Single Pixel Camera

Rahul Vaze

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Outline

- Modern Day Mega Pixel Cameras
- Riddle to Compact Storage
- Fundamental Principles
- Single-Pixel Camera
- Conclusions

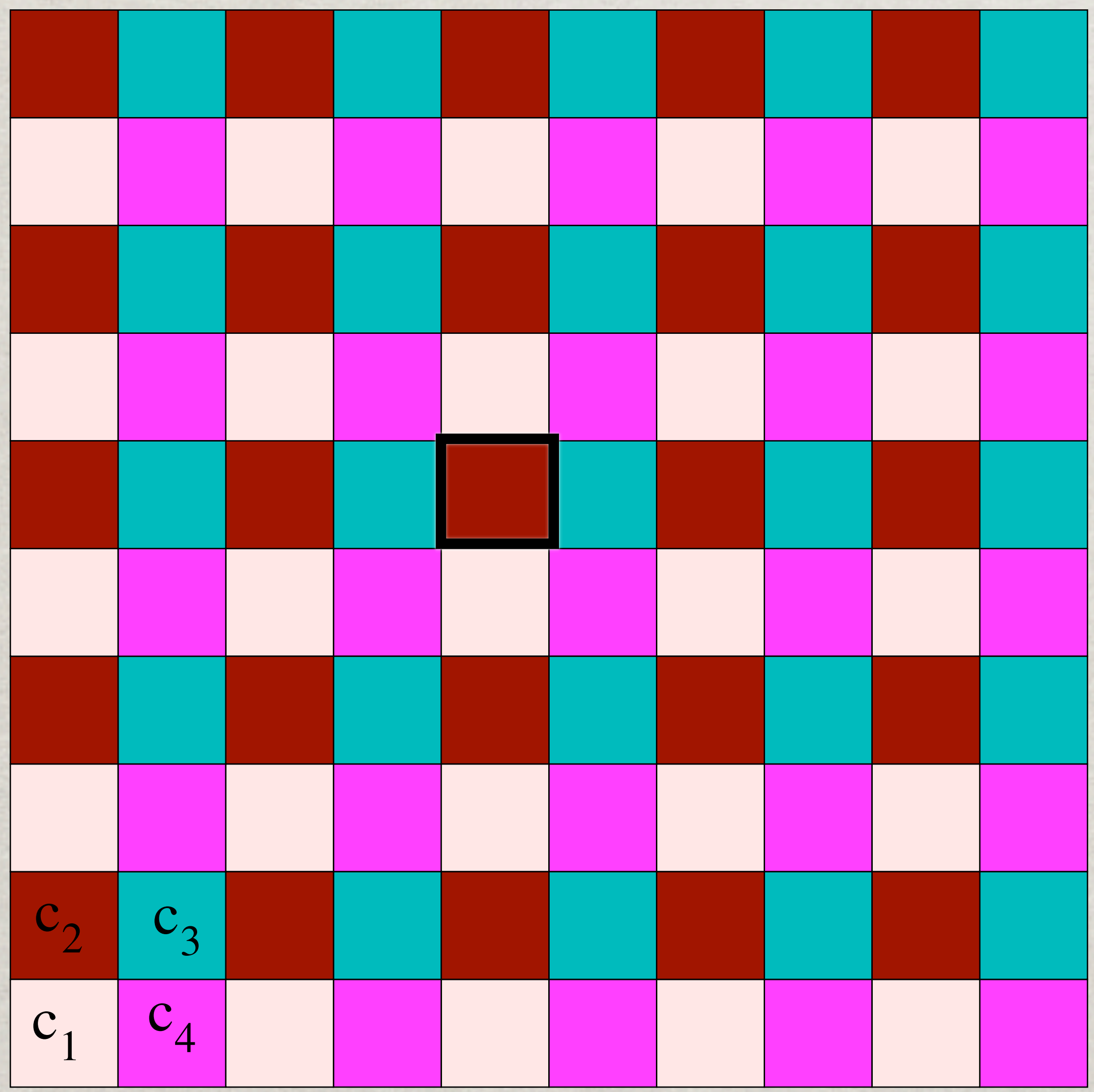
Modern Cameras

- 5 MP is common

- 10-15 MP is also popular

What is Pixel ?

- Smallest visual-sensor resolution



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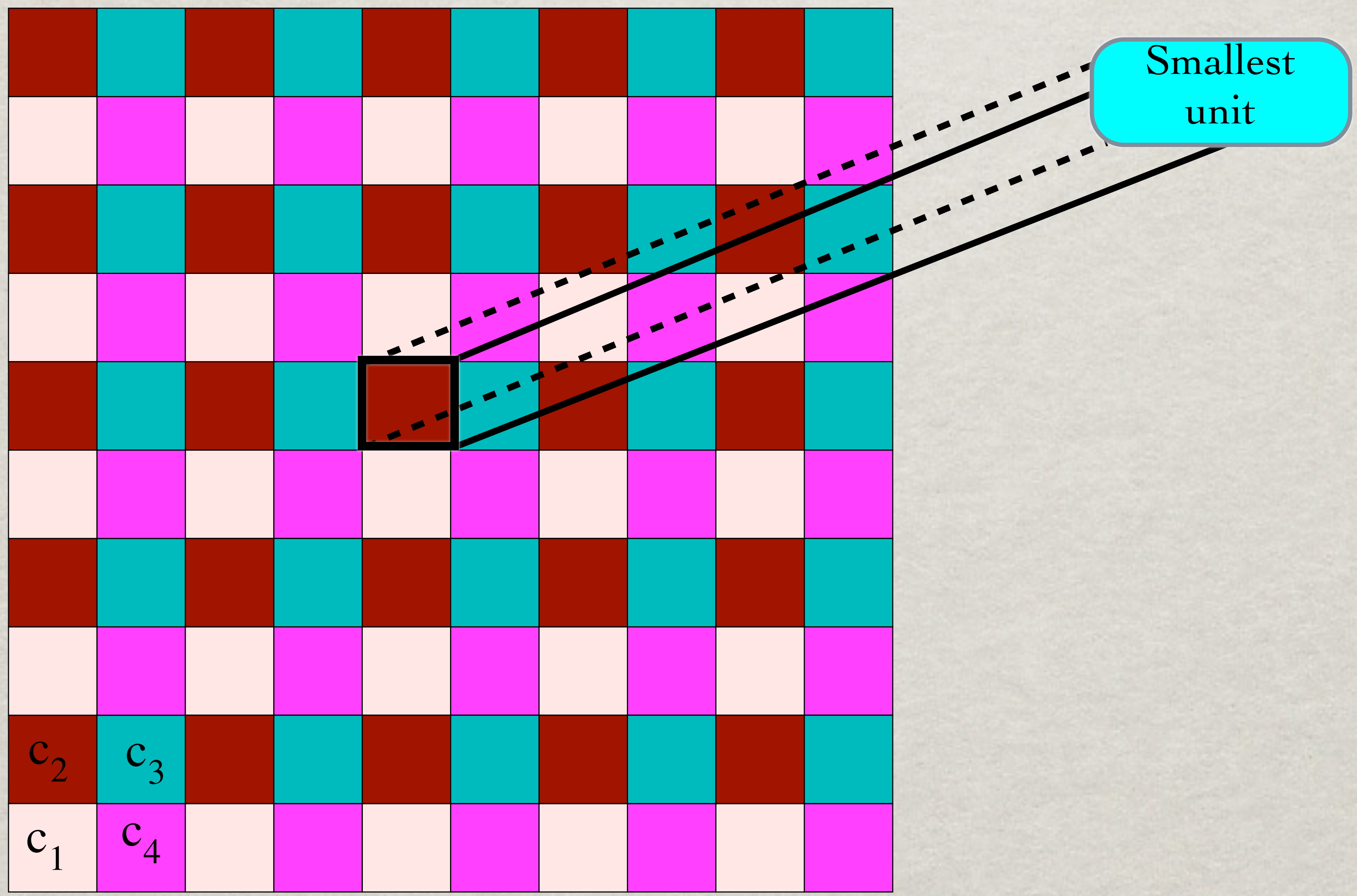

