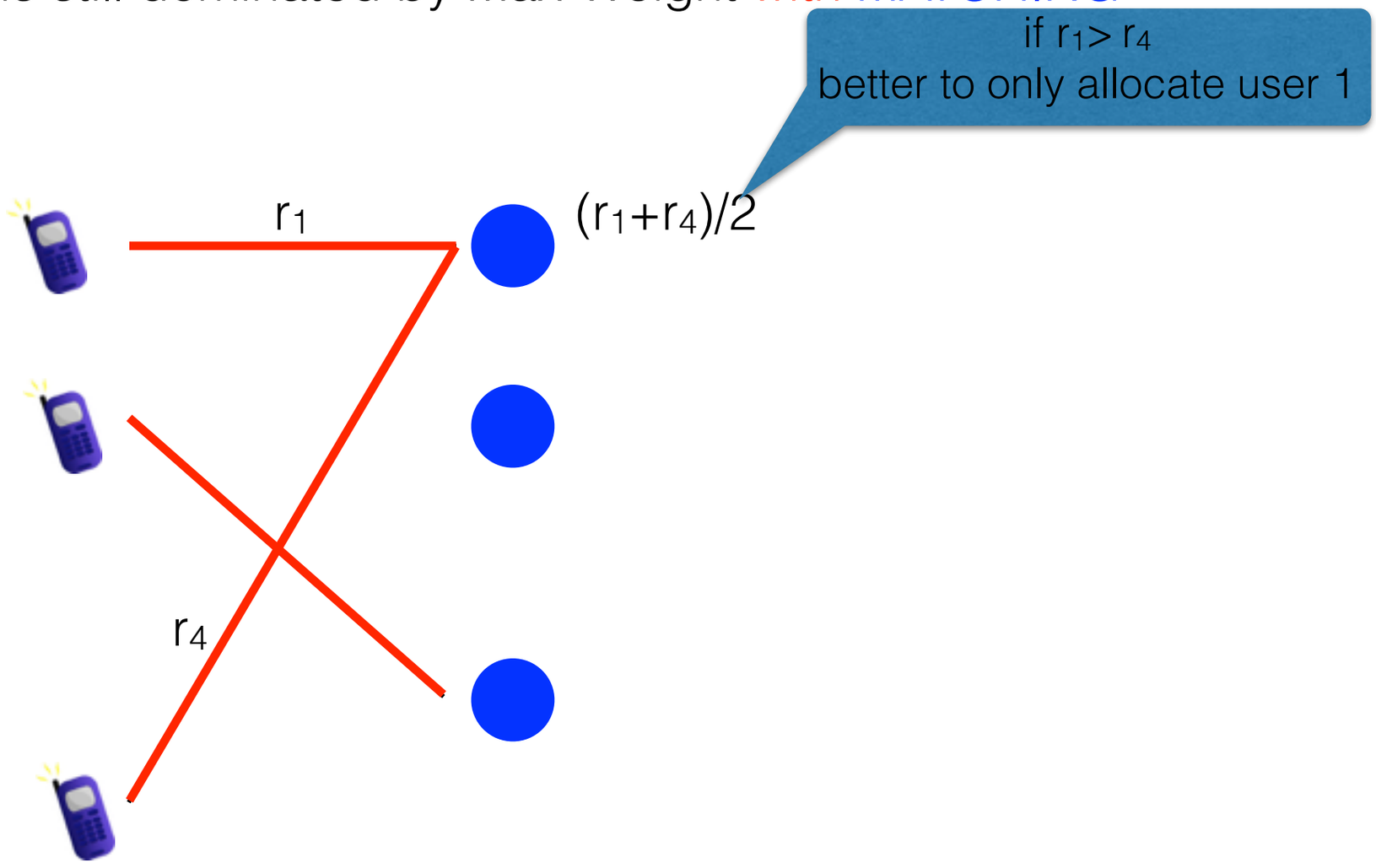

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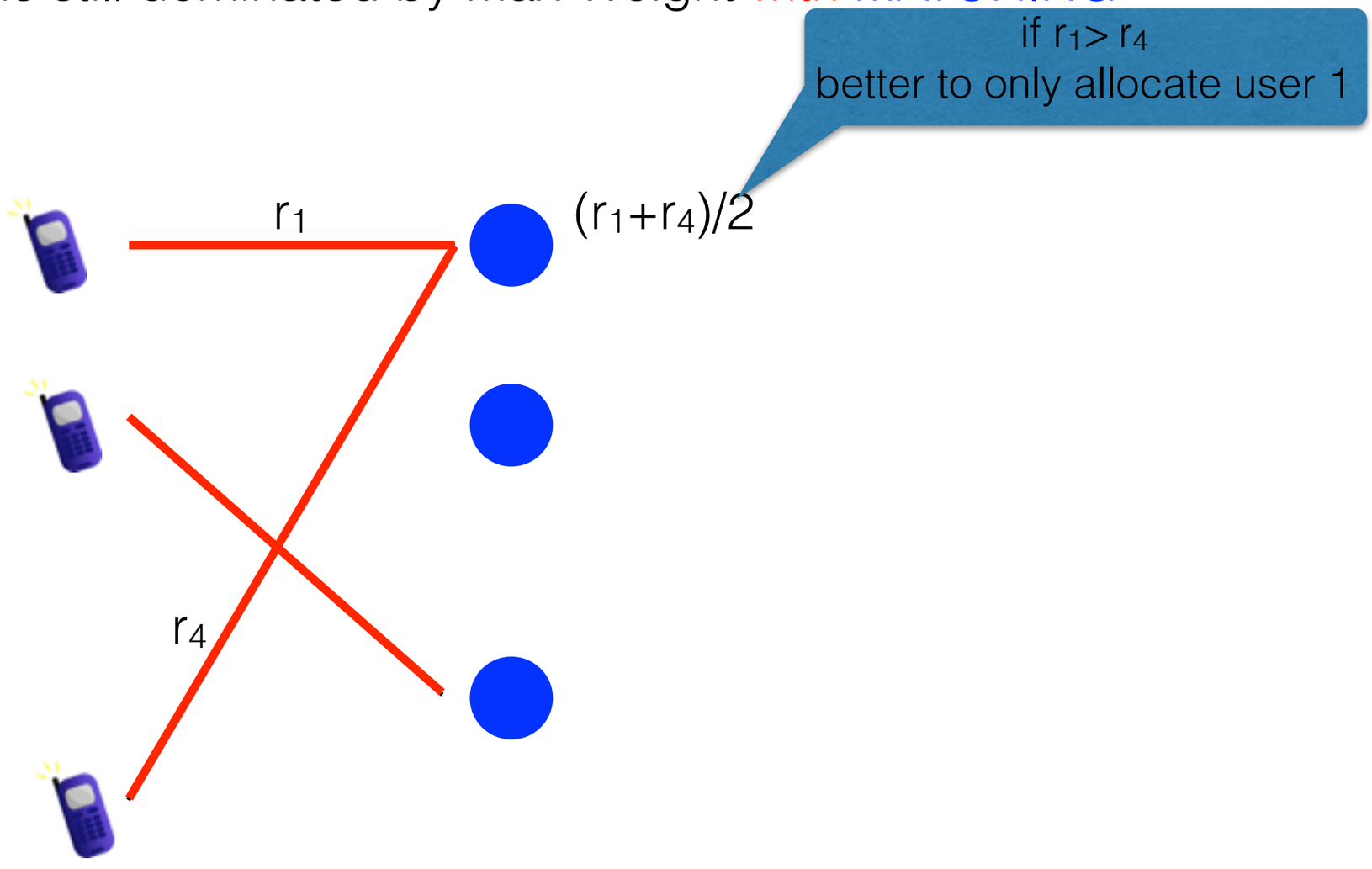
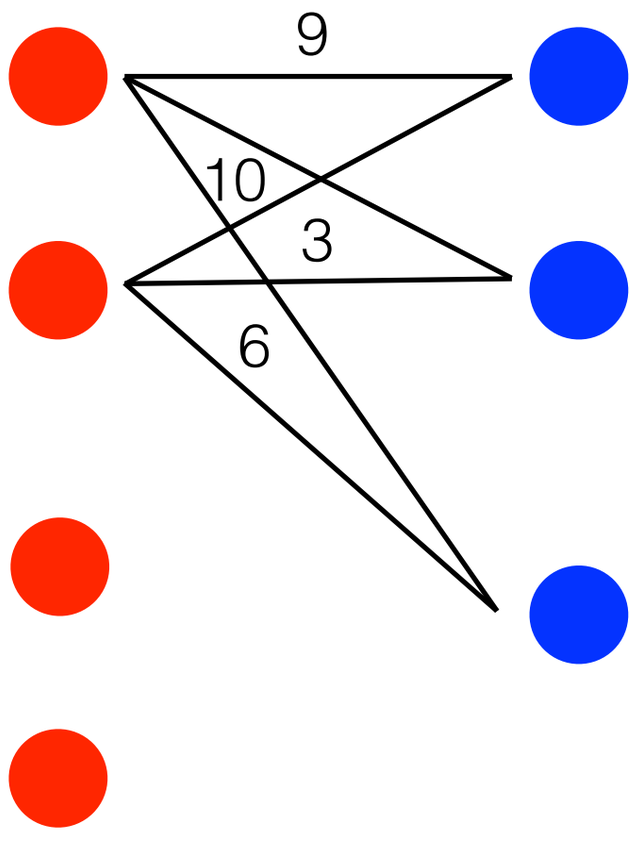


Upper Bound : Max-Weight with **Matching**

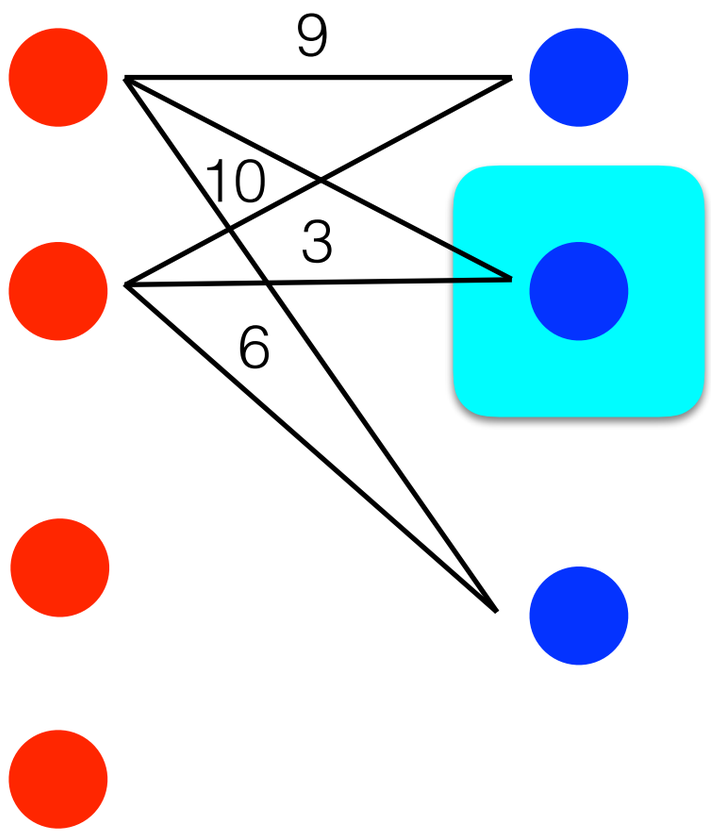
How to solve this ONLINE



How to solve this ONLINE

Sampling idea as before

Choose one BS randomly and associate all users rejected by Matching



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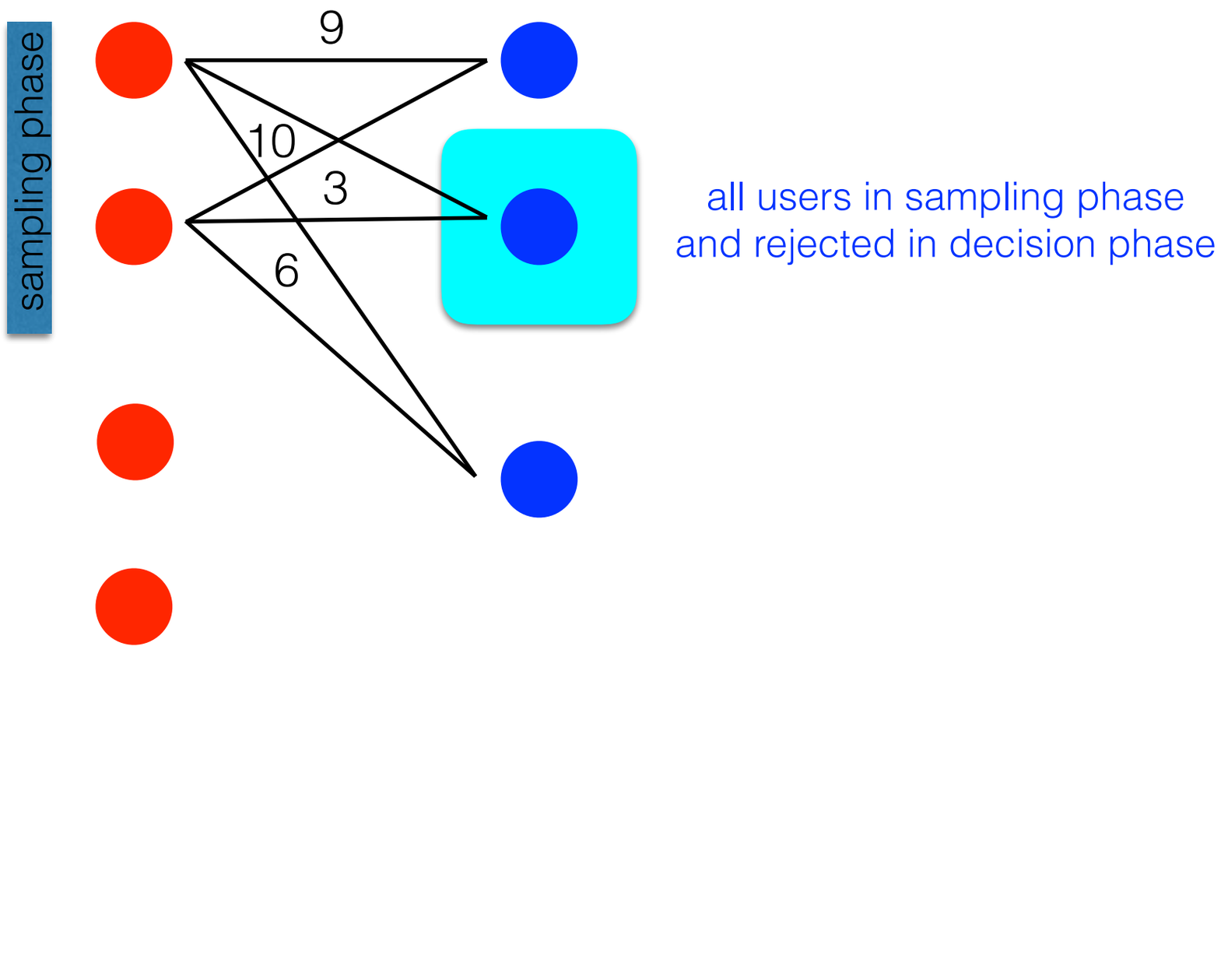

all users in sampling phase
and rejected in decision phase

How to solve this ONLINE

Sampling idea as before

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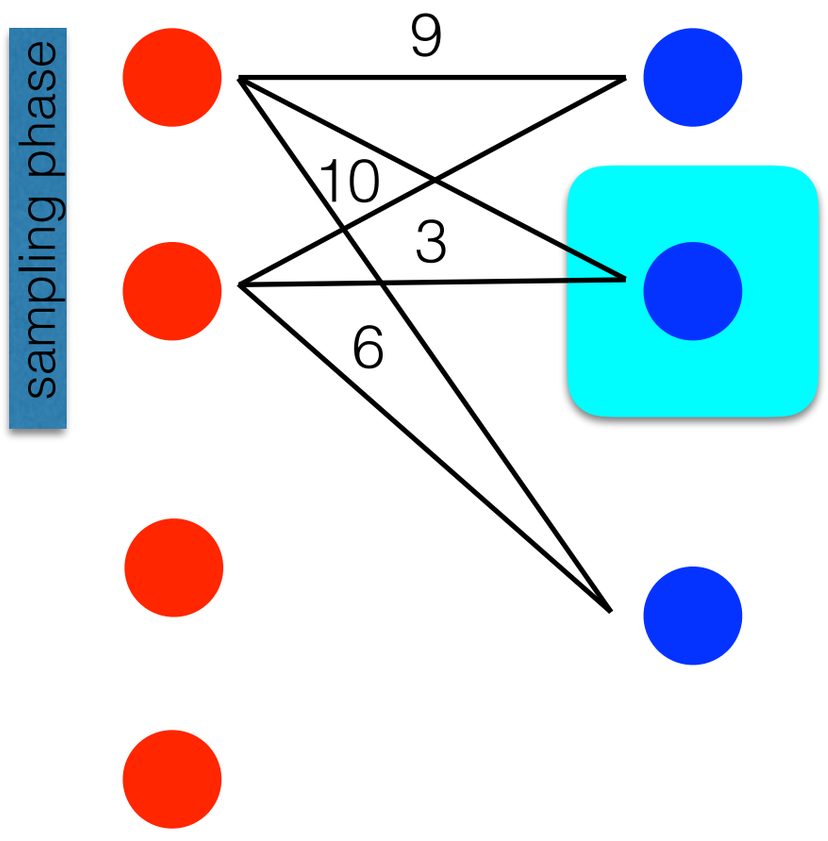
Find best matching



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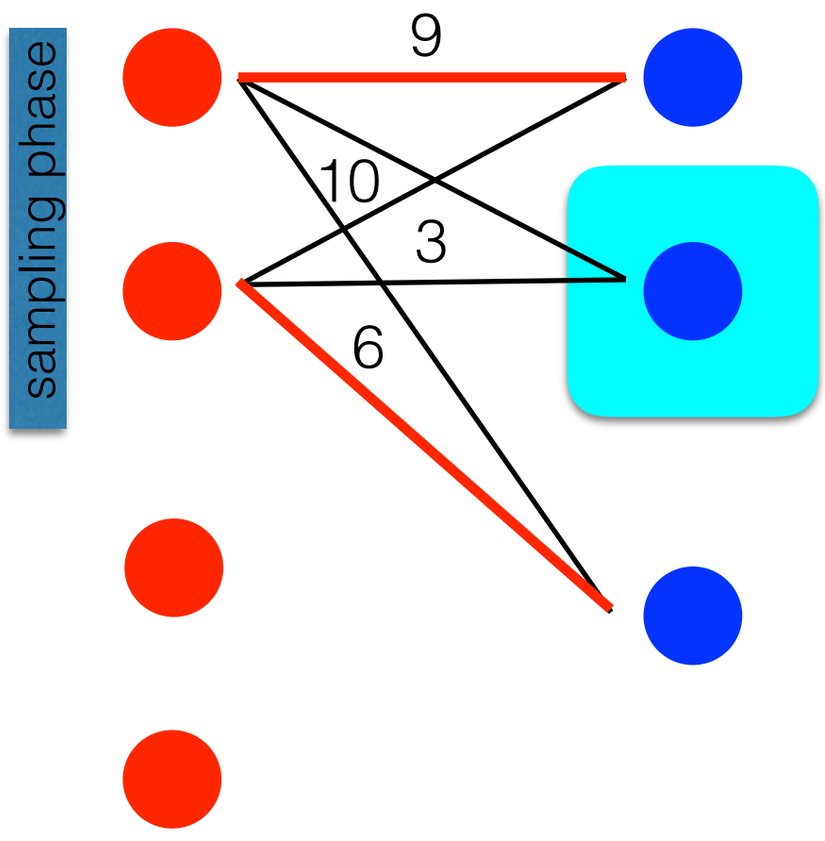