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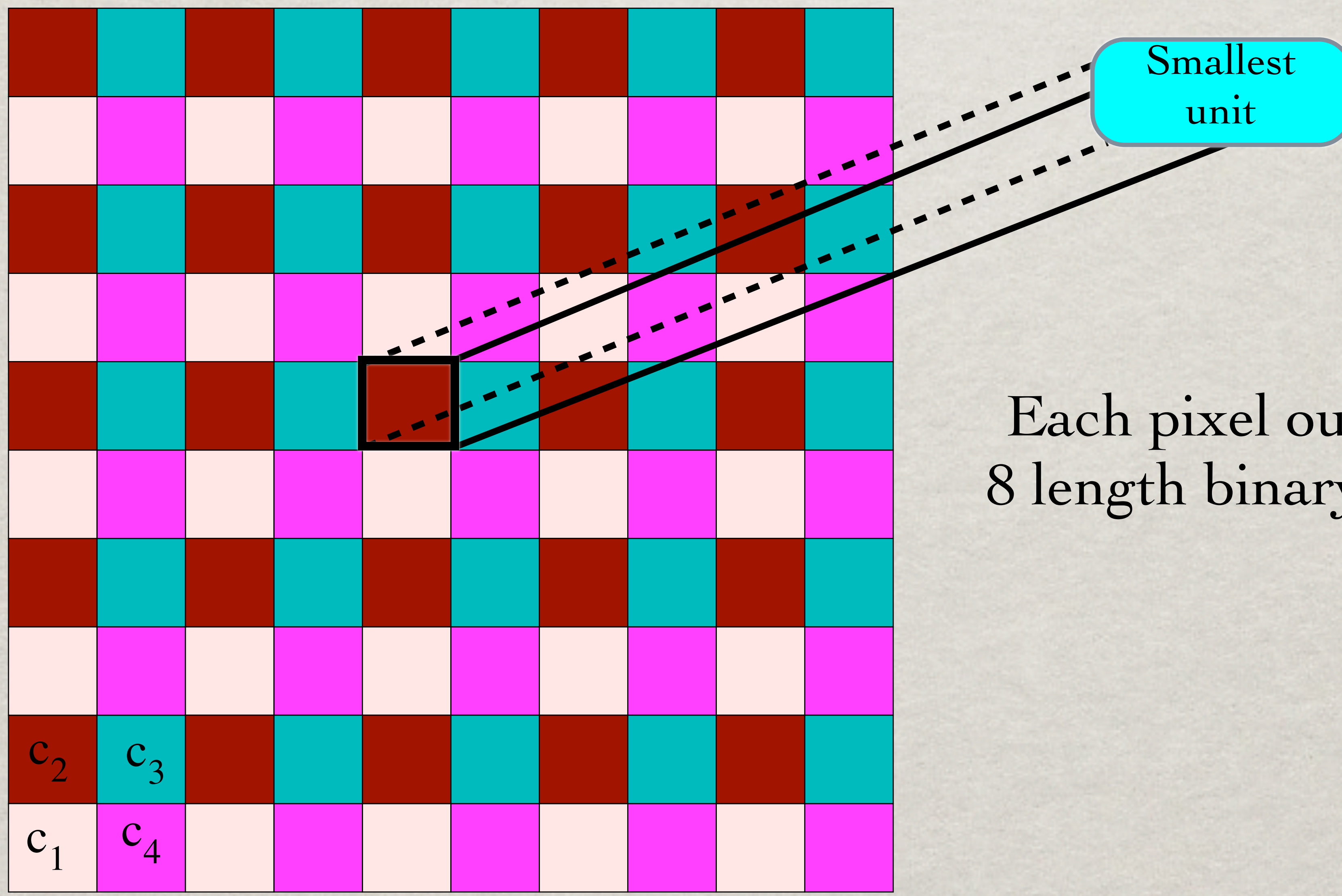

Each pixel output is
8 length binary string

Total Storage

- Each Pixel - roughly 8 bits
- 5MPixel - 5 MB of data per picture
- 1GB can store only 200 images
- But you can store many more
- Each picture - 200-440 KB
- Where is the MAGIC ?

Natural Images

Natural Images

Natural Images

- There is lot of local image correlation(similarity)
- Some simpler representation

History of this Image

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Alexander Sawchuk in summer of *73*, then an assistant professor of electrical engineering at the University of Southern California was hurriedly searching the lab for a good image to scan for a colleague's conference paper. They got tired of their stock of usual test images, dull stuff dating back to television standards work in the early 1960s. They wanted something glossy to ensure good output dynamic range, and they wanted a human face. Just then, somebody happened to walk in with a recent issue of *Playboy*.

David C. Munson, editor-in-chief, January 1996 *IEEE Transactions on Image Processing*
First, the image contains a nice mixture of detail, flat regions, shading, and texture that do a good job of testing various image processing algorithms. It is a good test image! Second, the Lena image is a picture of an attractive woman.

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Scientists do have an eye for beauty!

Analogy

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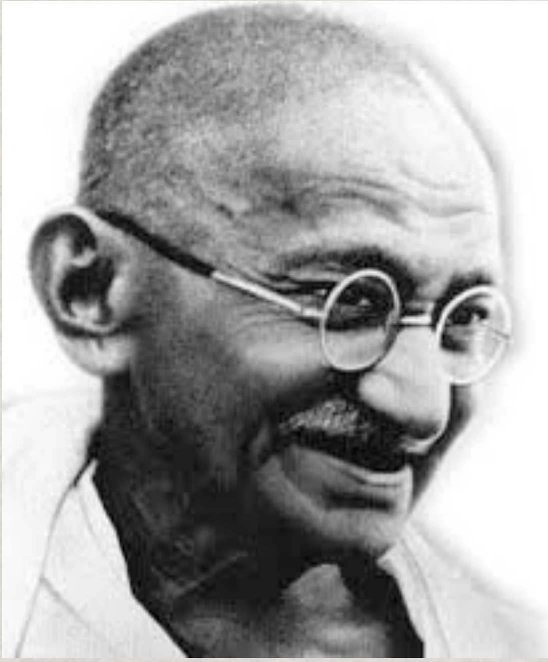

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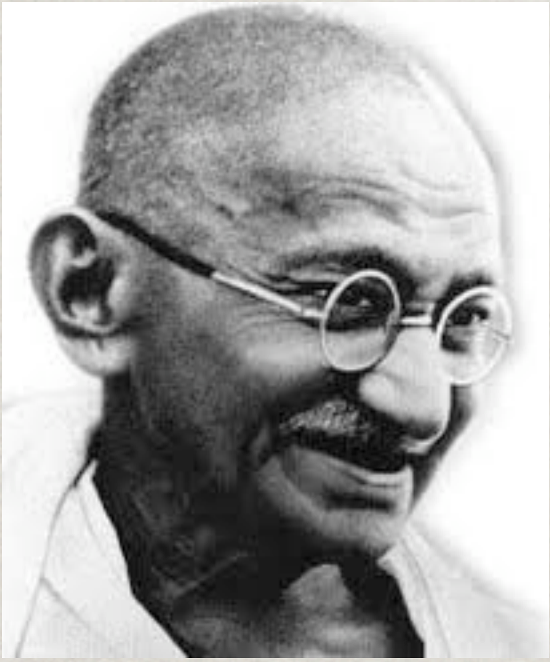


- Succinct Representation
- Reversible ?

Natural Images

Natural Images

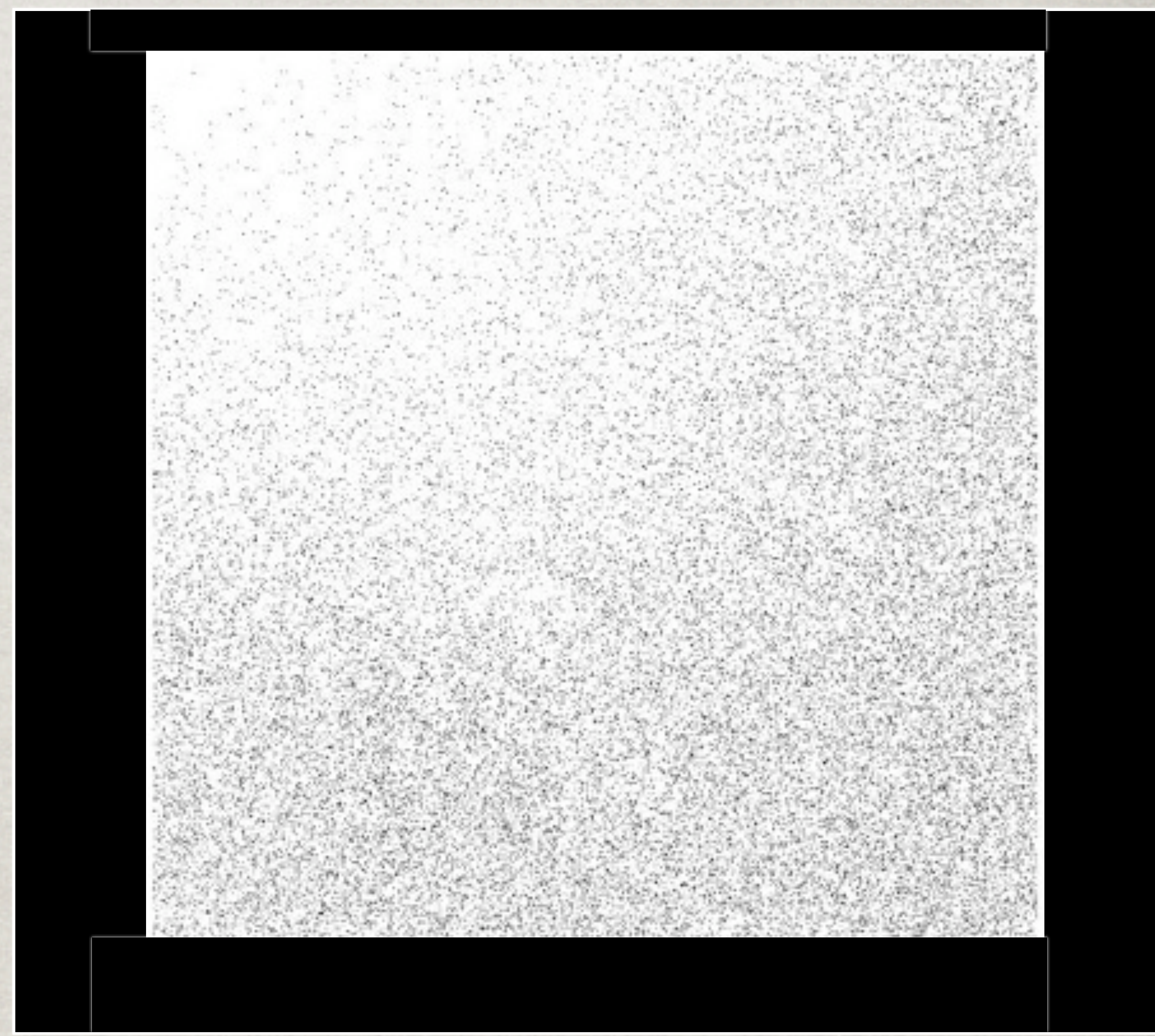
Natural Images

- There is lot of local image correlation(similarity)
- Some simpler representation
- Reversible