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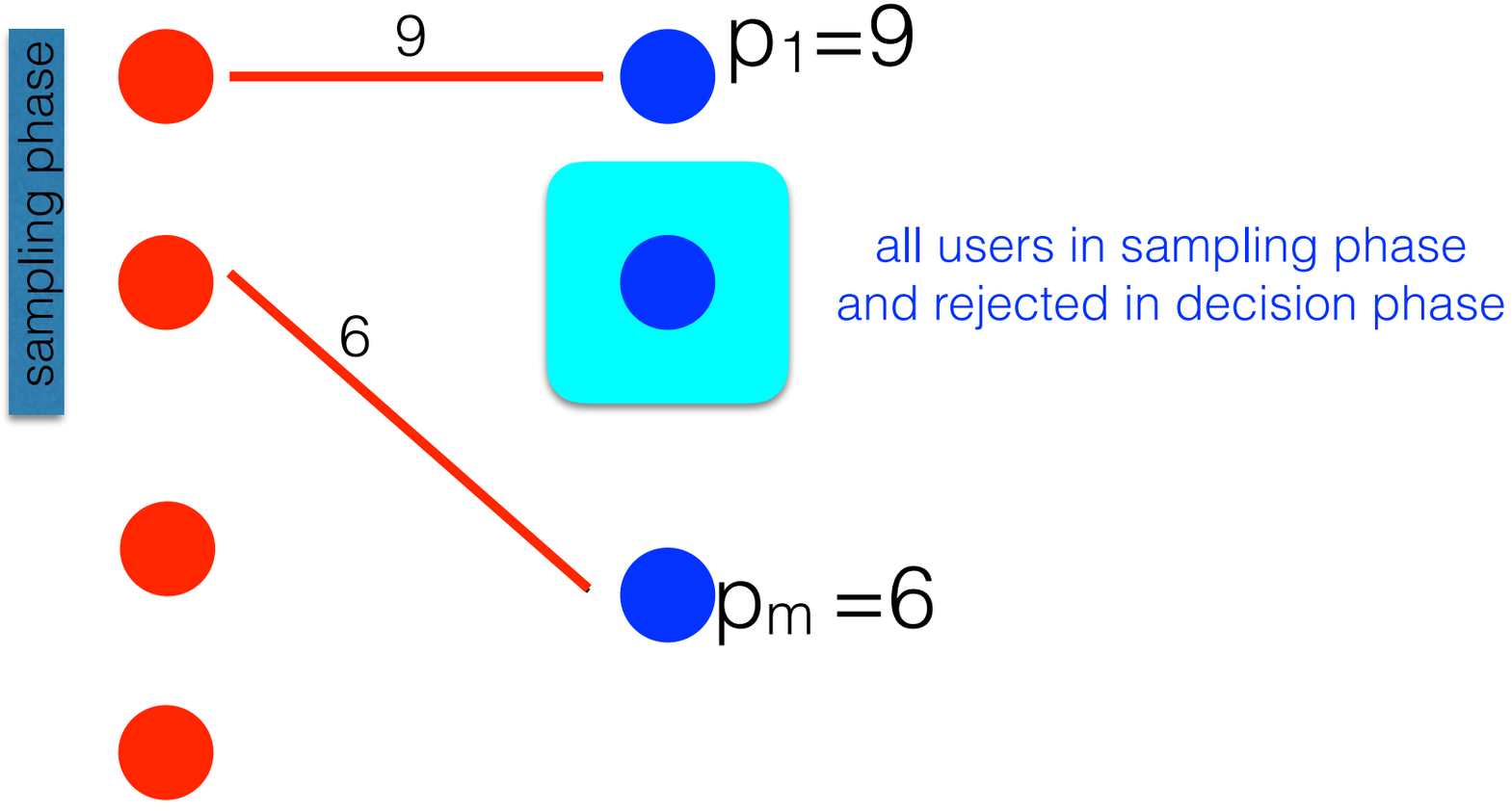

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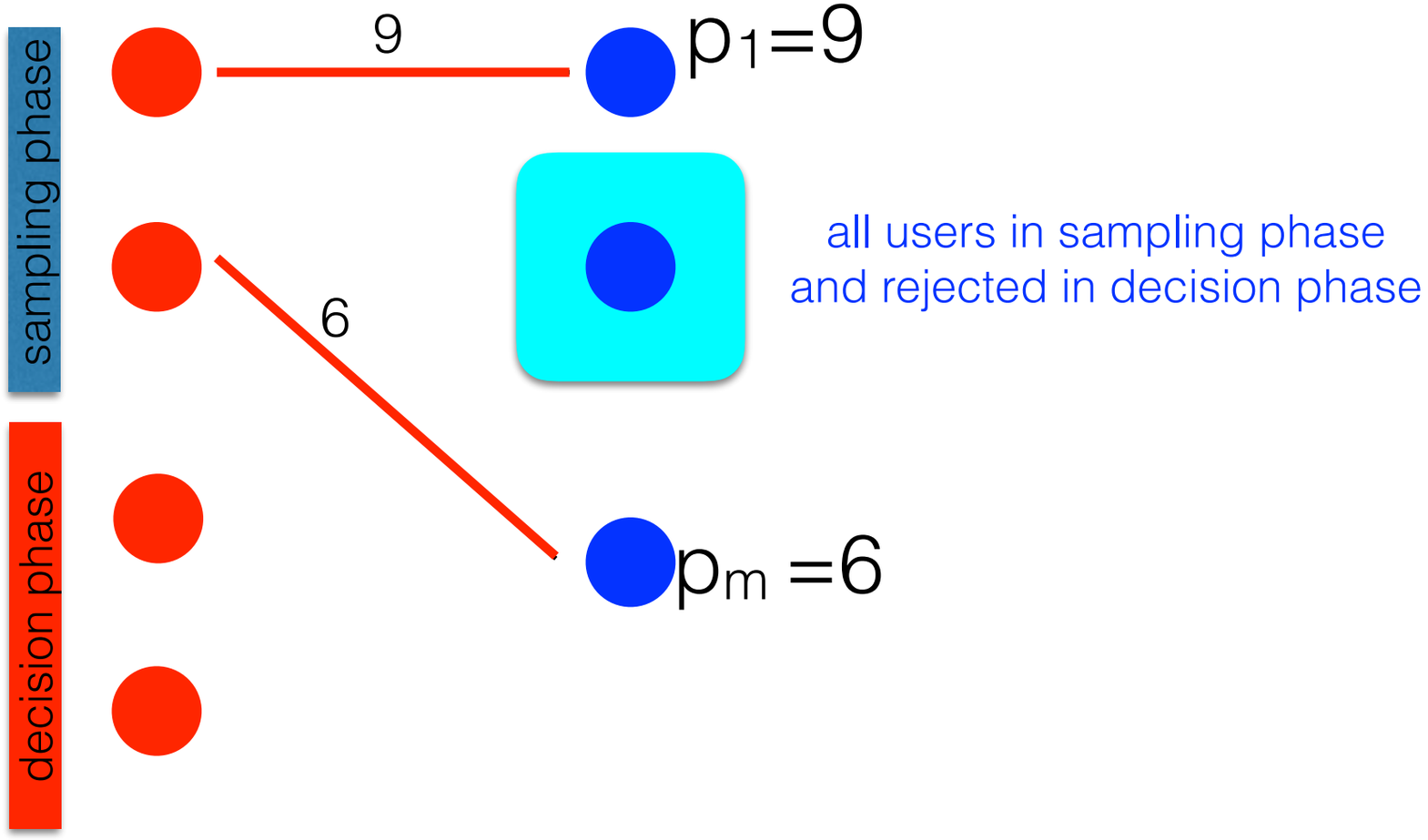

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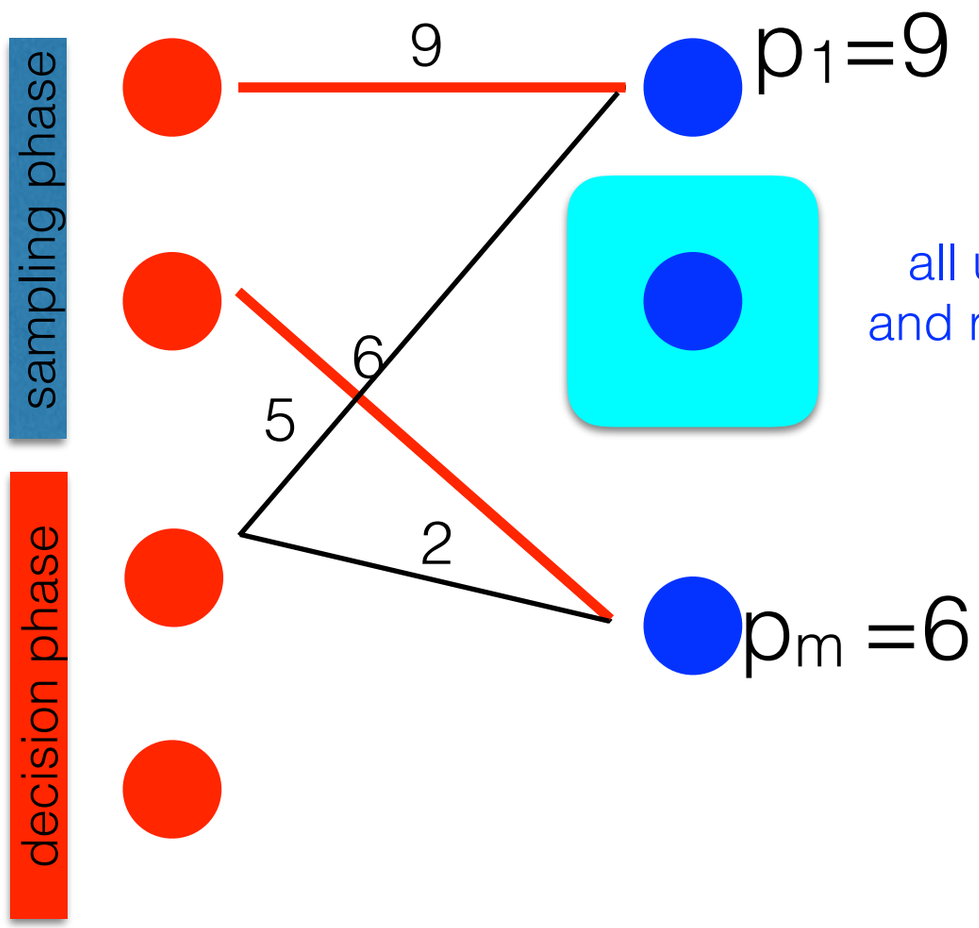


all users in sampling phase
and rejected in decision phase

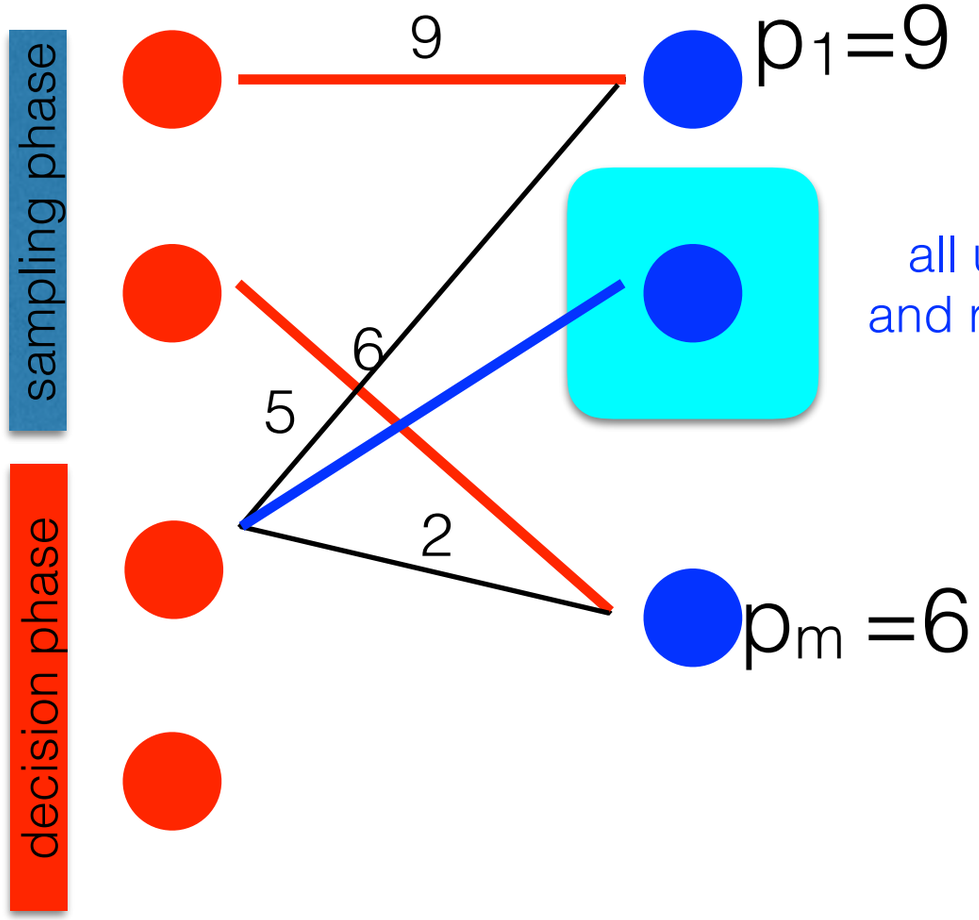
match the largest weight
more than price

How to solve this ONLINE

Sampling idea as before

Choose one BS randomly and associate all users rejected by Matching

Find best matching



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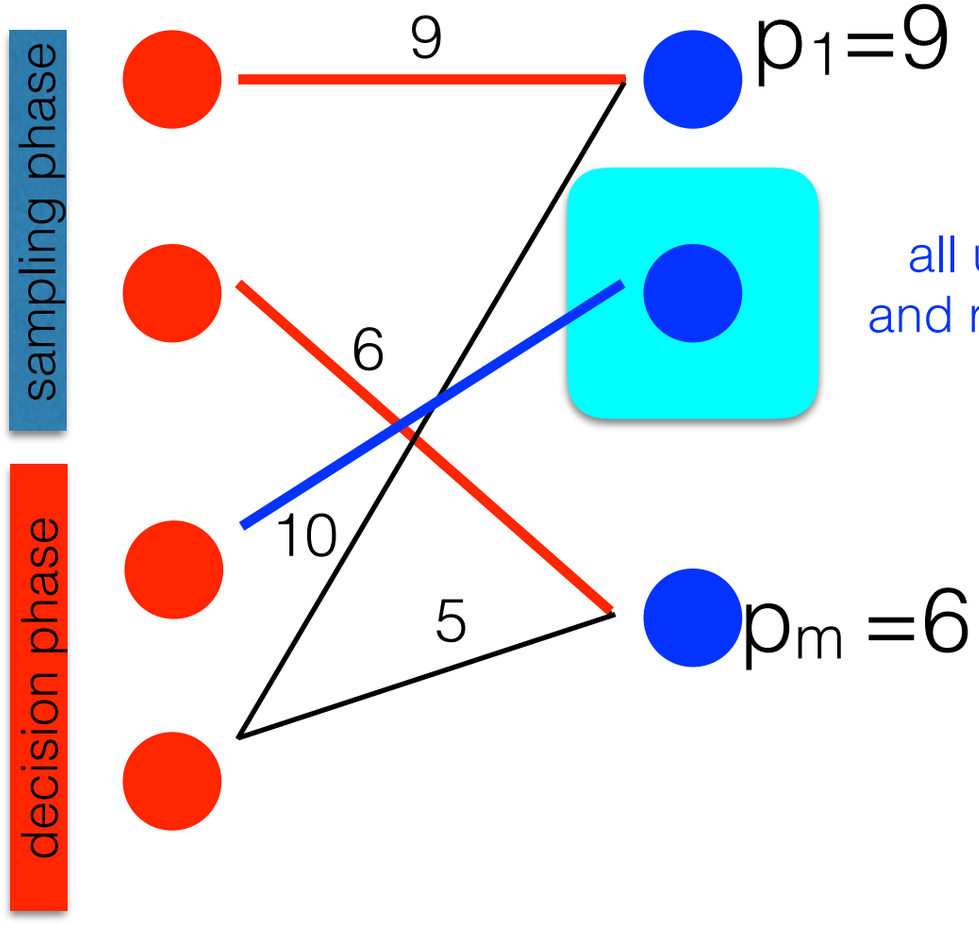
Find best matching



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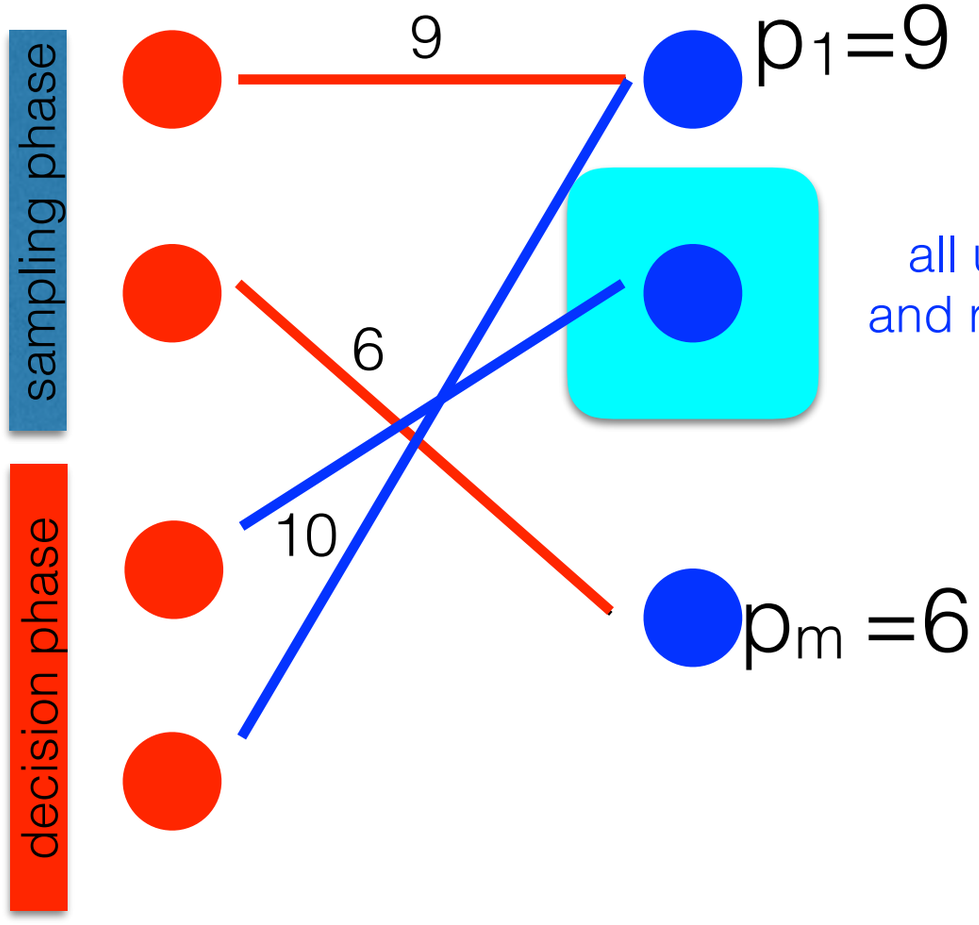

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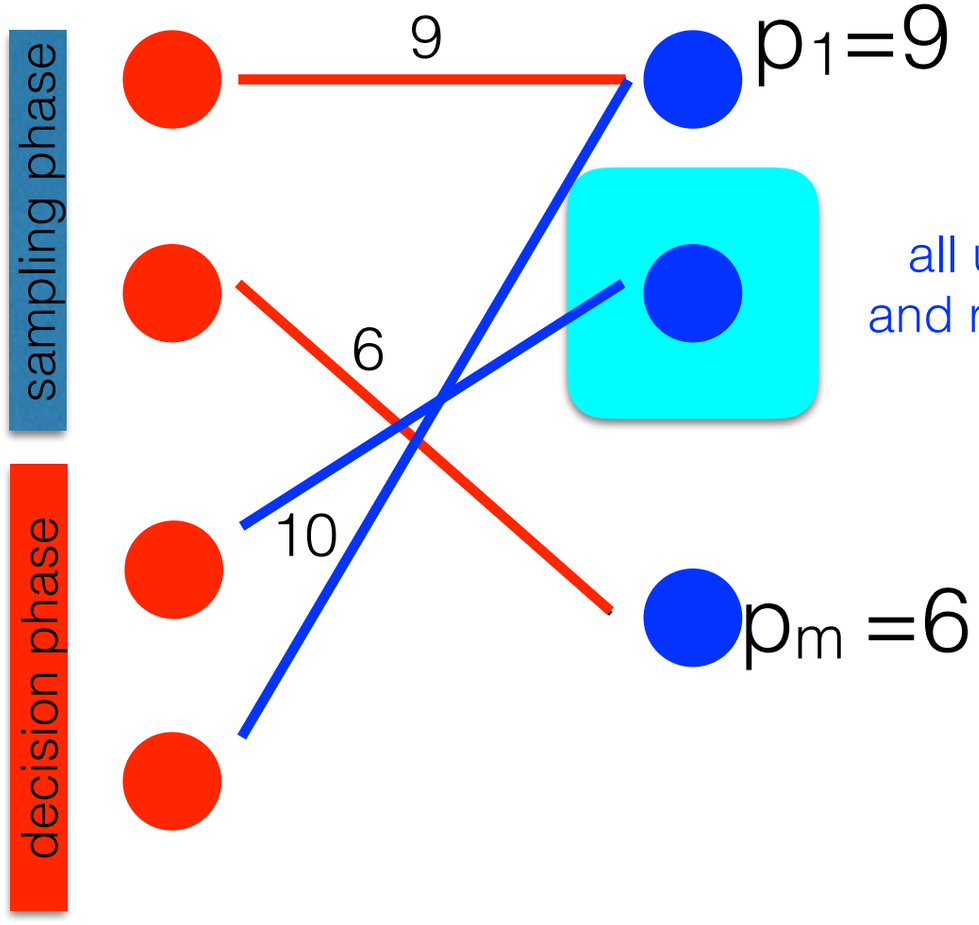
Choose one BS randomly and associate all users rejected by Matching