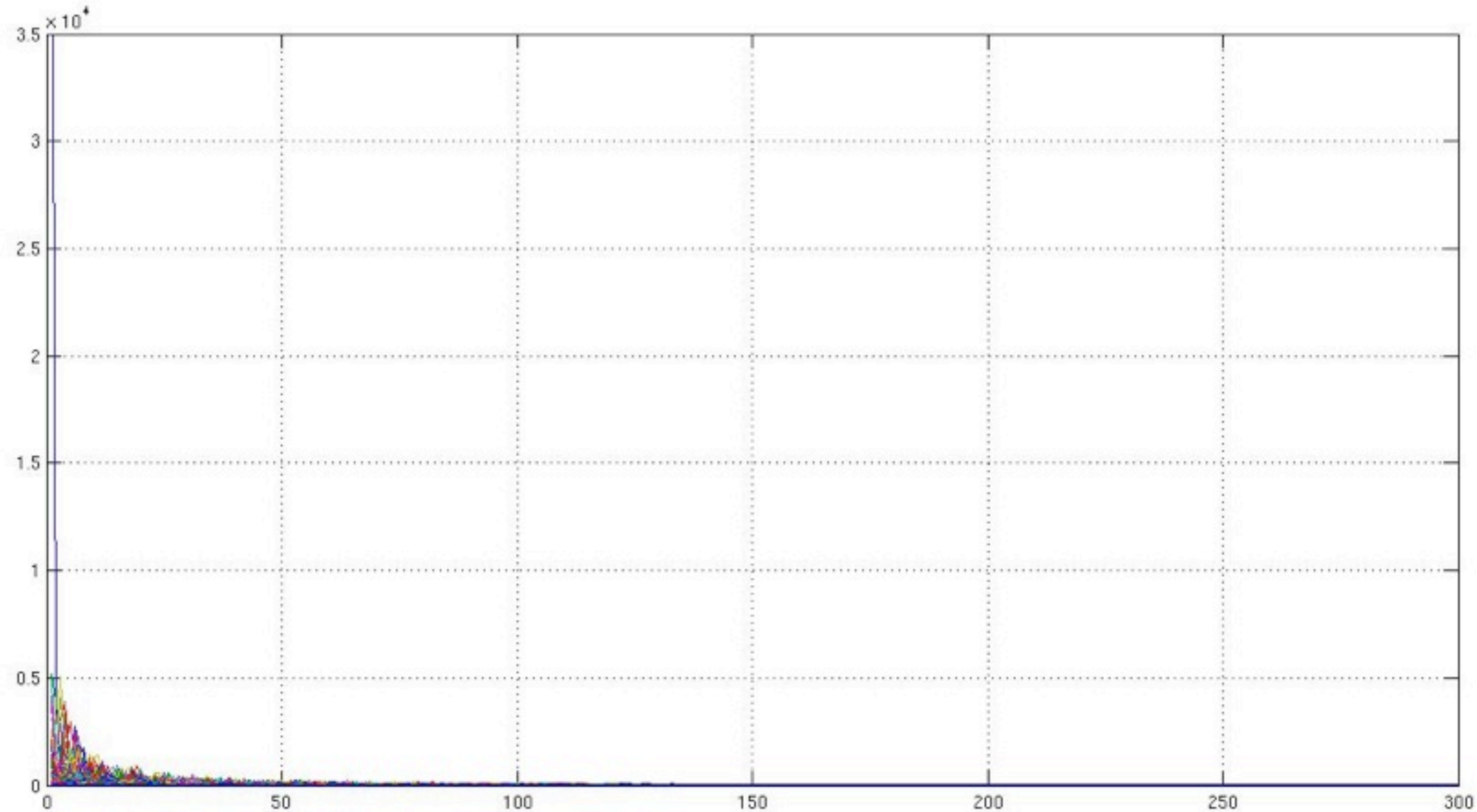
Magic Unravelled

- Discrete Cosine/Wavelet Transform

DCT



DCT



What is DCT ?

$$X_{k_1,k_2} = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{n_1,n_2} \cos\left[\frac{\pi}{N_1}\left(n_1+\frac{1}{2}\right)k_1\right] \cos\left[\frac{\pi}{N_2}\left(n_2+\frac{1}{2}\right)k_2\right].$$

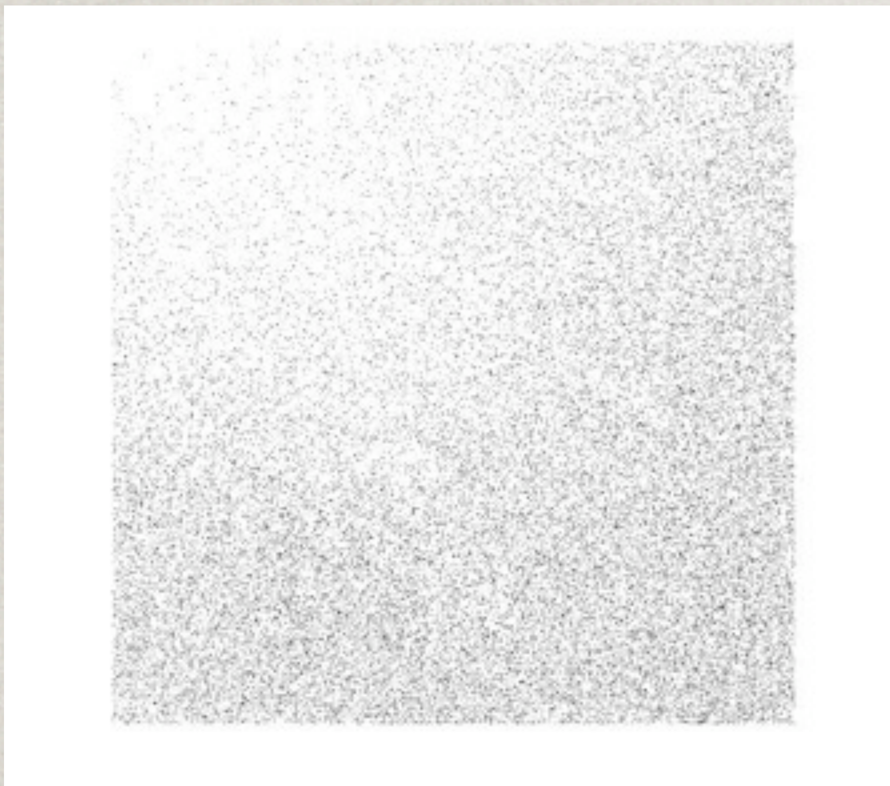
- Similar to Fourier Transform
- With no sine pulses
- Uses only real values (comp. to FT)
- Easier that K-L Transform (optimal)

Why DCT ?

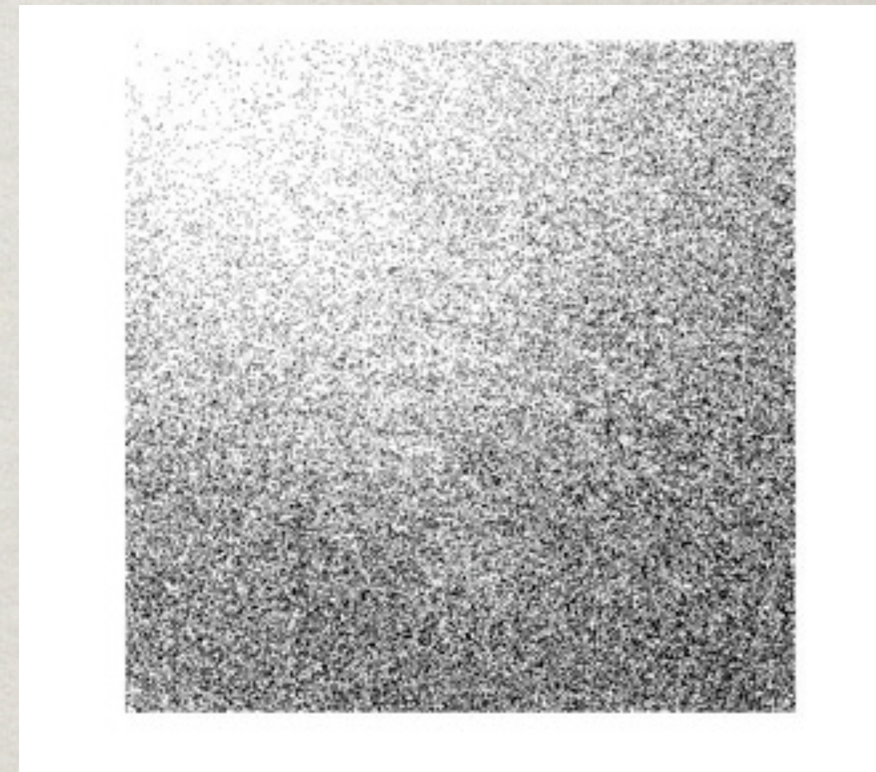
- Decorrelates (dis-similar) the data
- Energy compaction property
- Reduces entropy
- Useful for audio/image compression

Compressed DCT

Orig.



Orig.
<1 removed



Orig.
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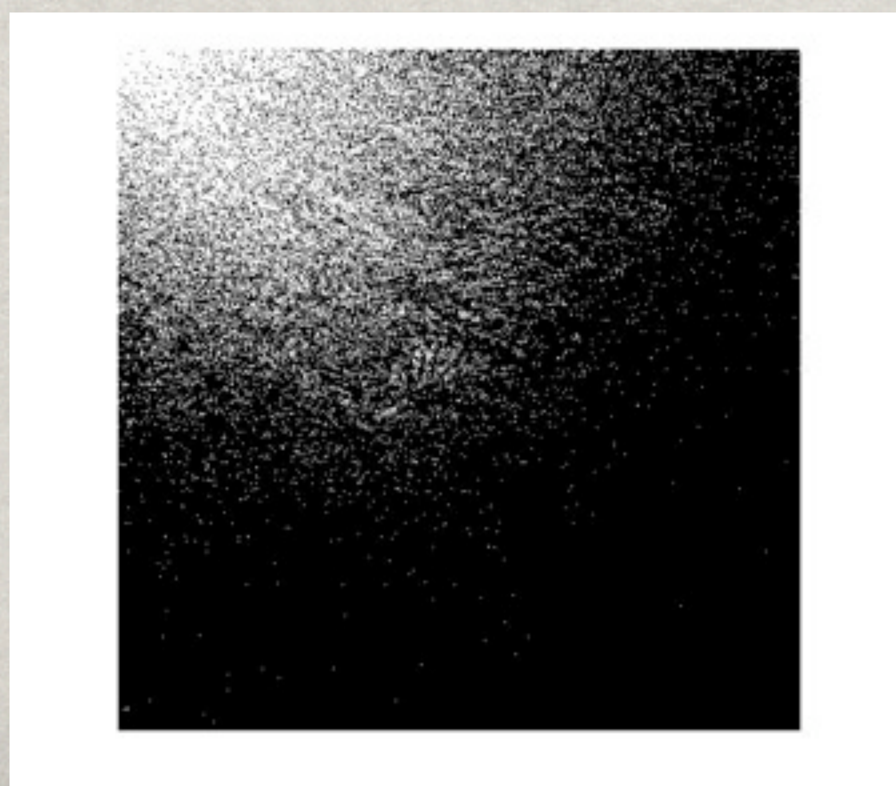
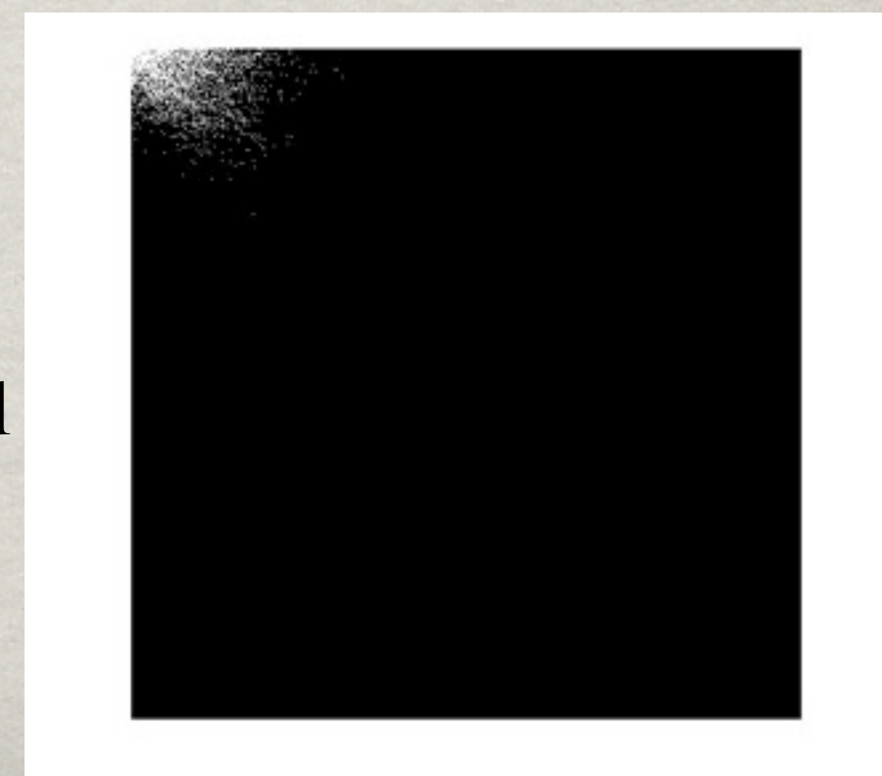


Orig.
<100 removed



Uncompressed-IDCT

Orig.