Find best matching



all users in sampling phase
and rejected in decision phase

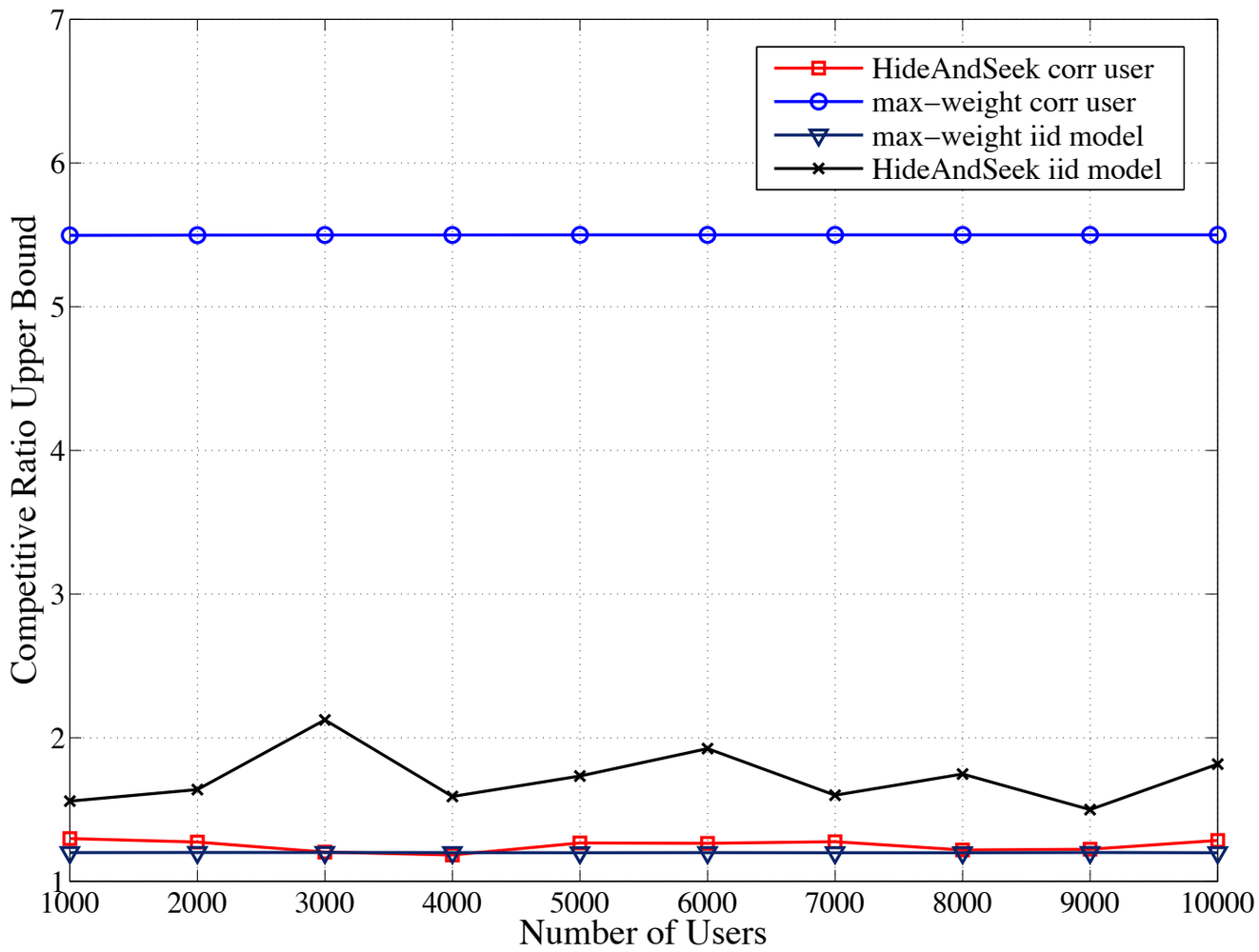
match the largest weight
more than price

Result: 8m/(m-1)—competitive/optimal [*V, Thangaraj' 13*]

Implication

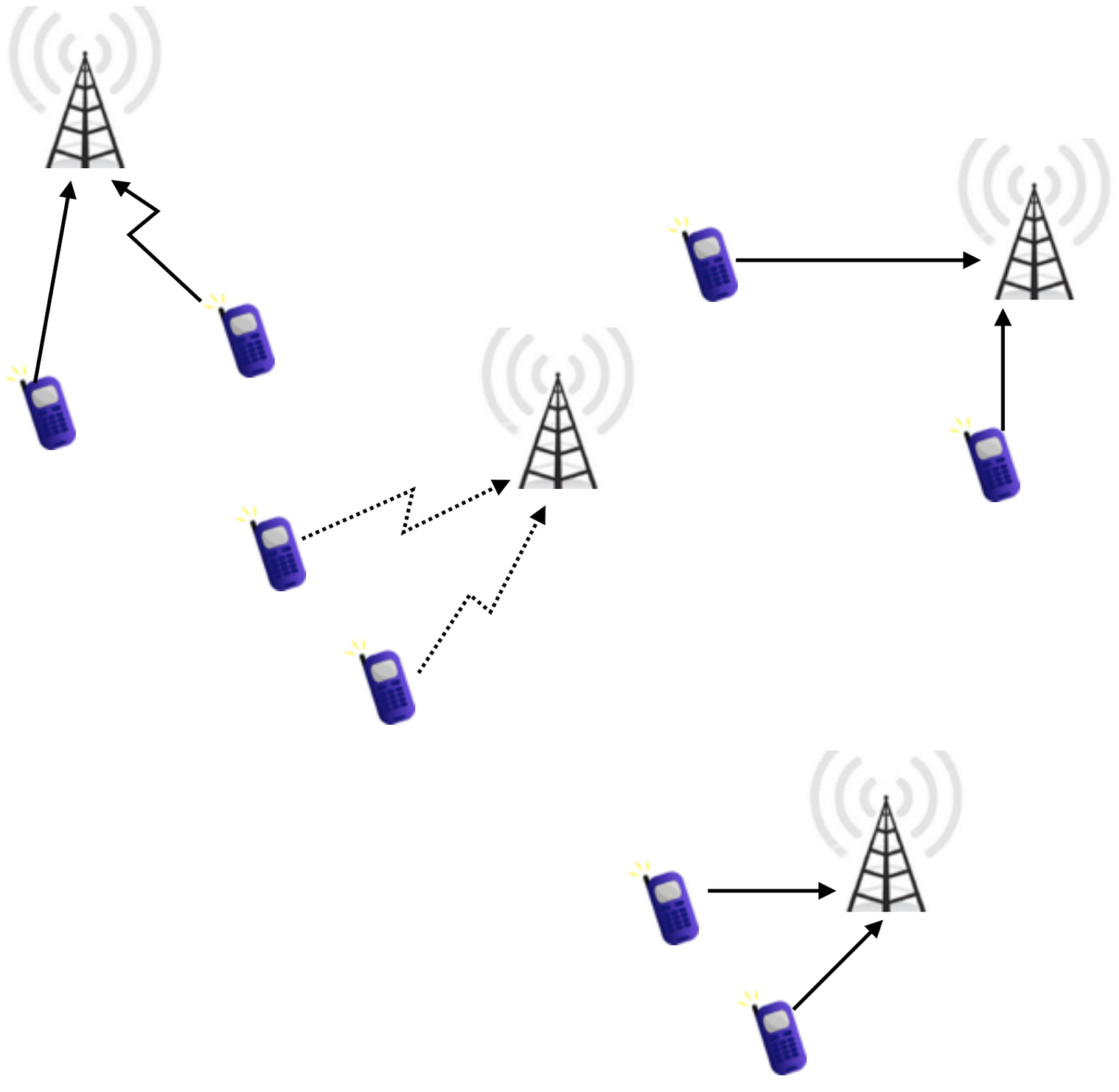
Lot of users get associated to just one BS

Still better than natural algorithm of connecting to the strongest BS

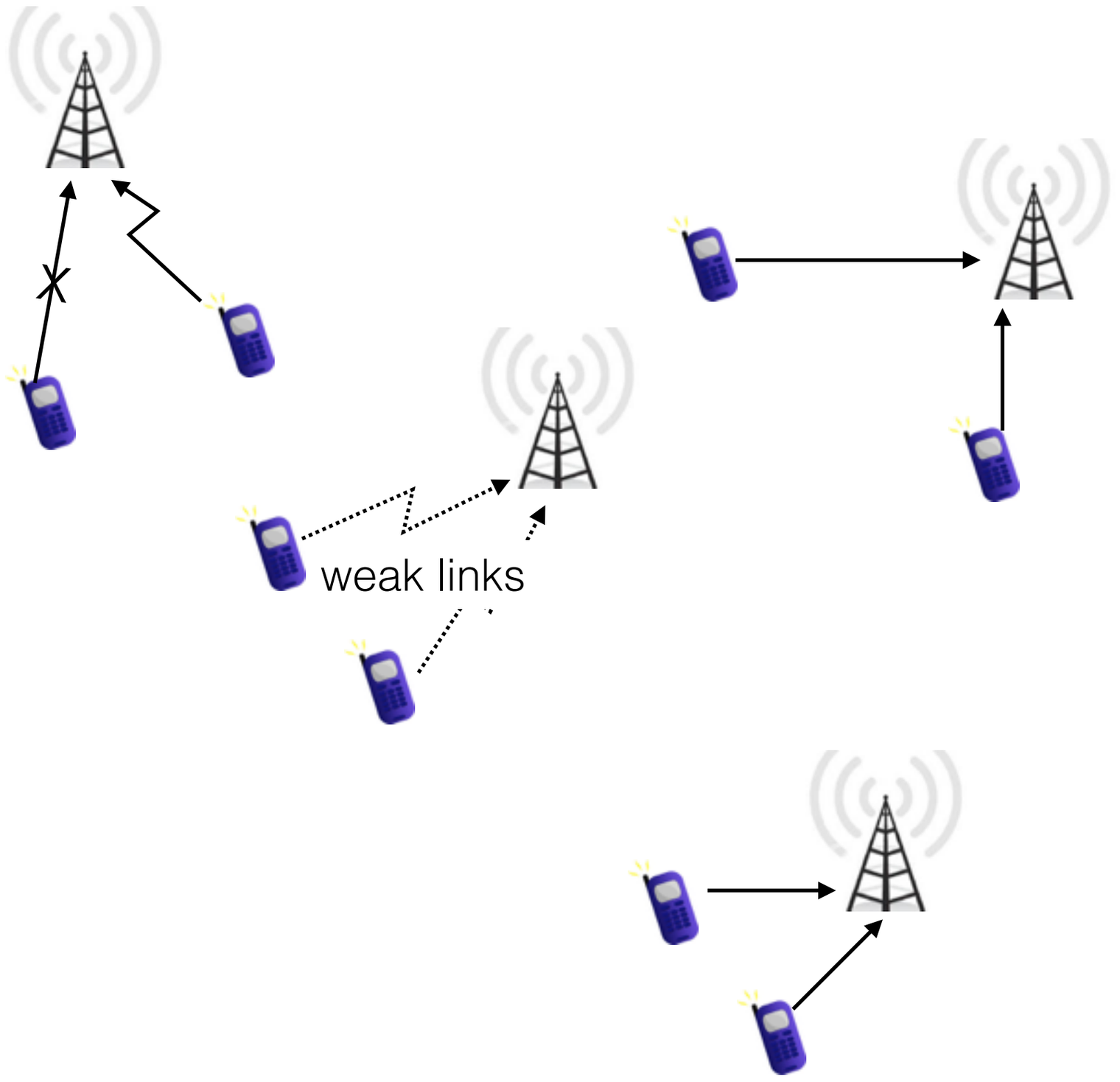


5G

Modern Problem Device-2-Device Communication



Modern Problem Device-2-Device Communication



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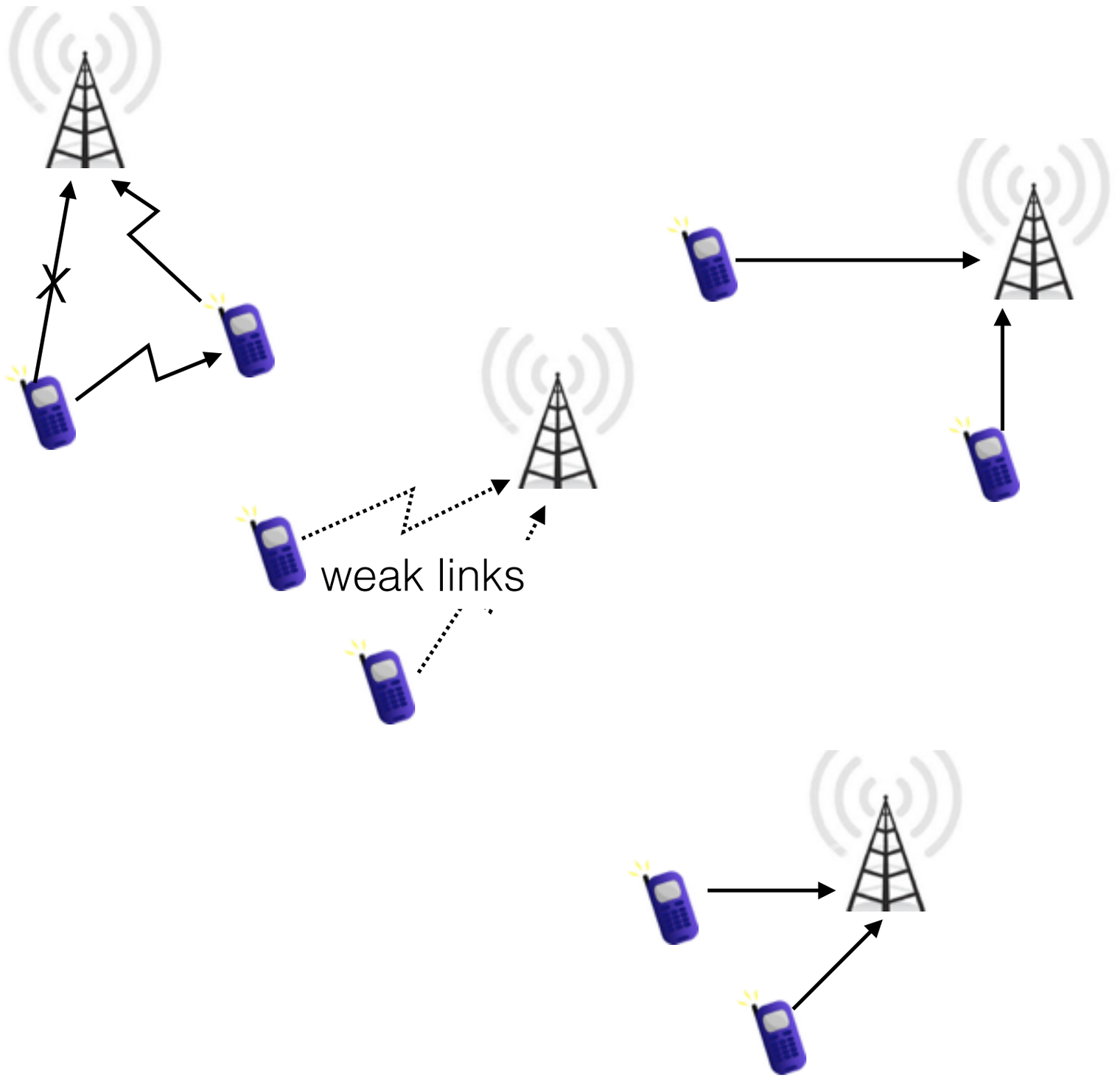

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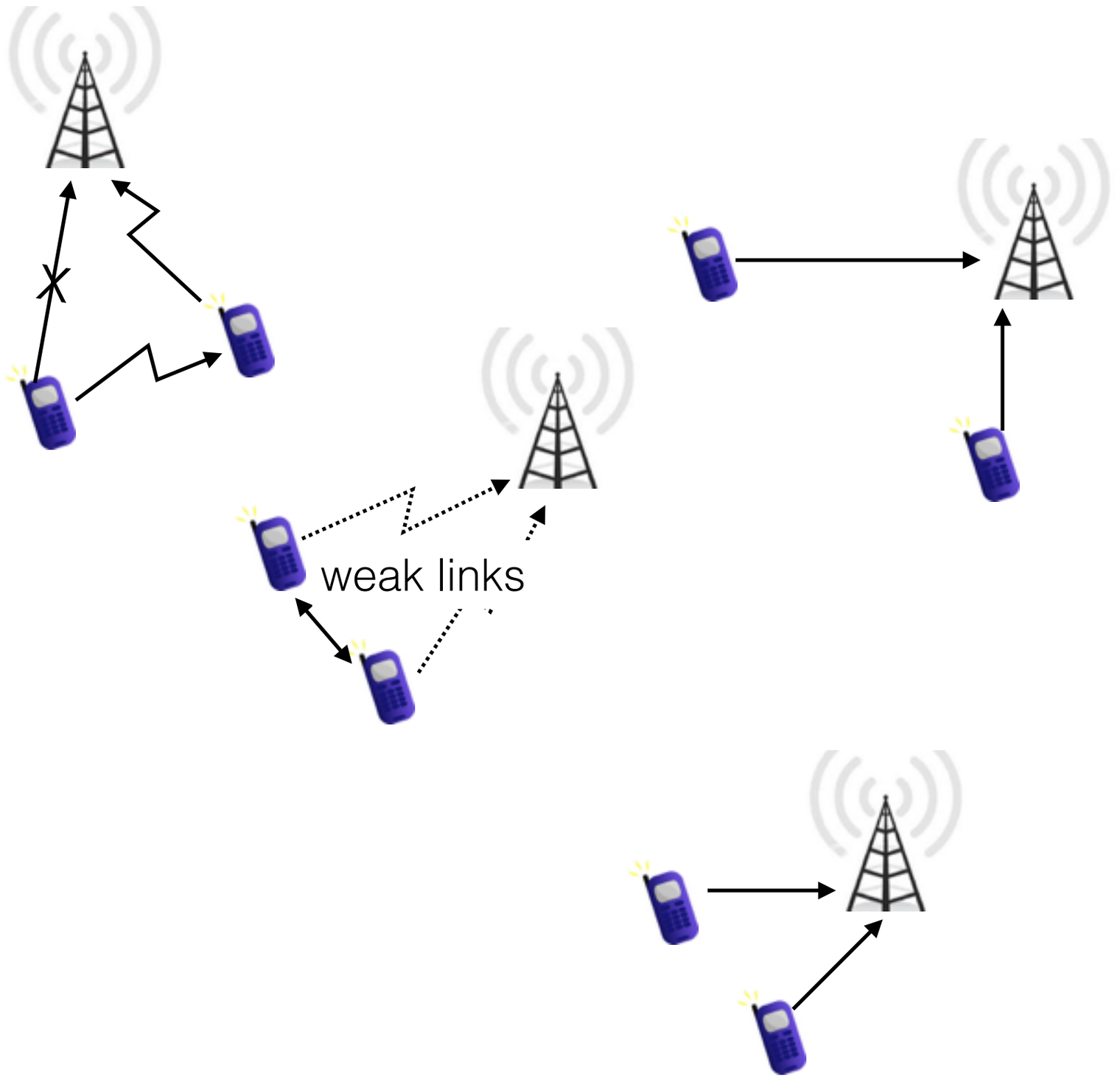