Rec.
<1 removed

Rec.
<10 removed

Rec.
<100 removed

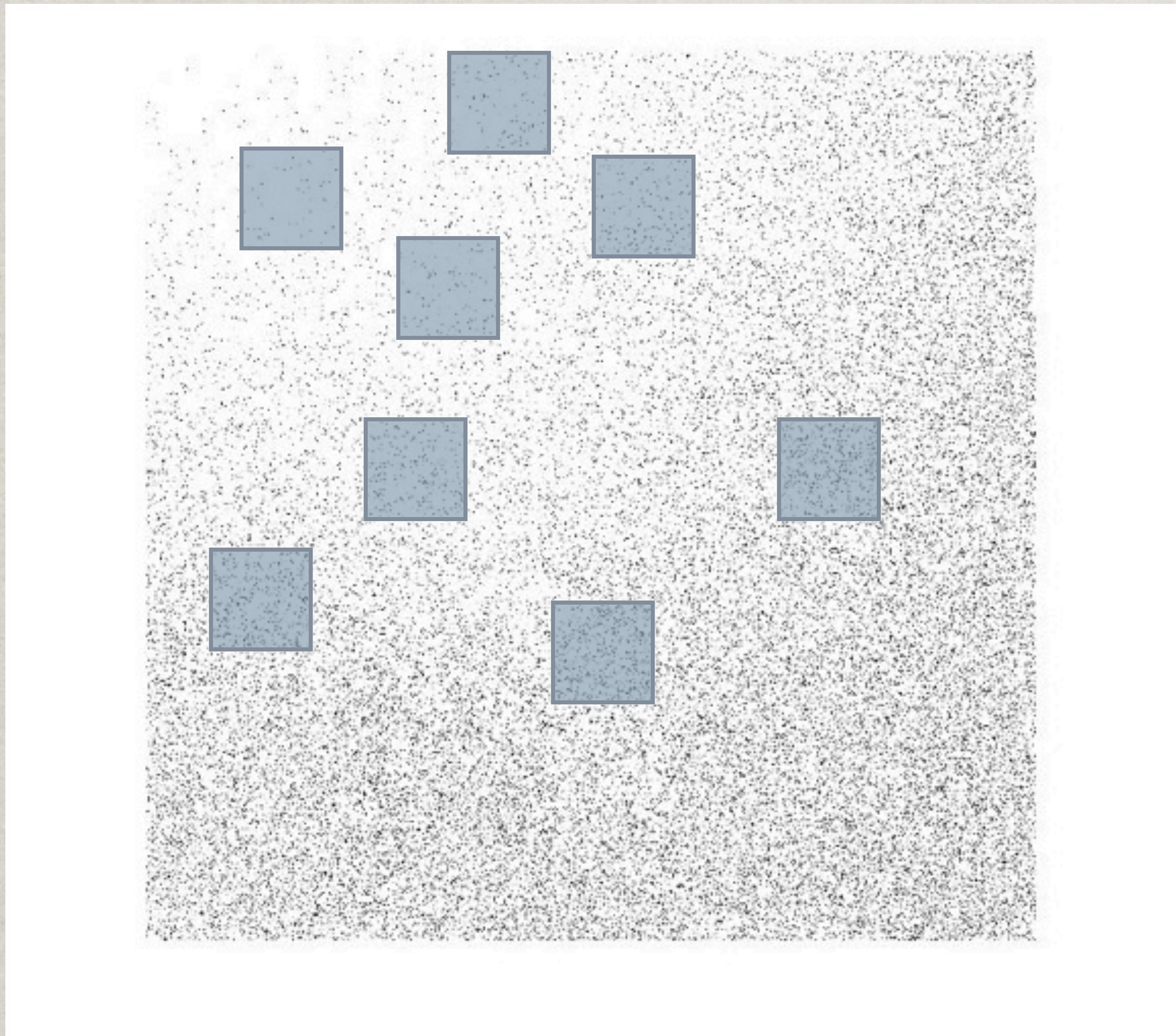
Digital Image Storage

- Store only the significant DCT components
- Use IDCT to recover the image
- Significant savings in storage
- Almost no perceptible image quality loss

Philosophical Lesson ?

- Conventional photography
  - collects a lot
  - throws away a lot
- Can we collect as much as we need
  - Ans: Compressed sensing

Naive Approach For Compression



Select some fixed number of entries of DCT

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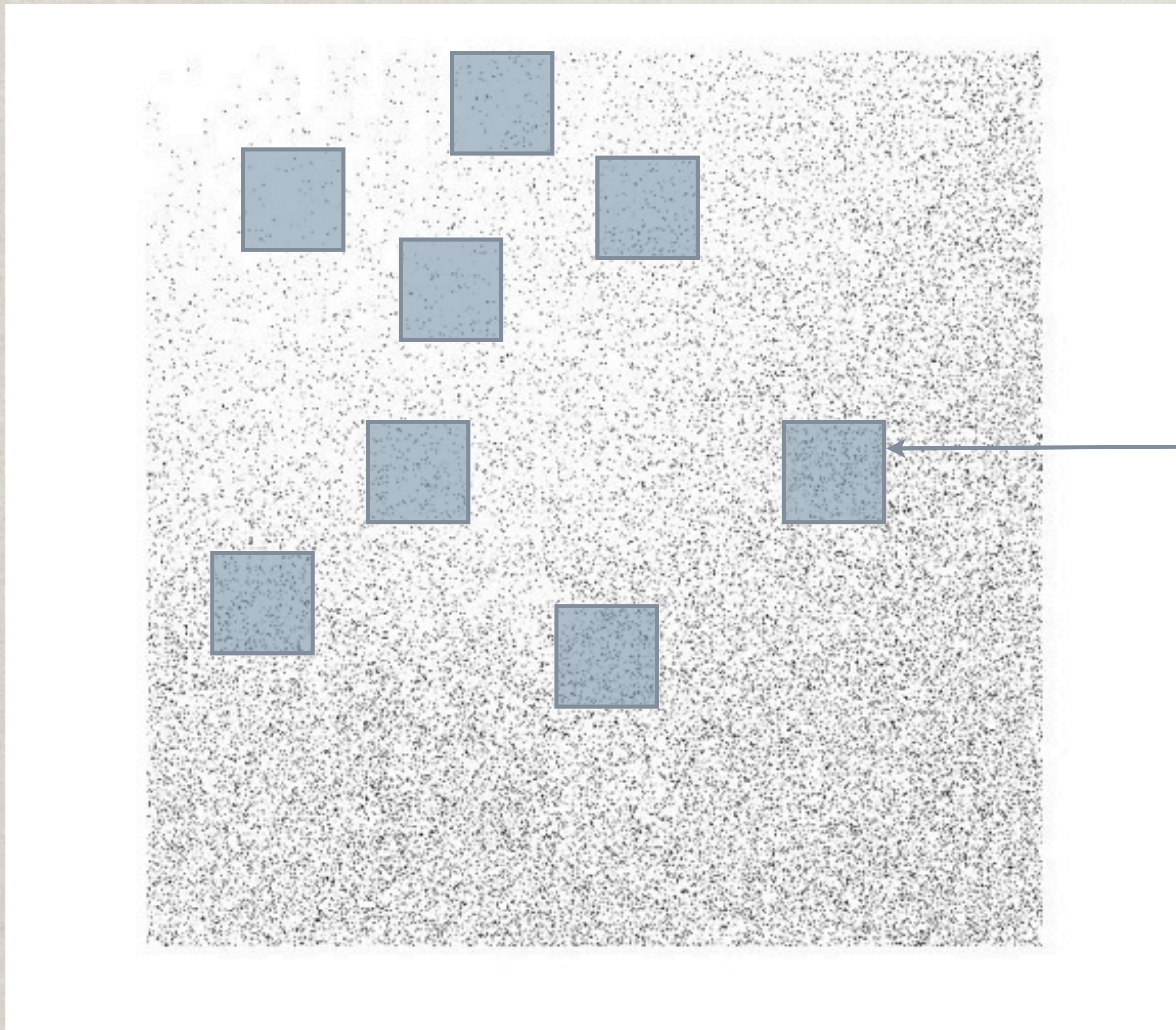

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where to place them ?