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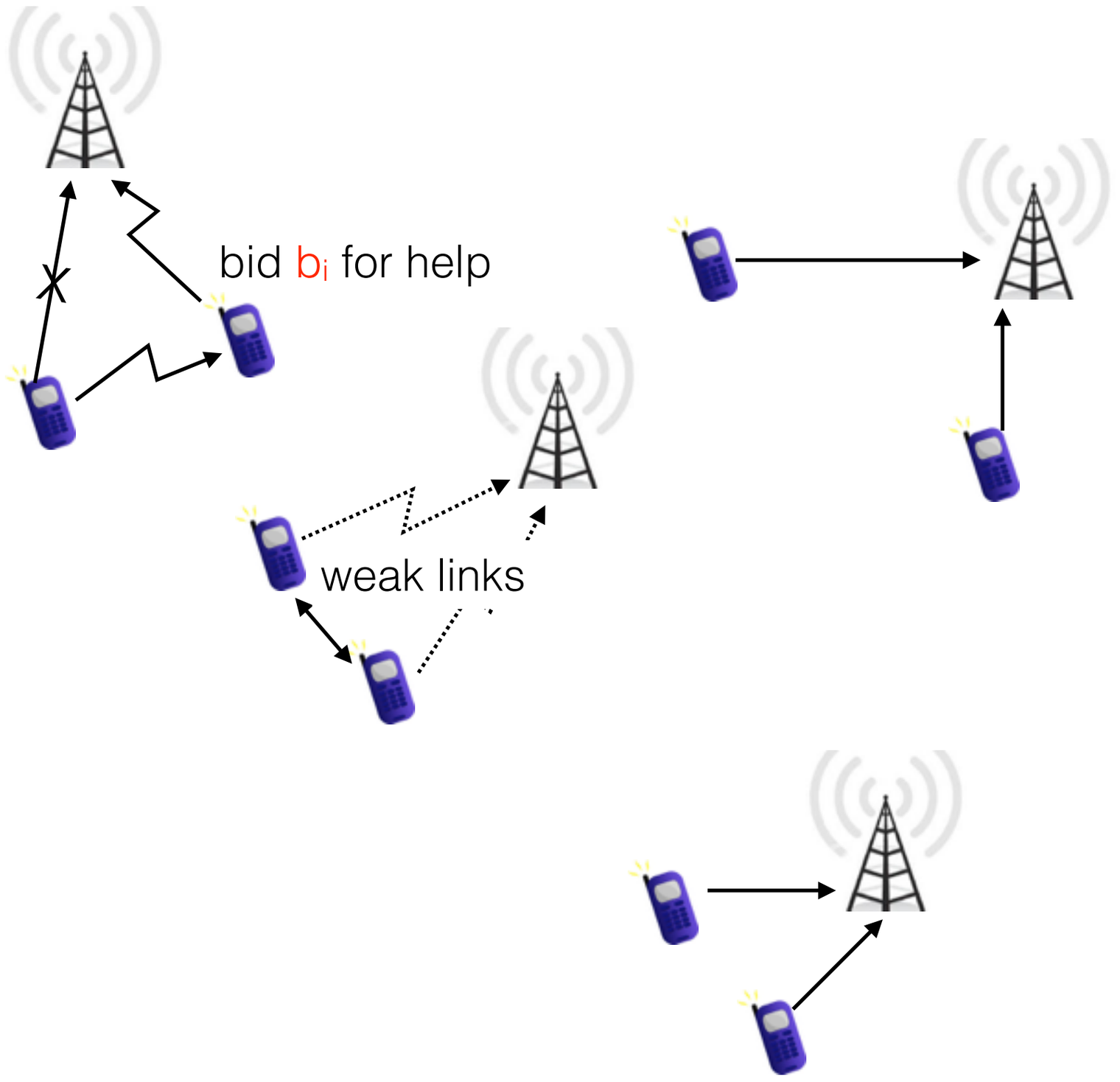

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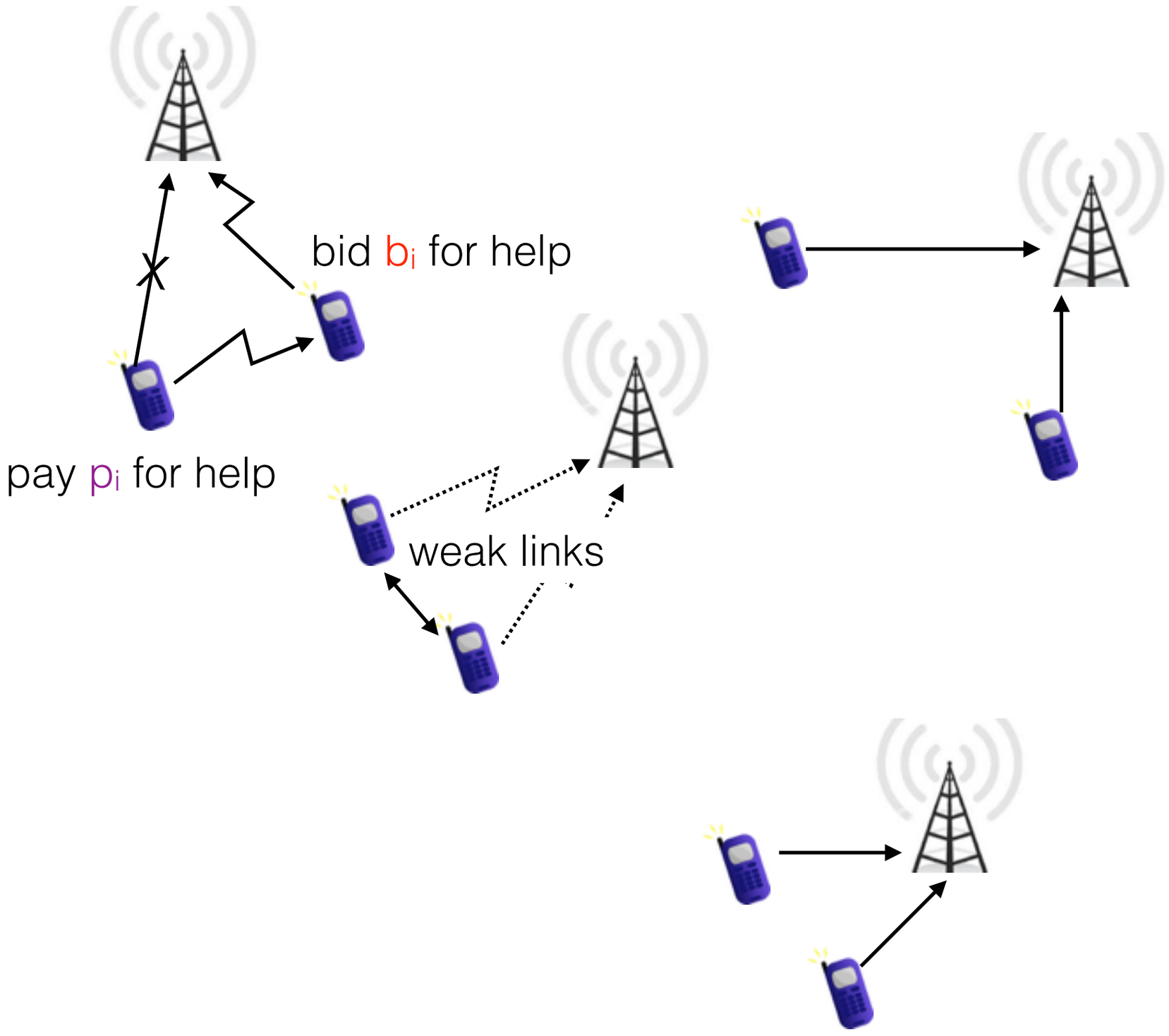
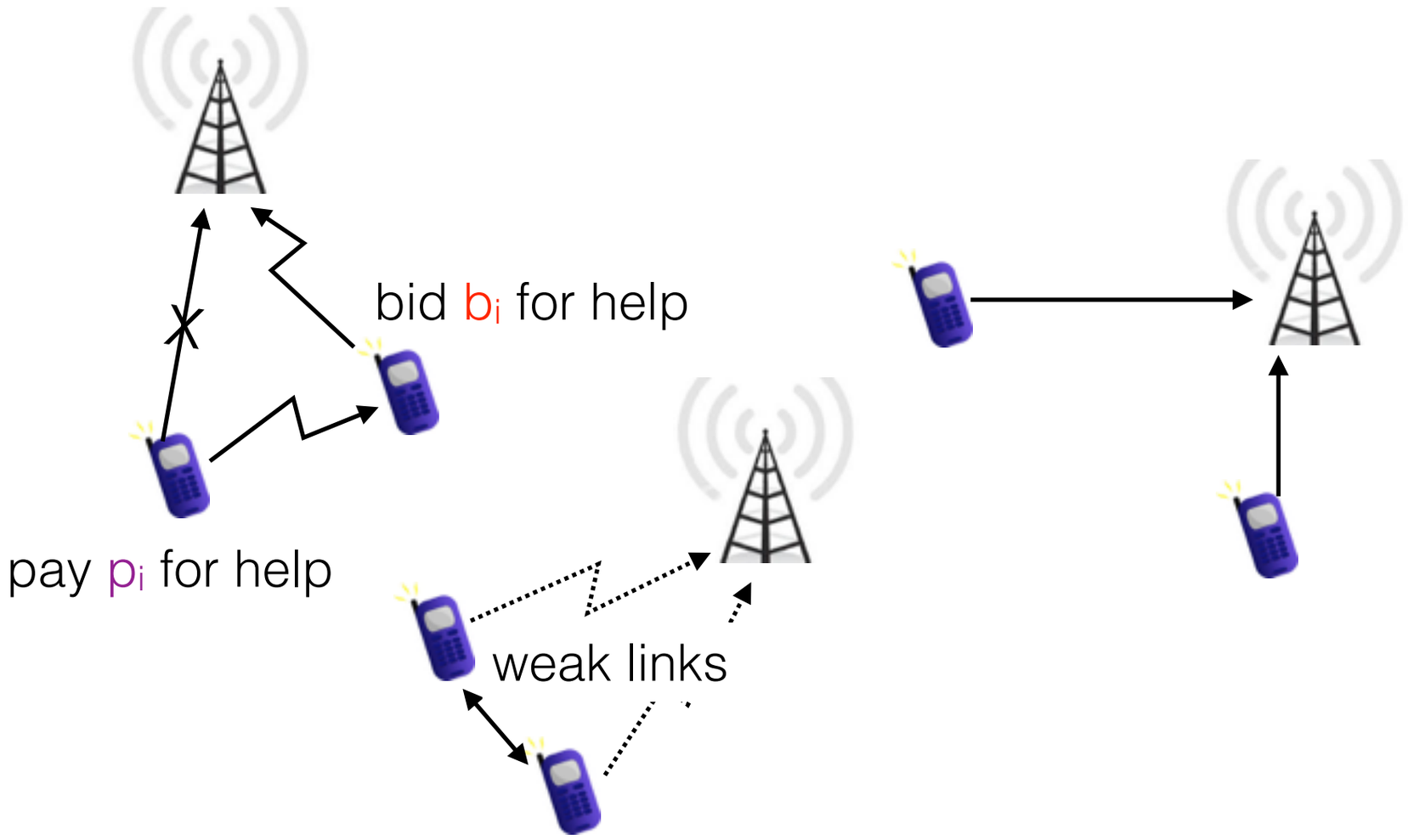
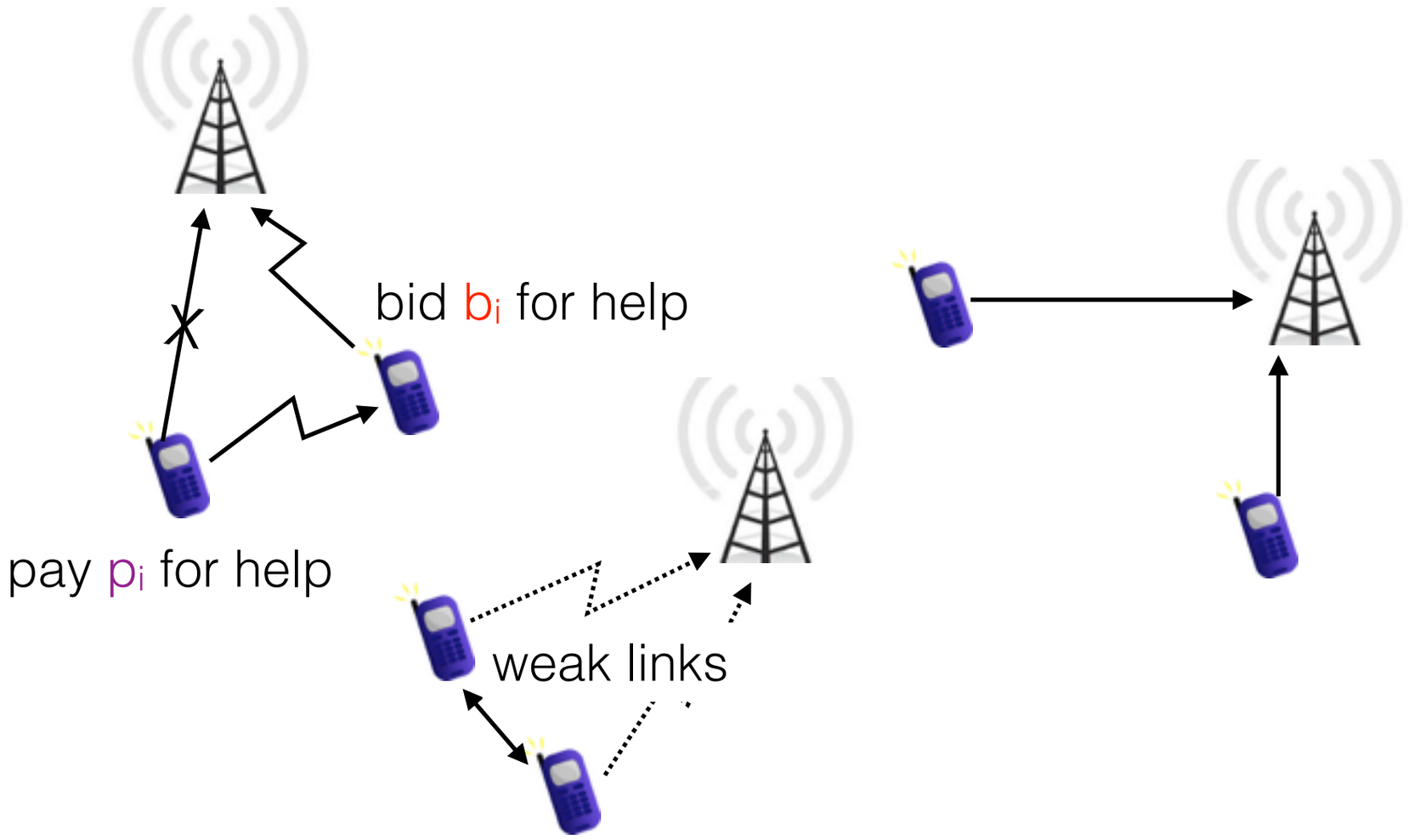


Modern Problem Device-2-Device Communication



mechanism to avoid cheating

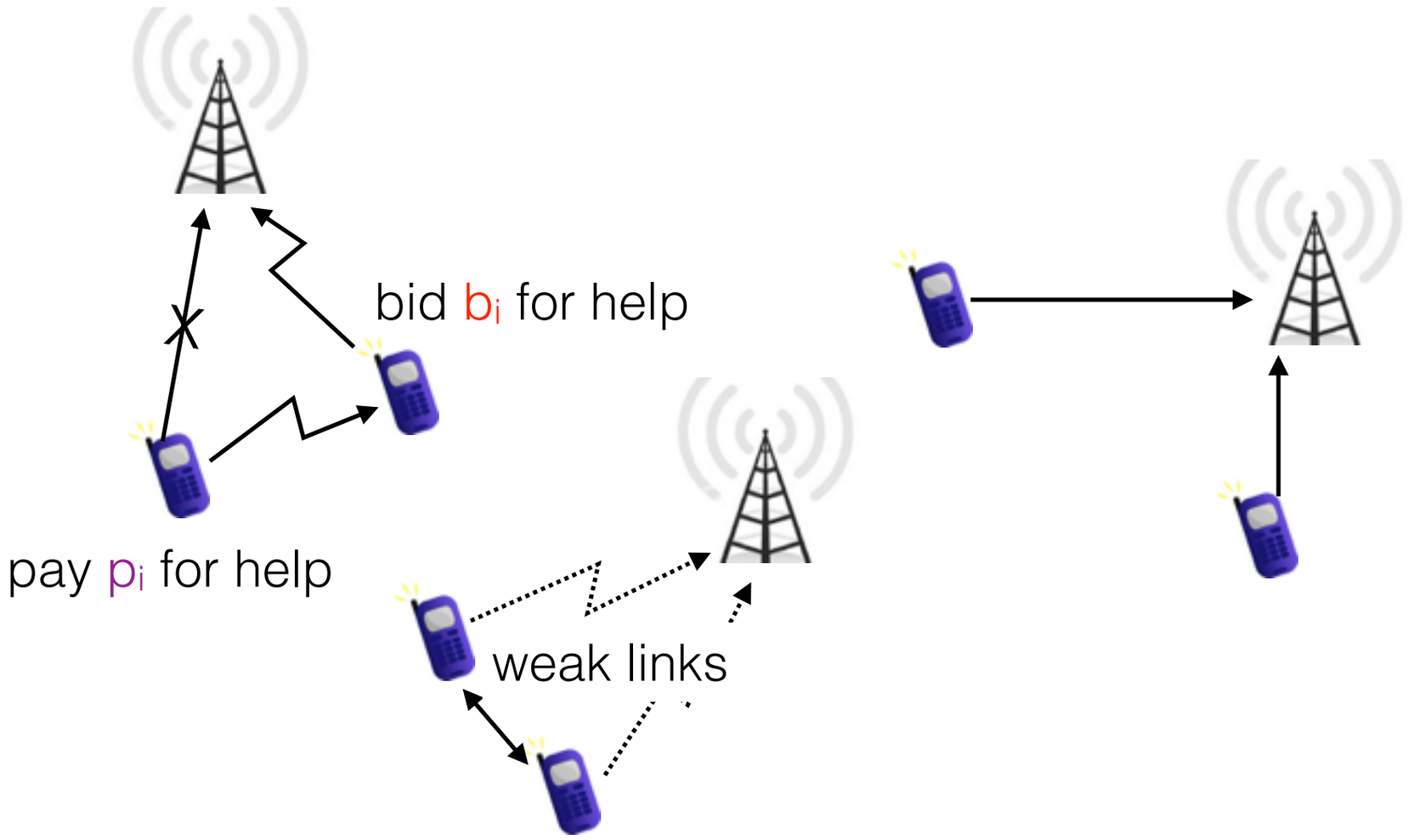
Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

Find optimal helper association and incentive rule that is truthful

NATION, IN OTHER NEWS

Kerala IAS officer lures public with biryani to clean lake

DECCAN CHRONICLE

Published Jan 27, 2016, 5:54 pm IST **Updated** Jan 27, 2016, 5:57 pm IST

Volunteers cleaned up the 14-acre lake and were rewarded with a plate of Malabar biryani.

IAS officer and collector Prasanth Nair (Photo Courtesy: Facebook.com/Prasanth Nair)

Running an Election Campaign

Running an Election Campaign

Running an Election Campaign

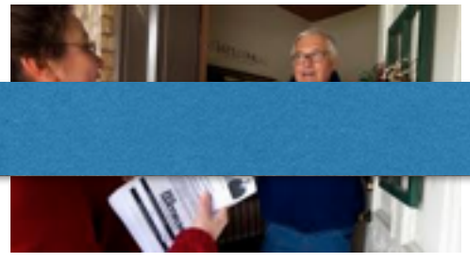
bid

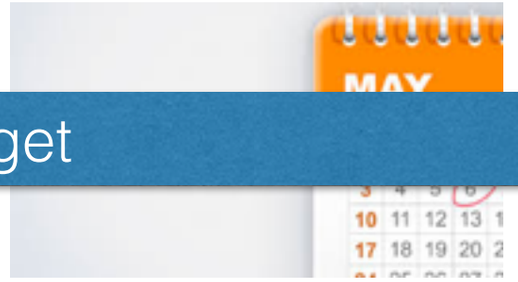
Running an Election Campaign

bid

payment

Running an Election Campaign



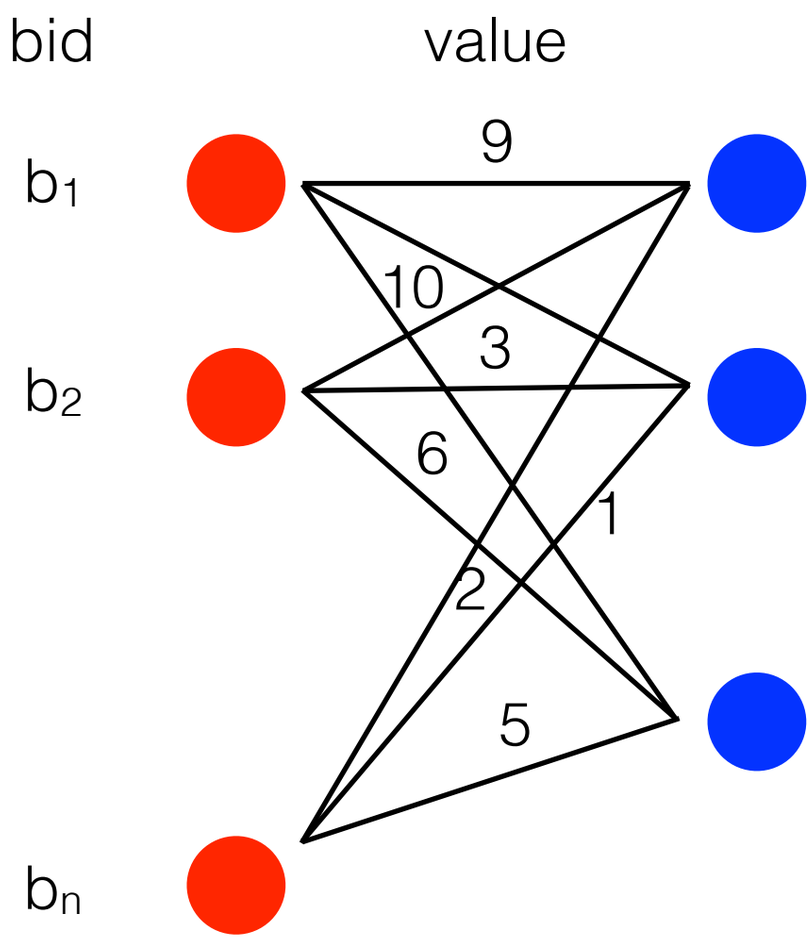


Total Budget

bid

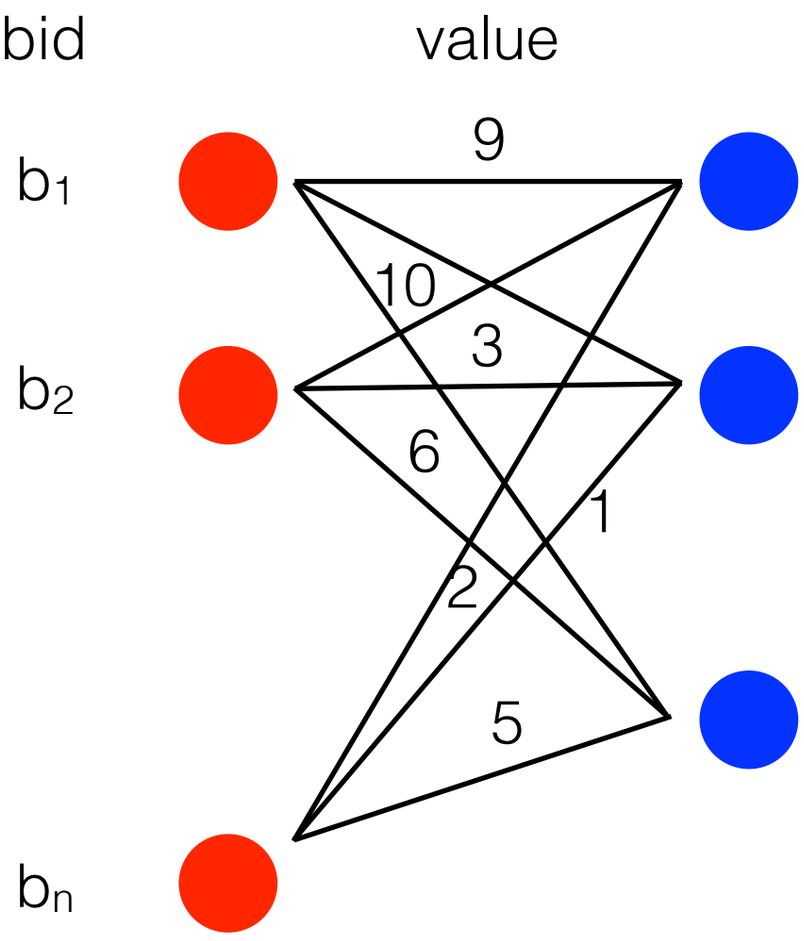
payment

Example



Example

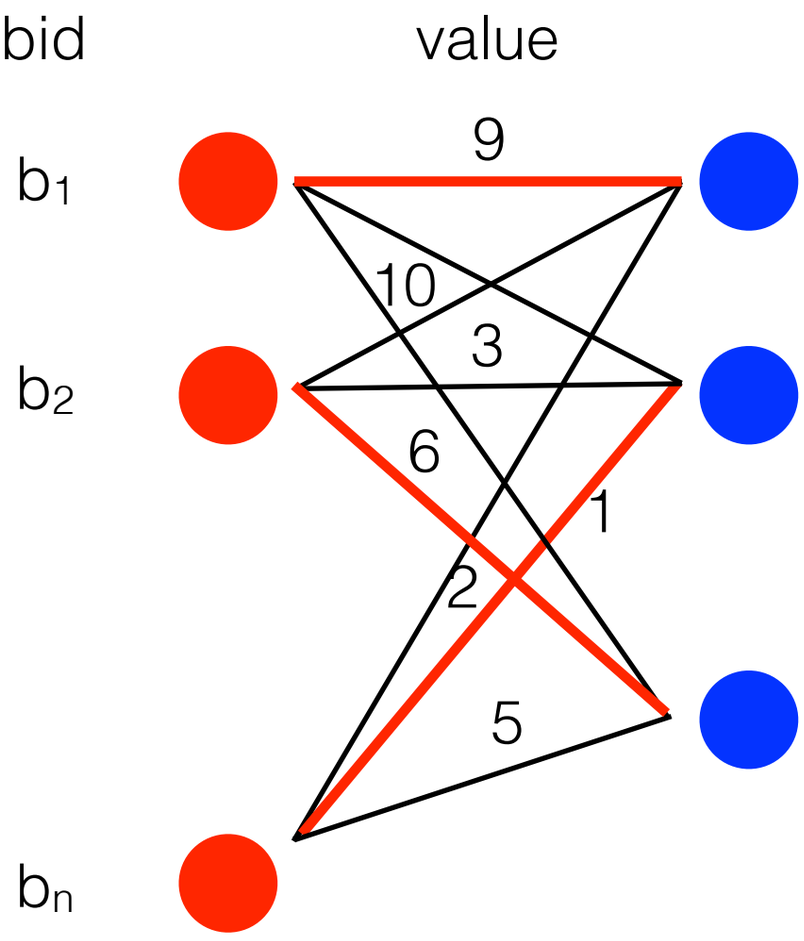
For simplicity at most one accepted edge



Objective: Truthful Matching with largest sum weight under a budget constraint

Example

For simplicity at most one accepted edge



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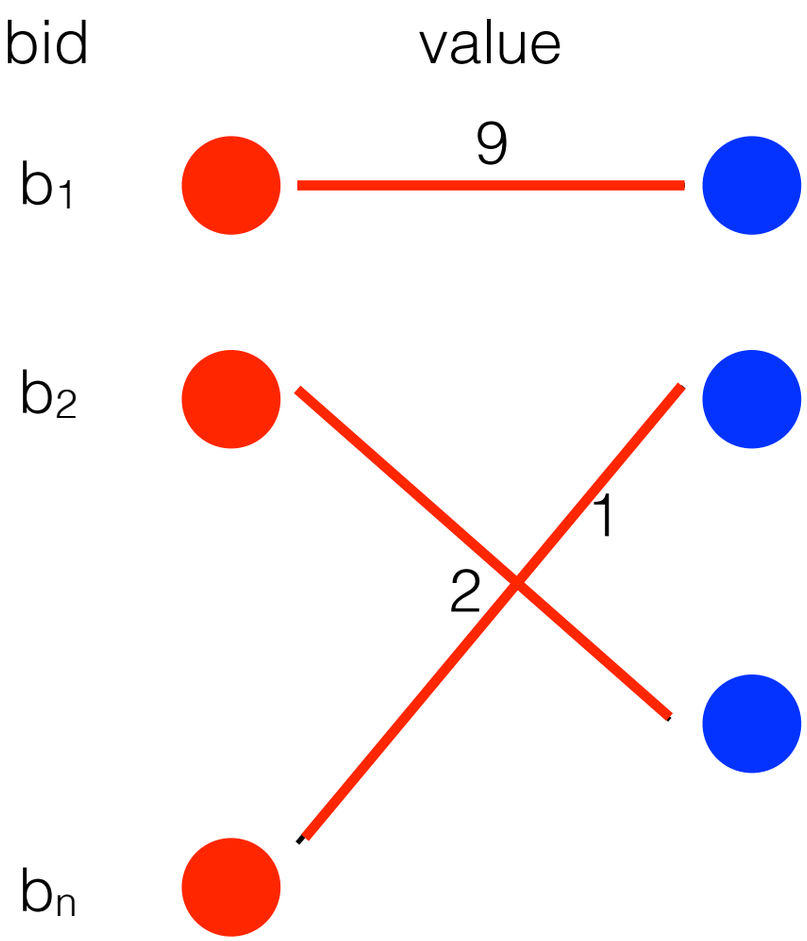

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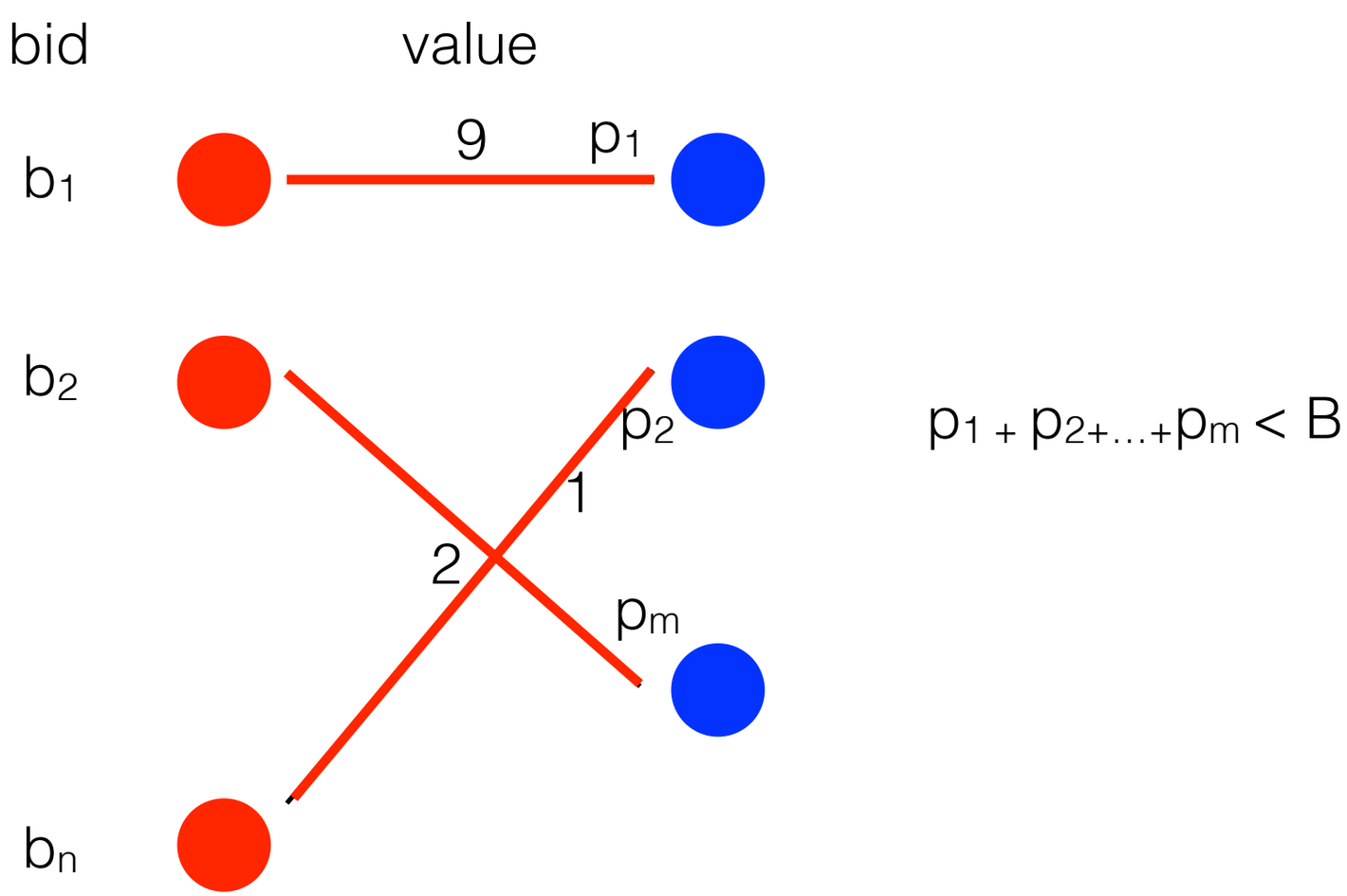
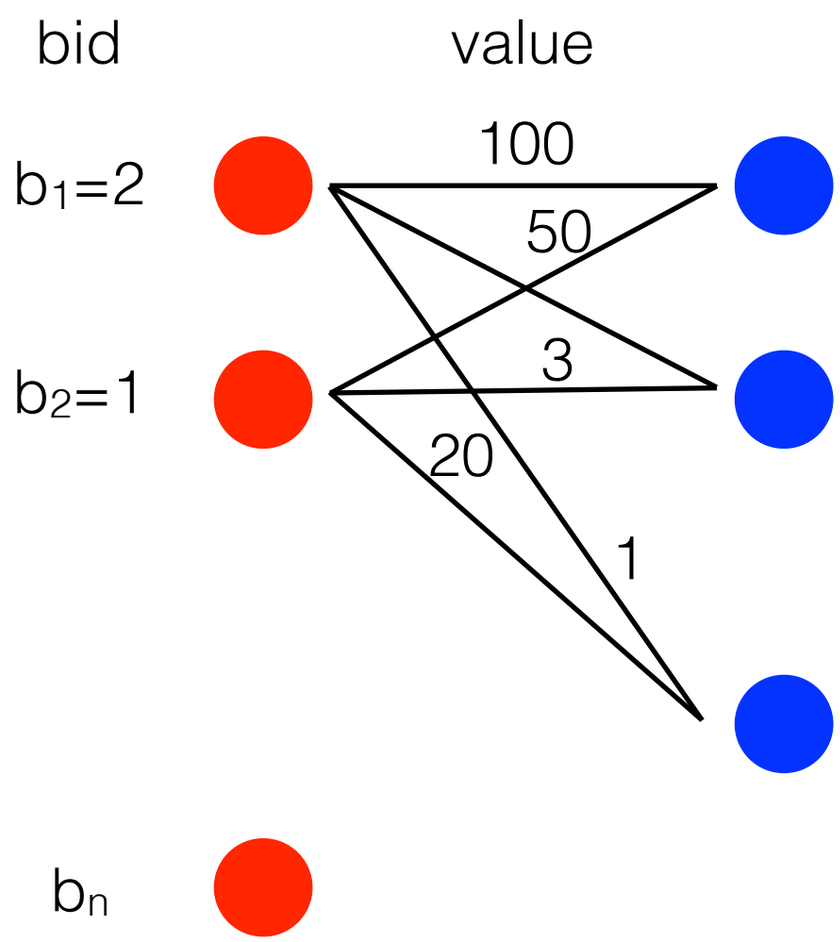


Objective: Truthful Matching with largest sum weight under a budget constraint

Algorithm

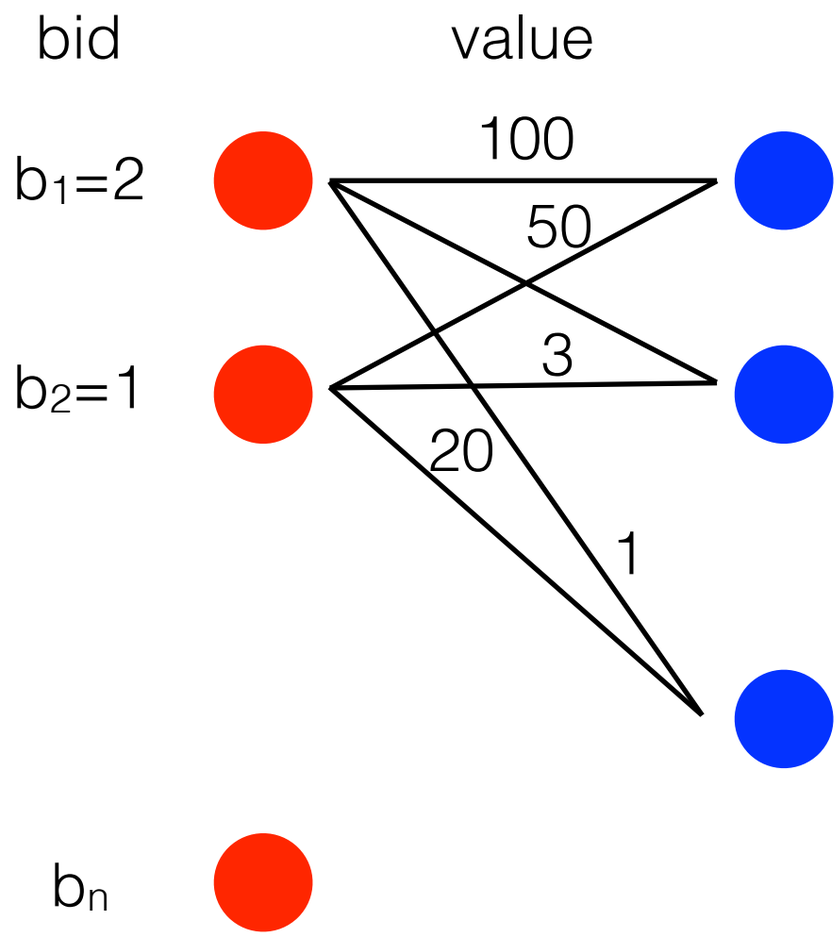


$p_1 + p_{2+\ldots+}p_m < B$

$\mathrm{M}(\gamma)$

Algorithm

same idea as before sampling and decision phase



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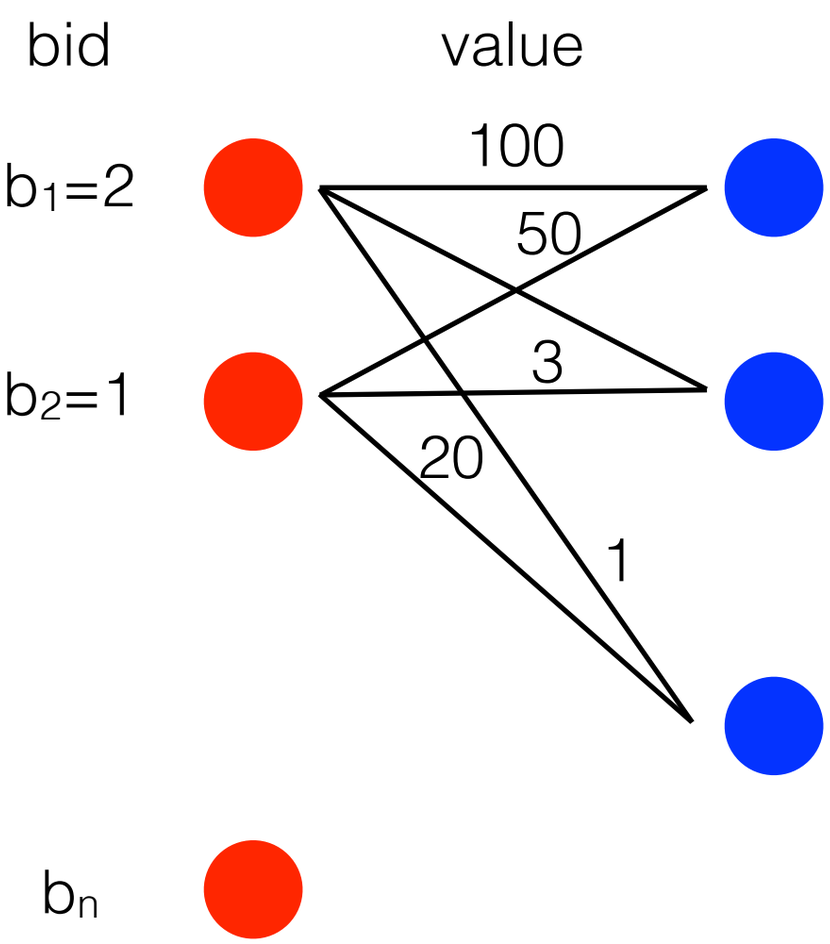