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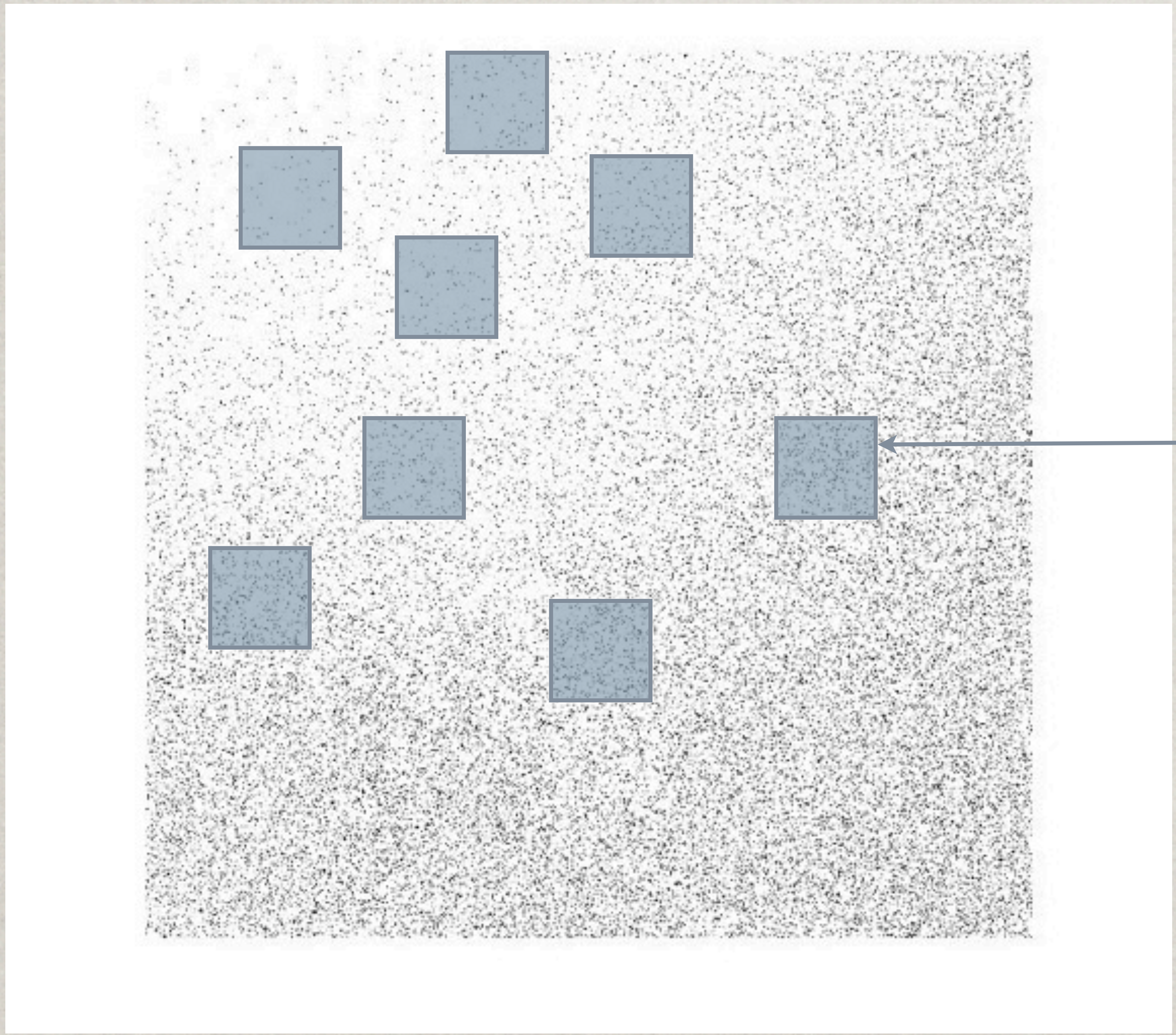

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it has to be image dependent

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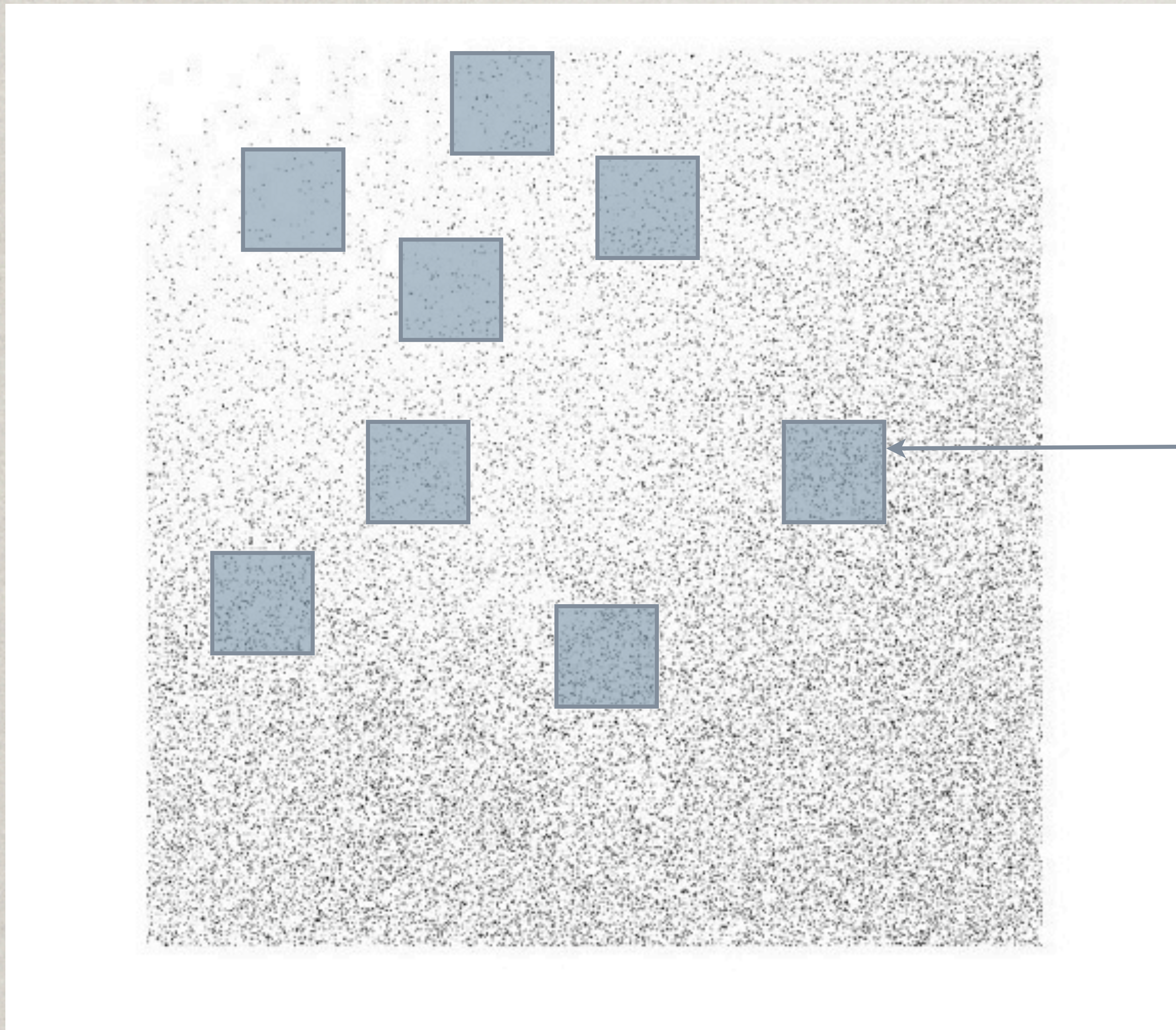