In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

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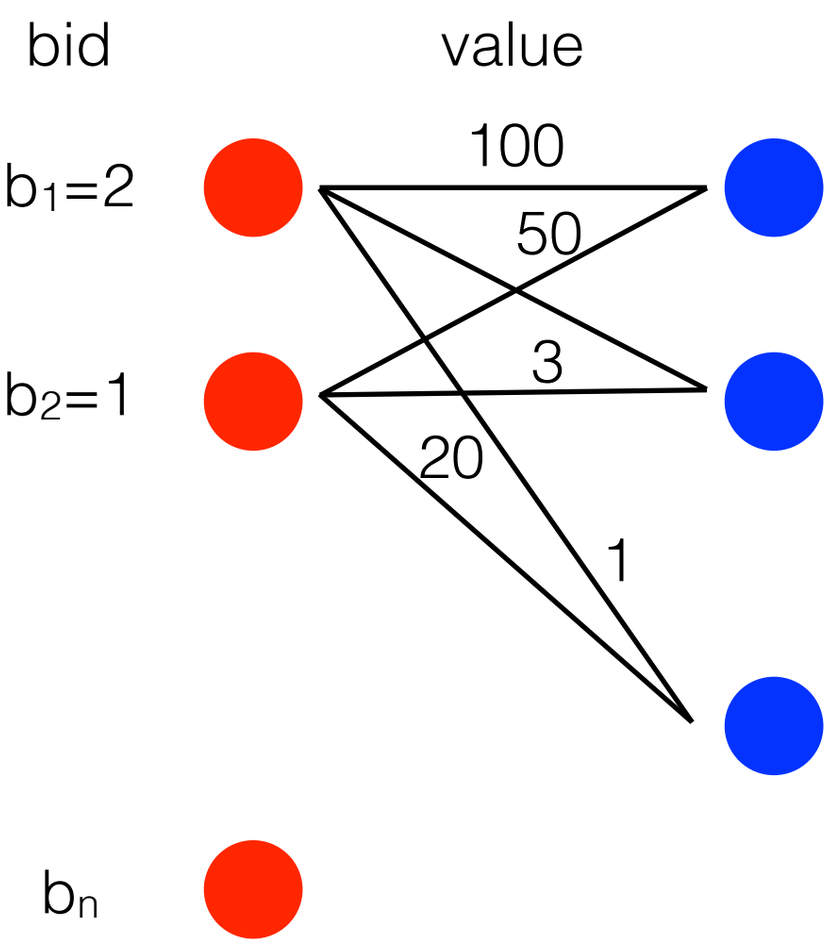
Algorithm

same idea as before sampling and decision phase



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In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

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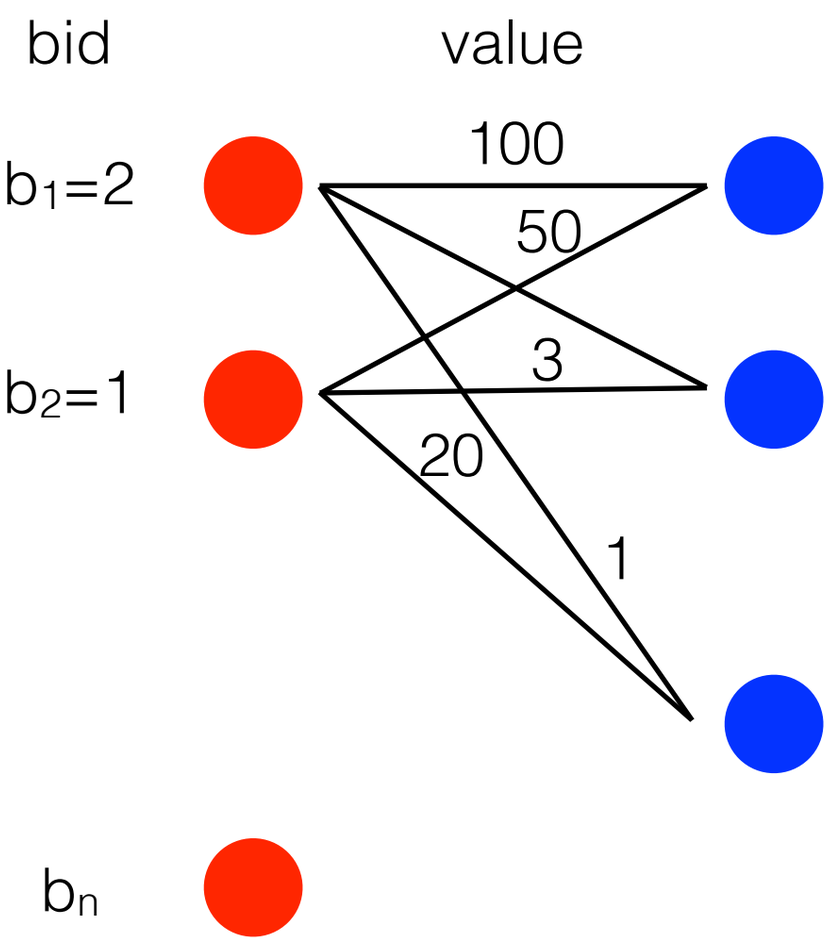

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$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

$\mathrm{M}(\gamma)$

Algorithm

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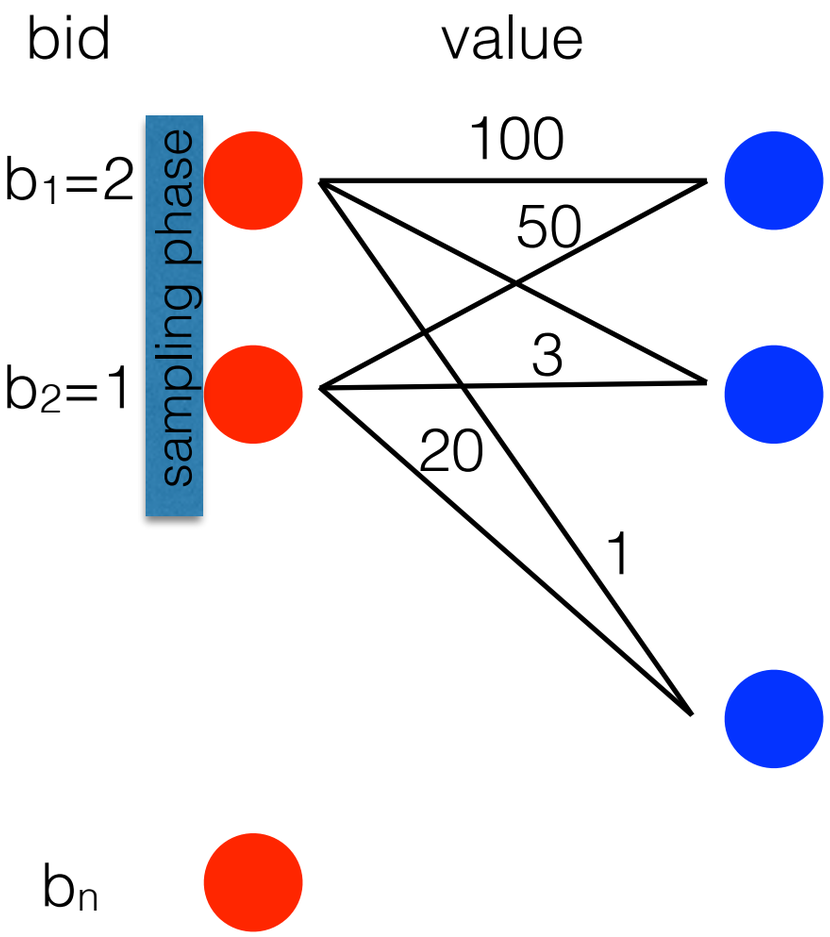

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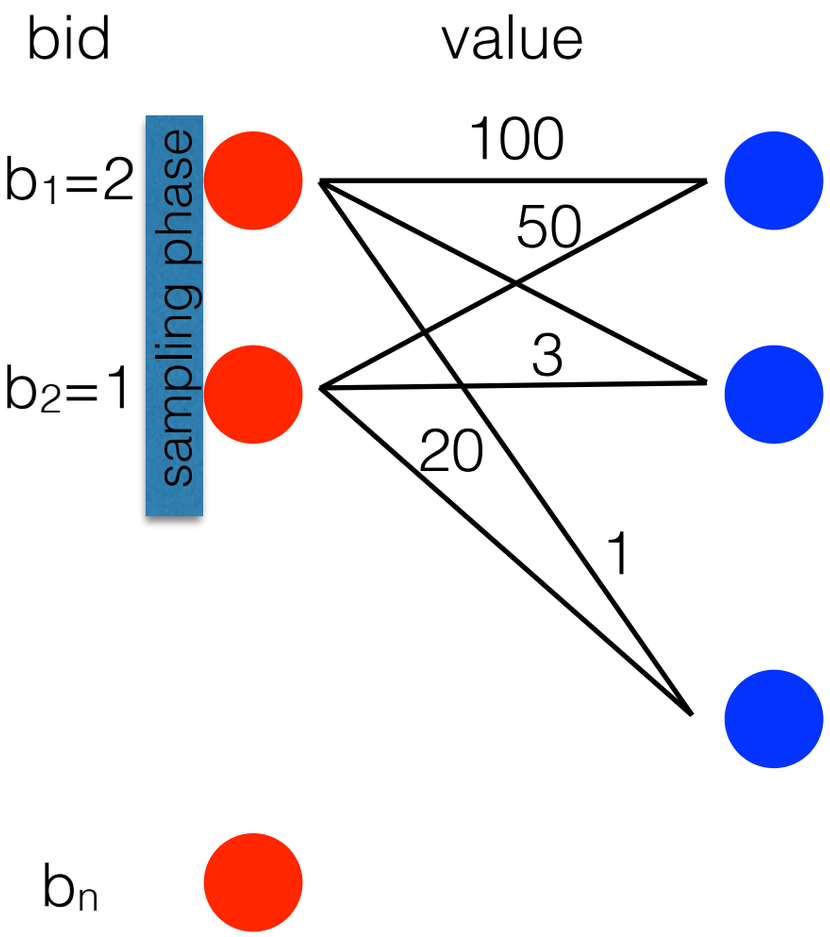

$\gamma = 0.1$

In Sampling Phase

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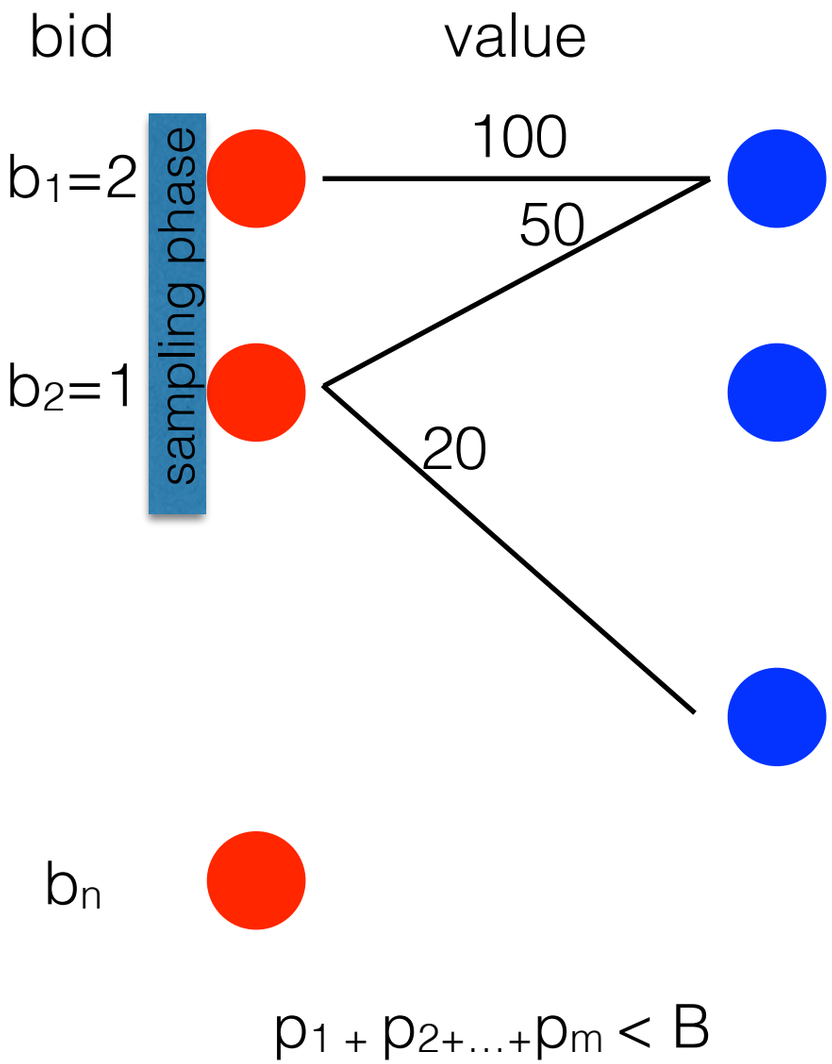

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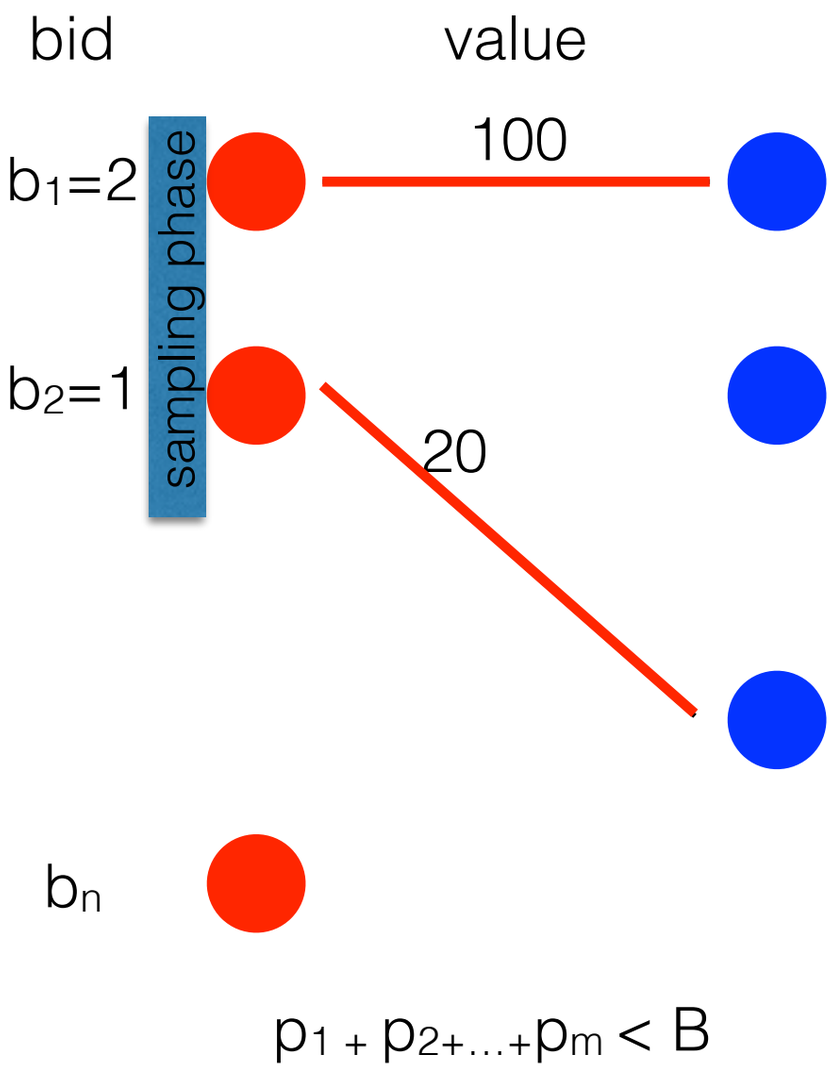

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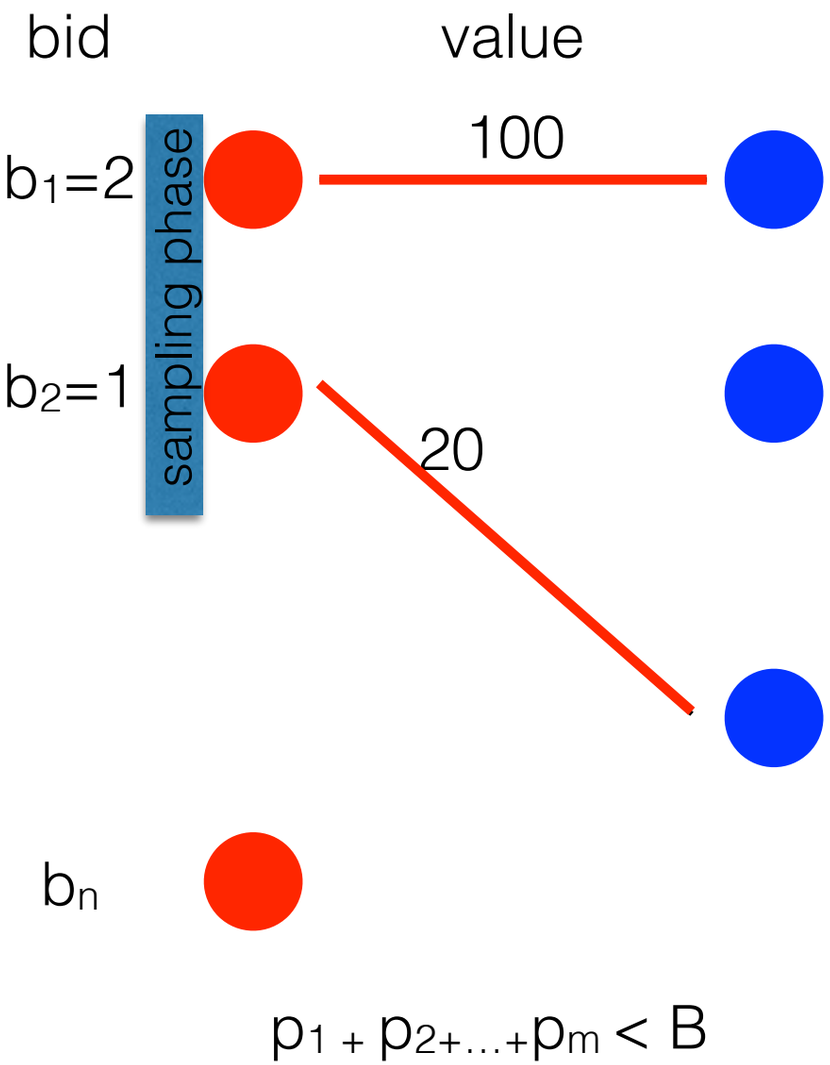

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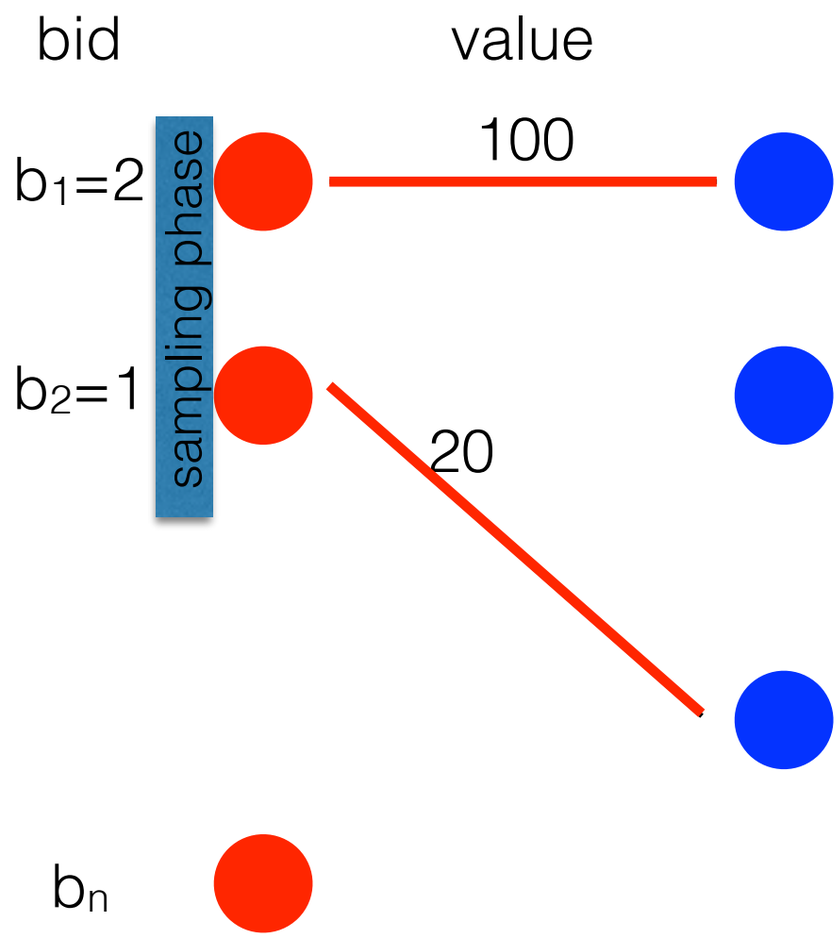
Find largest γ

$$\gamma \sum_{e \in M(\gamma)} v(e) \leq B$$

$\mathrm{M}(\gamma)$

Algorithm

same idea as before sampling and decision phase



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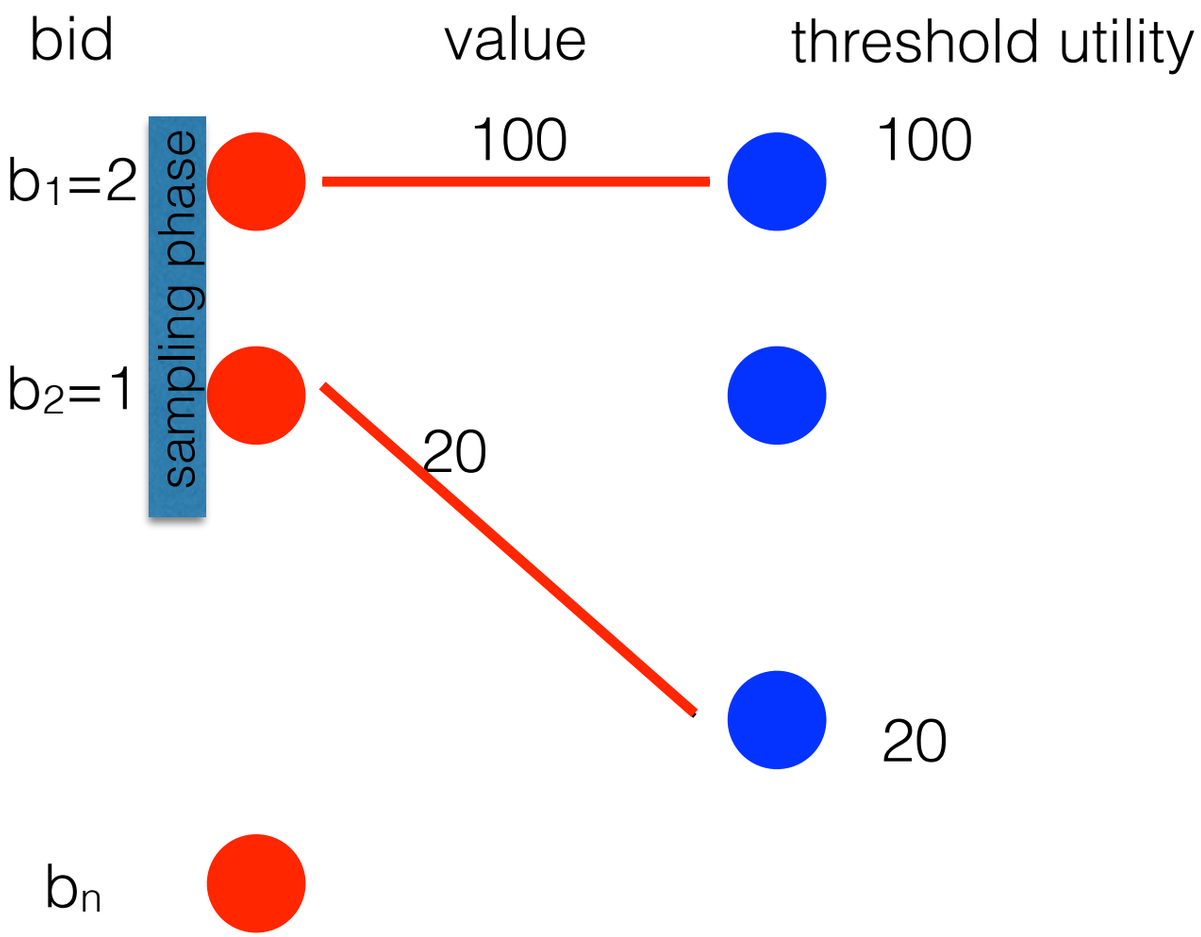
For decision phase utility threshold of each blue node to be value in Matching $\mathrm{M}(\gamma)$

Algorithm

same idea as before sampling and decision phase



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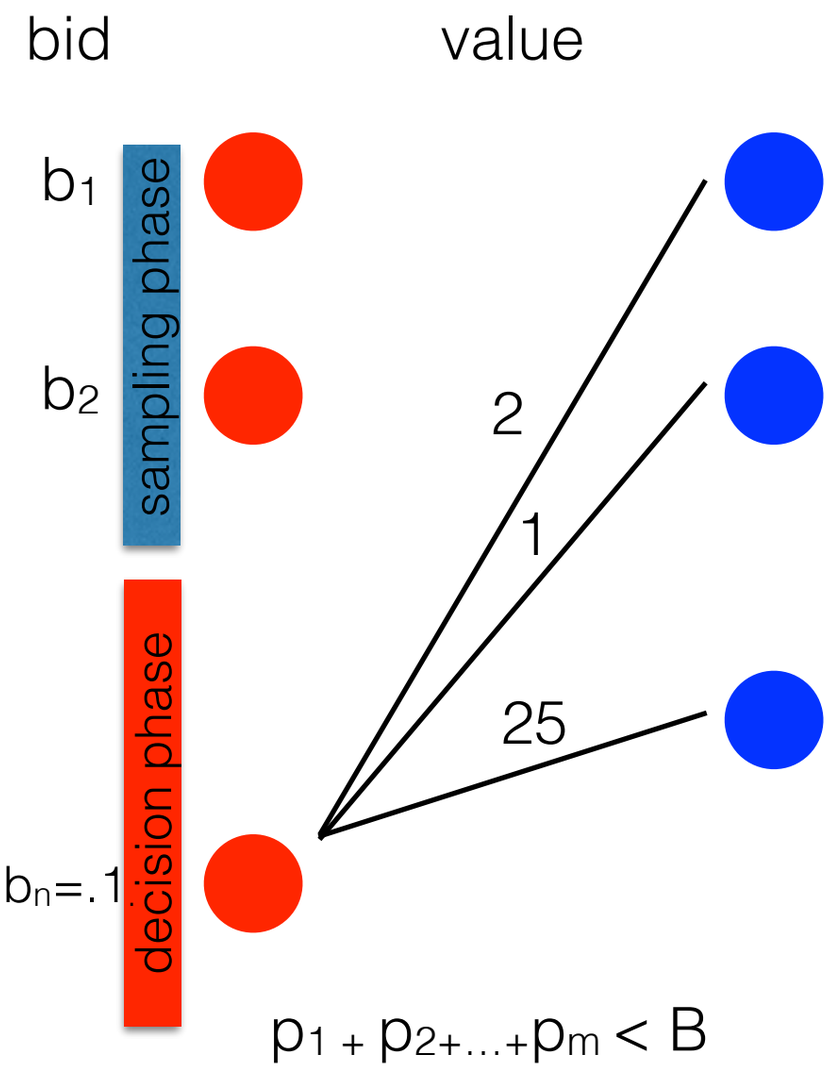
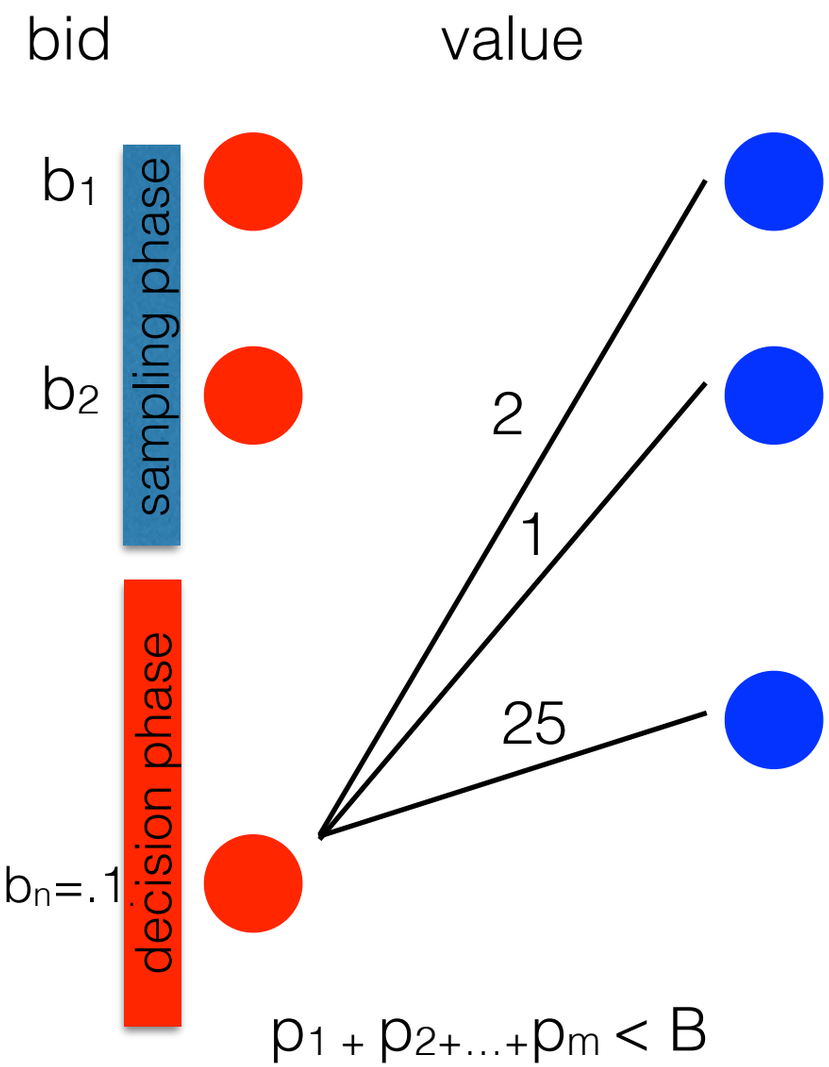
Algorithm - decision phase



Algorithm - decision phase

γ and threshold utility obtained from sampling phase



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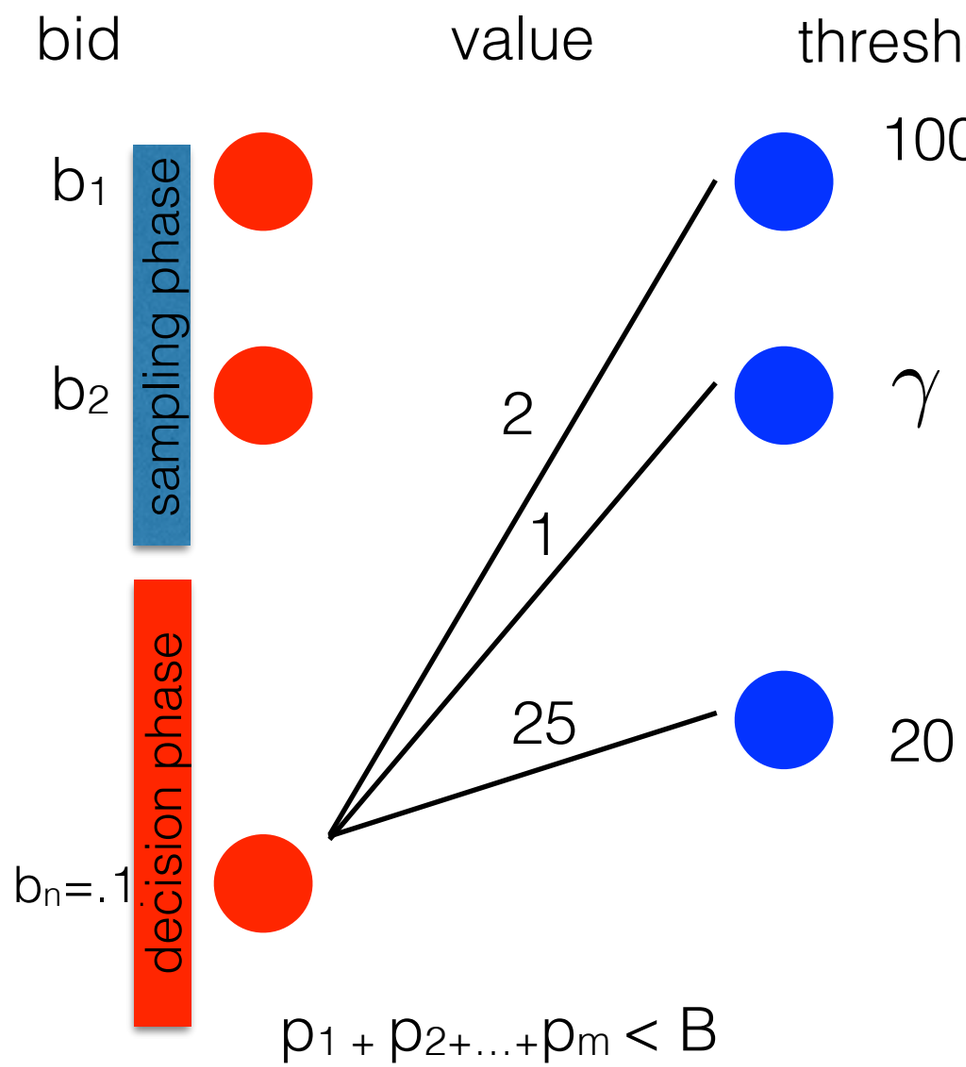

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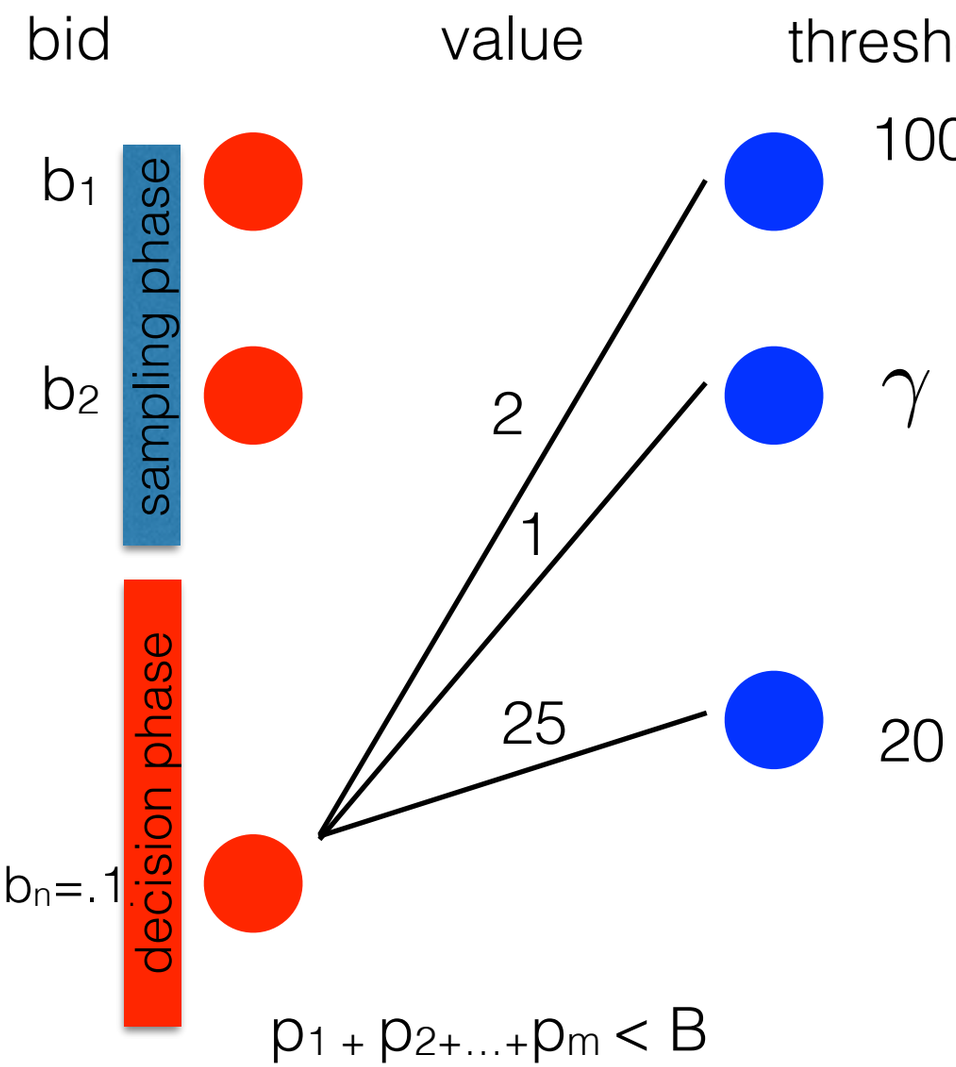
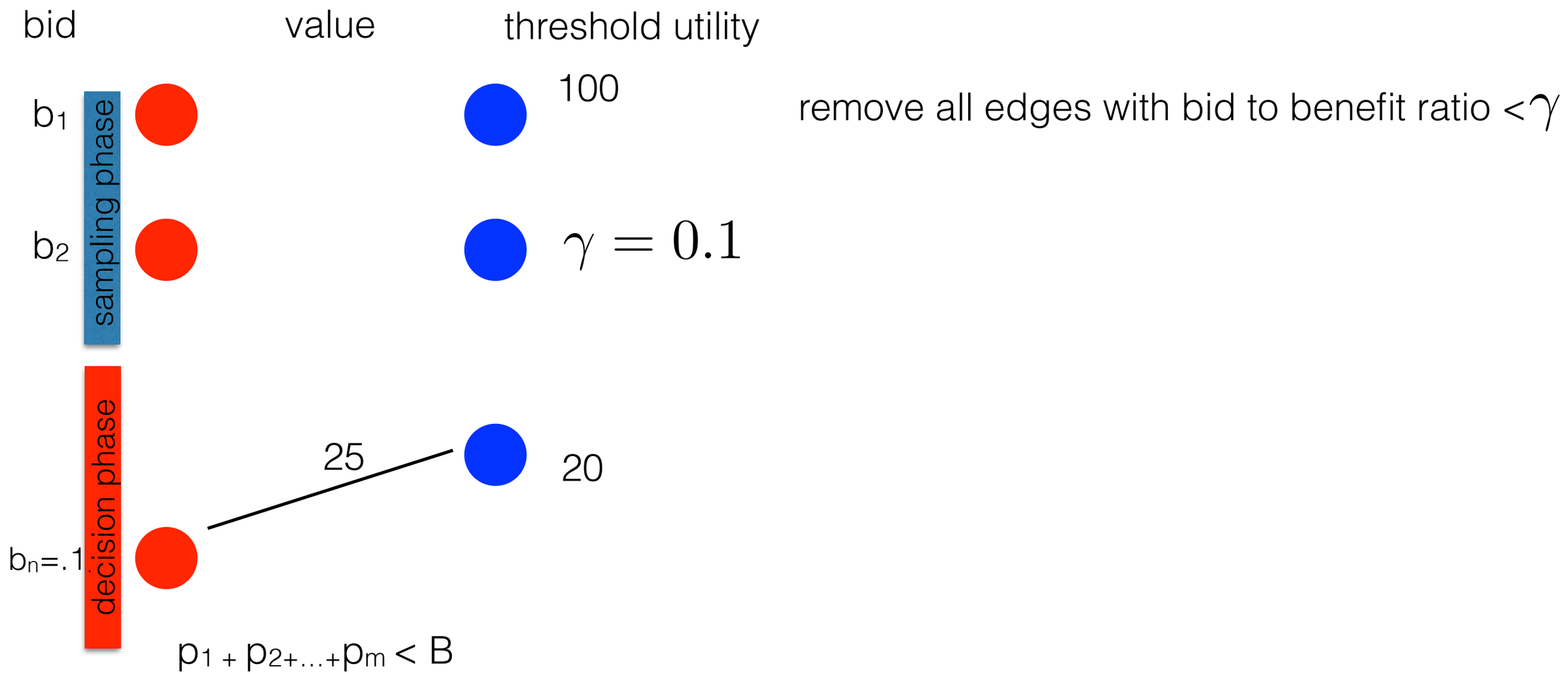


remove all edges with bid to benefit ratio $<\gamma$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



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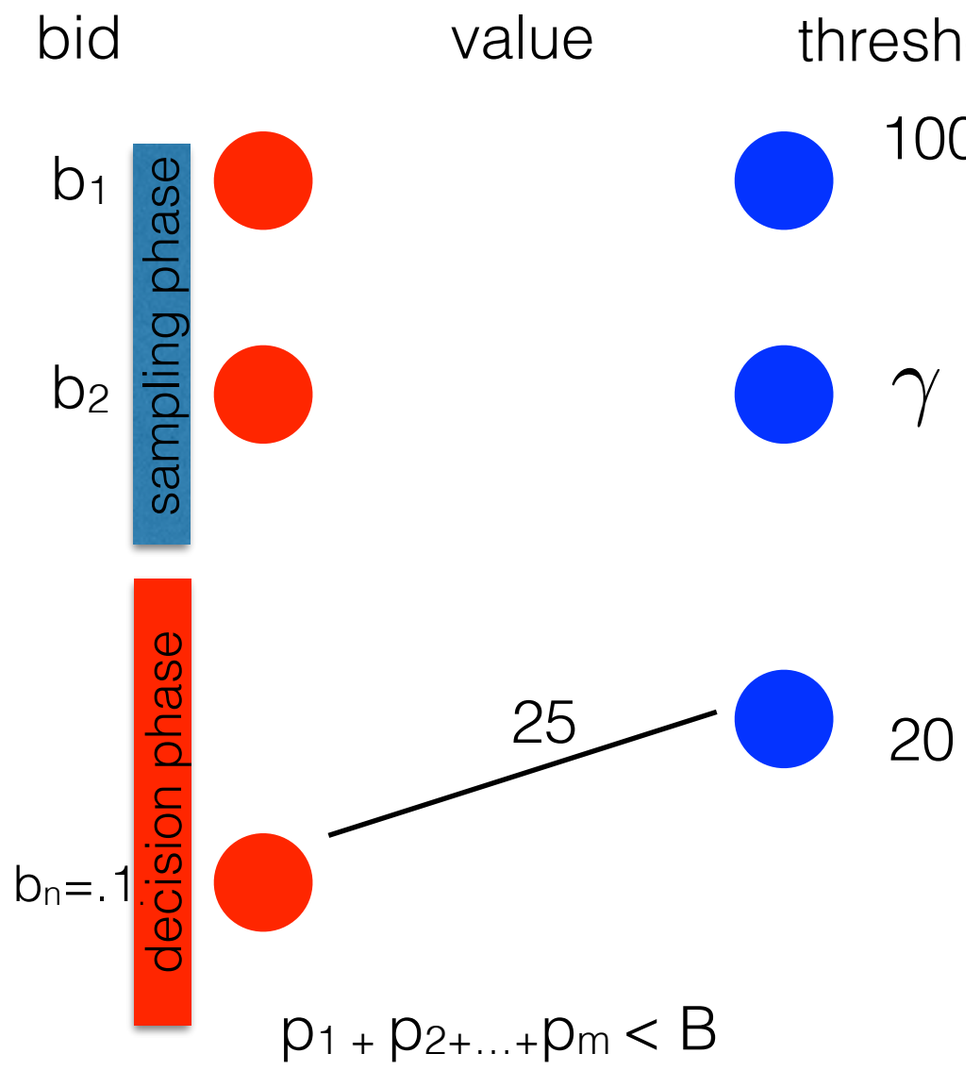
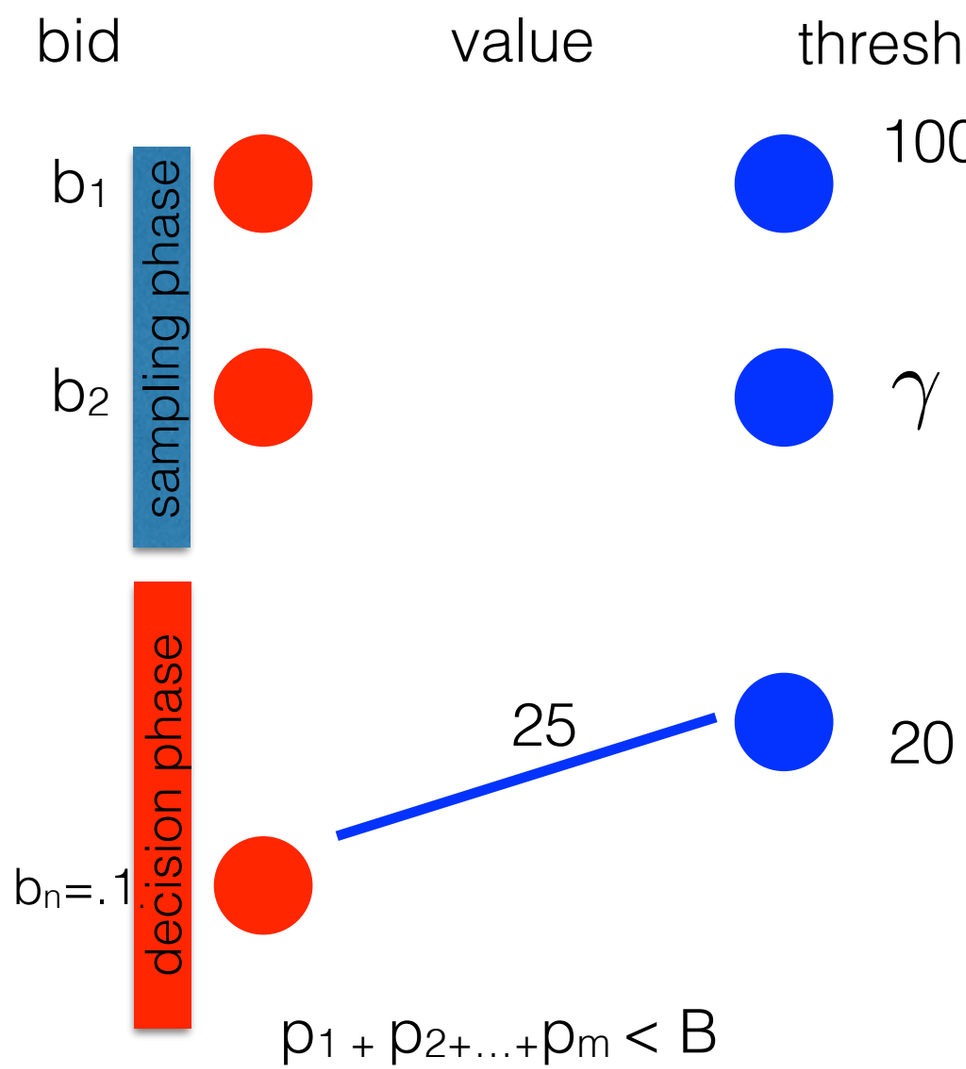


remove all edges with bid to benefit ratio $<\gamma$

match the largest weight more than utility threshold

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



In Decision Phase

remove all edges with bid to benefit ratio $< \gamma$

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Algorithm - decision phase

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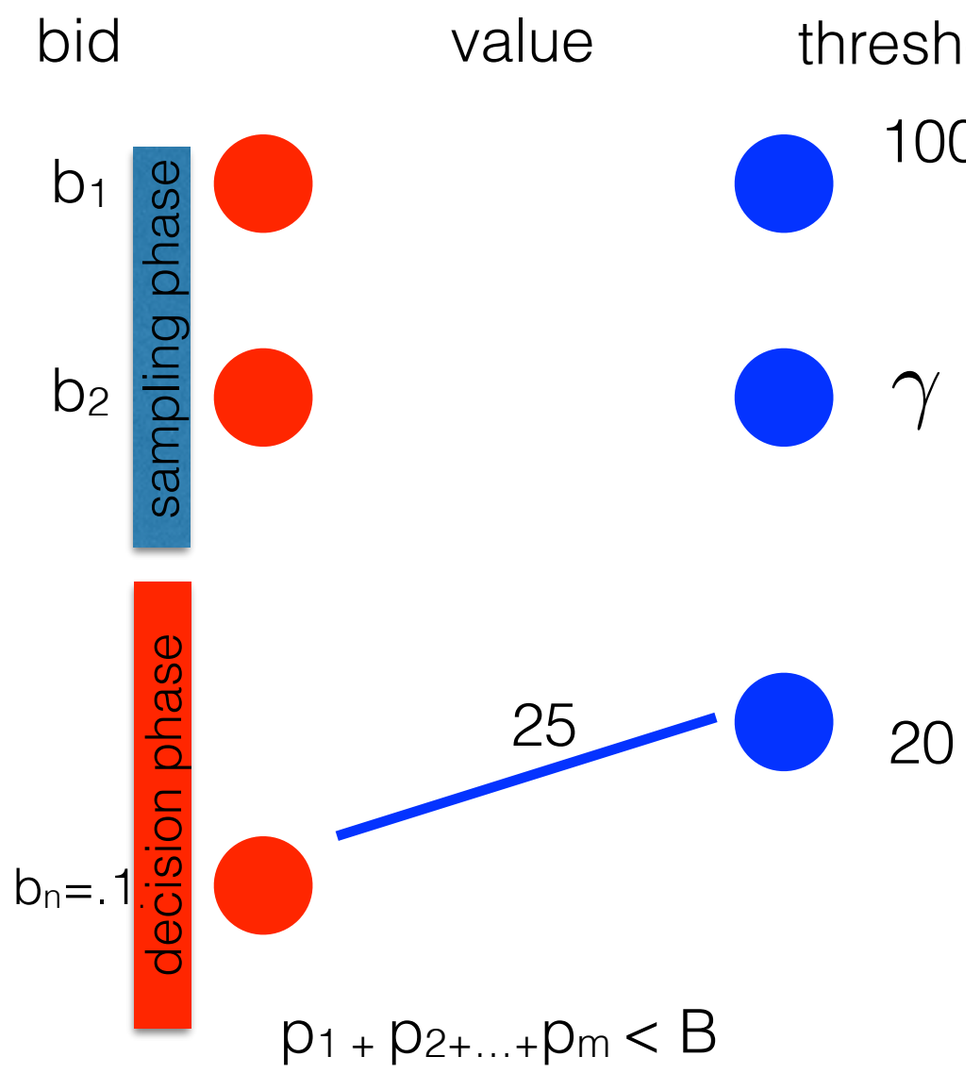


In Decision Phase

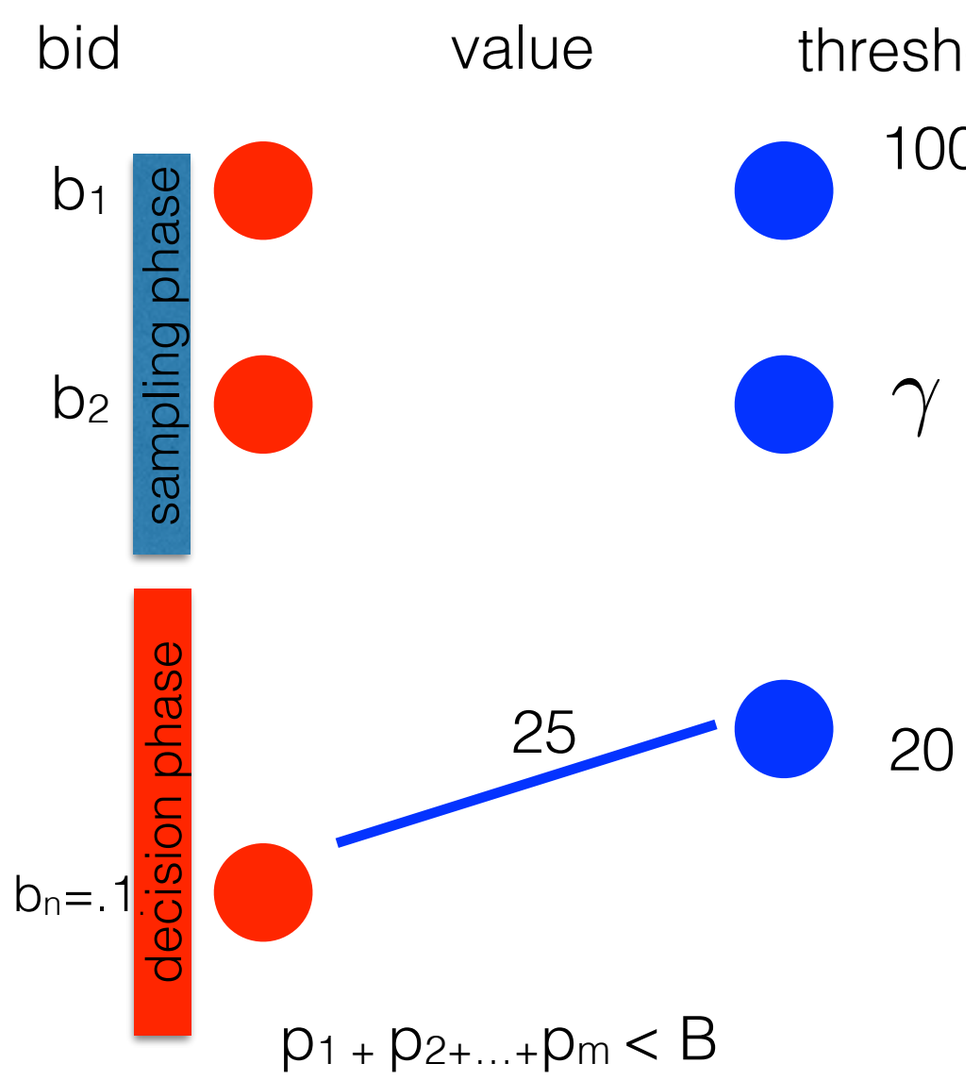
remove all edges with bid to benefit ratio $< \gamma$

match the largest weight more than utility threshold

Payment for each selected e, $p(e) = \gamma v(e)$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



In Decision Phase

remove all edges with bid to benefit ratio $<\gamma$

match the largest weight more than utility threshold

Payment for each selected e, $p(e) = \gamma v(e)$

Result: 144—competitive/optimal and truthful [*V*, *Coupechoux*]

when is a reverse auction truthful ?

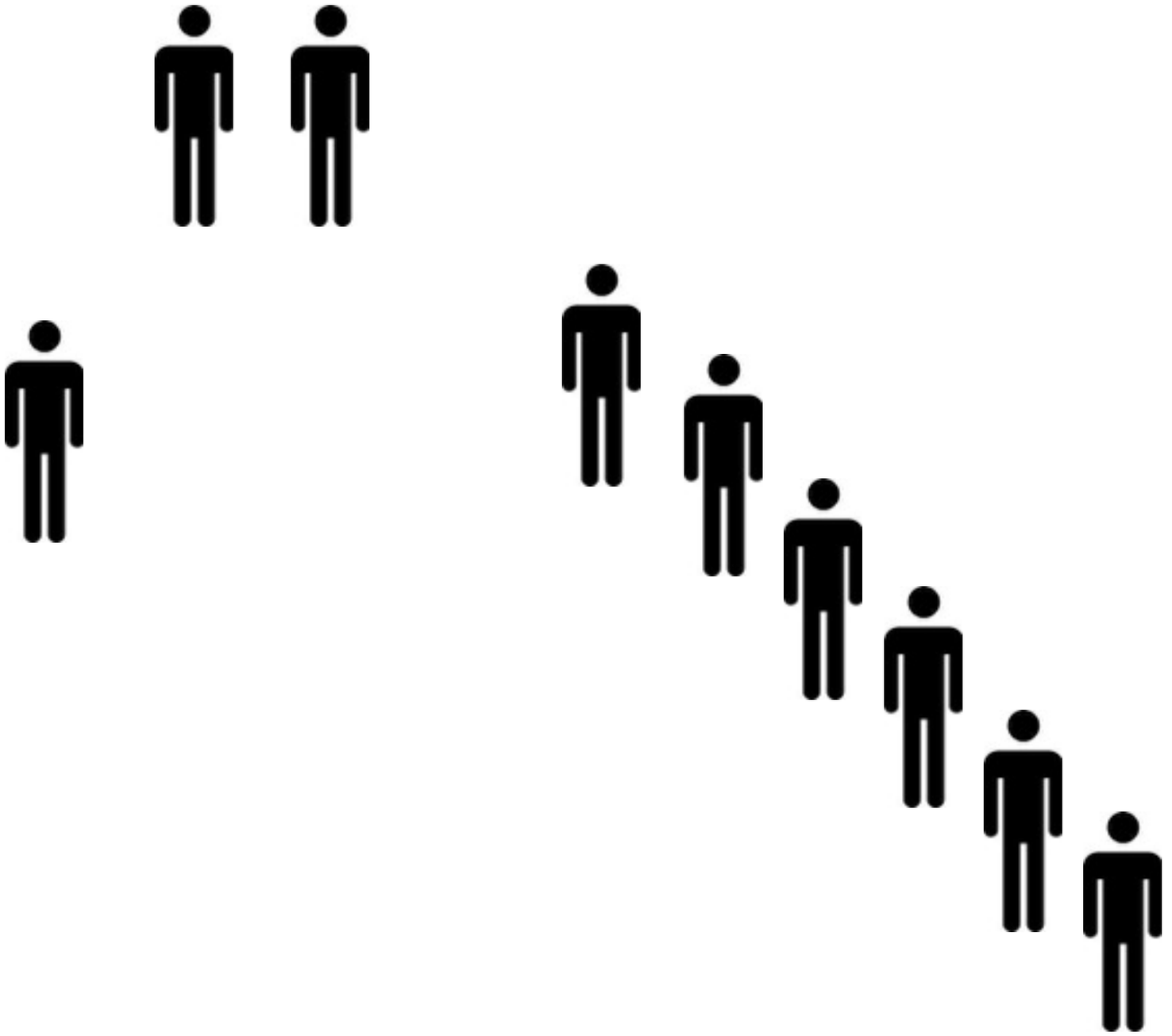
R. Myerson

Monotonicity - if an agent is selected with bid b, then
he is always selected if he bids below b

Critical Price - there exists a threshold price such that
if an agent bids above it he is never selected

I don't always fail

But when I do, I make sure that you're in the middle of something important.



Secretary Problem

1. Why arbit doesn't work
2. Randomized Model
3. Simple Algo 1/2

Sec. Prob as Matching with only one left vertex
Bipartite matching problem
Greedy 1/2 algo
Philosophy from Sec problem Hide the first half Set the price and select above the threshold

Wireless Problem -BS assoc
Equal weight case- Offline is to keep one per good BS
Use the same philosophy show that OFF < Max Weight
Designate one BS as garbage and do Online Matching on the rest Guarantee (M-1)/8M