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it has to be image dependent

Option: Take random linear combinations

Compressed Sensing

Compressed Sensing

vector x - DCT image

$$x = D \times I$$

Compressed Sensing

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mN} \end{bmatrix}}_{A}$$

vector x - DCT image

$x = D \times I$

Matrix A for
random linear combinations
of x

Compressed Sensing

$$y = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mN} \end{bmatrix}}_{A}$$

$$m << N$$

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Matrix A for random linear combinations of x

Observation vector y

Compressed Sensing

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vector x - DCT image

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Matrix A for random linear combinations of x

Observation vector y

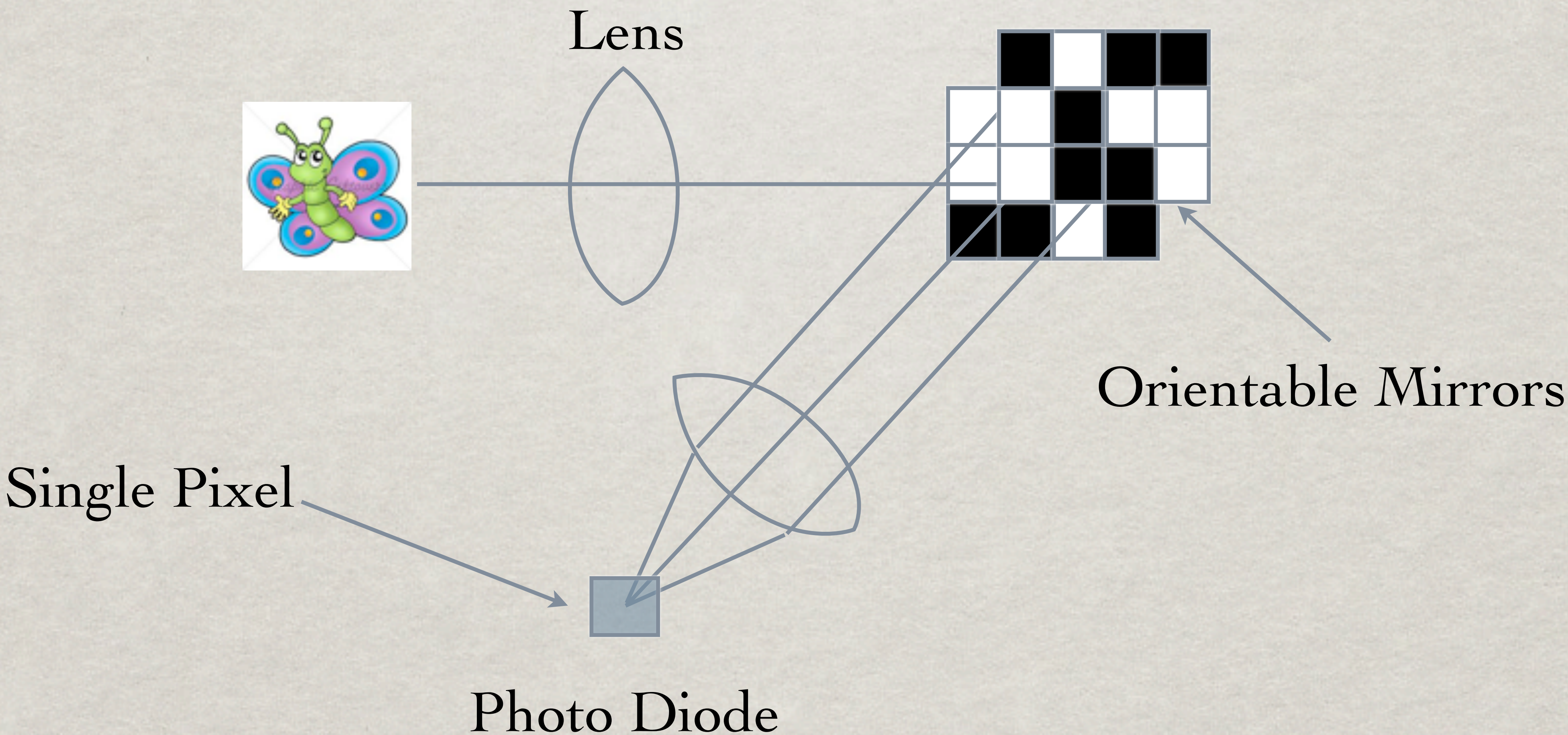
Recovery

find x

$$y = Ax$$

$$|x|_0 \leq s$$

Single Pixel Camera Assembly

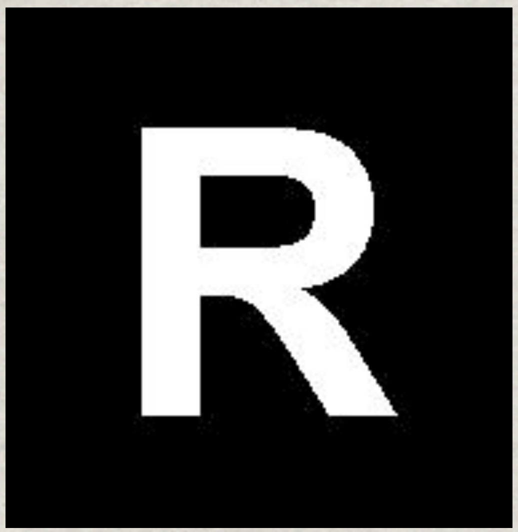


No. of Measurements m ~ s log N/s

Single Pixel Camera

- Single pixel used m times
- Each time taking a different linear combination of x
- Hope : We can recover I !

Image Reconstruction

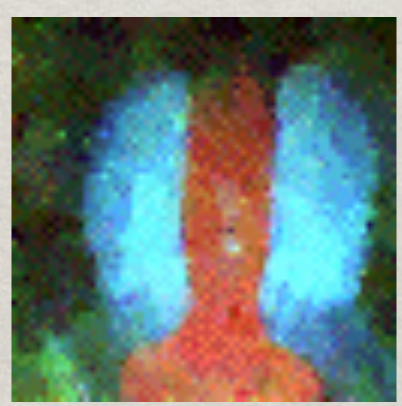


Original

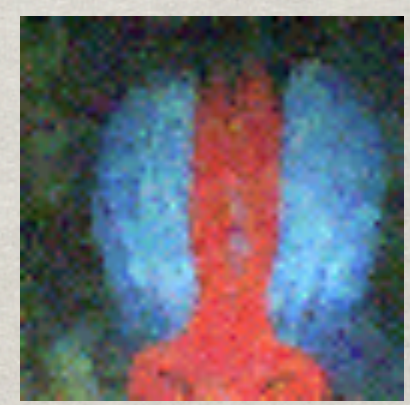
65536 Pixels
m=1300

65536 Pixels
m=3300

Original

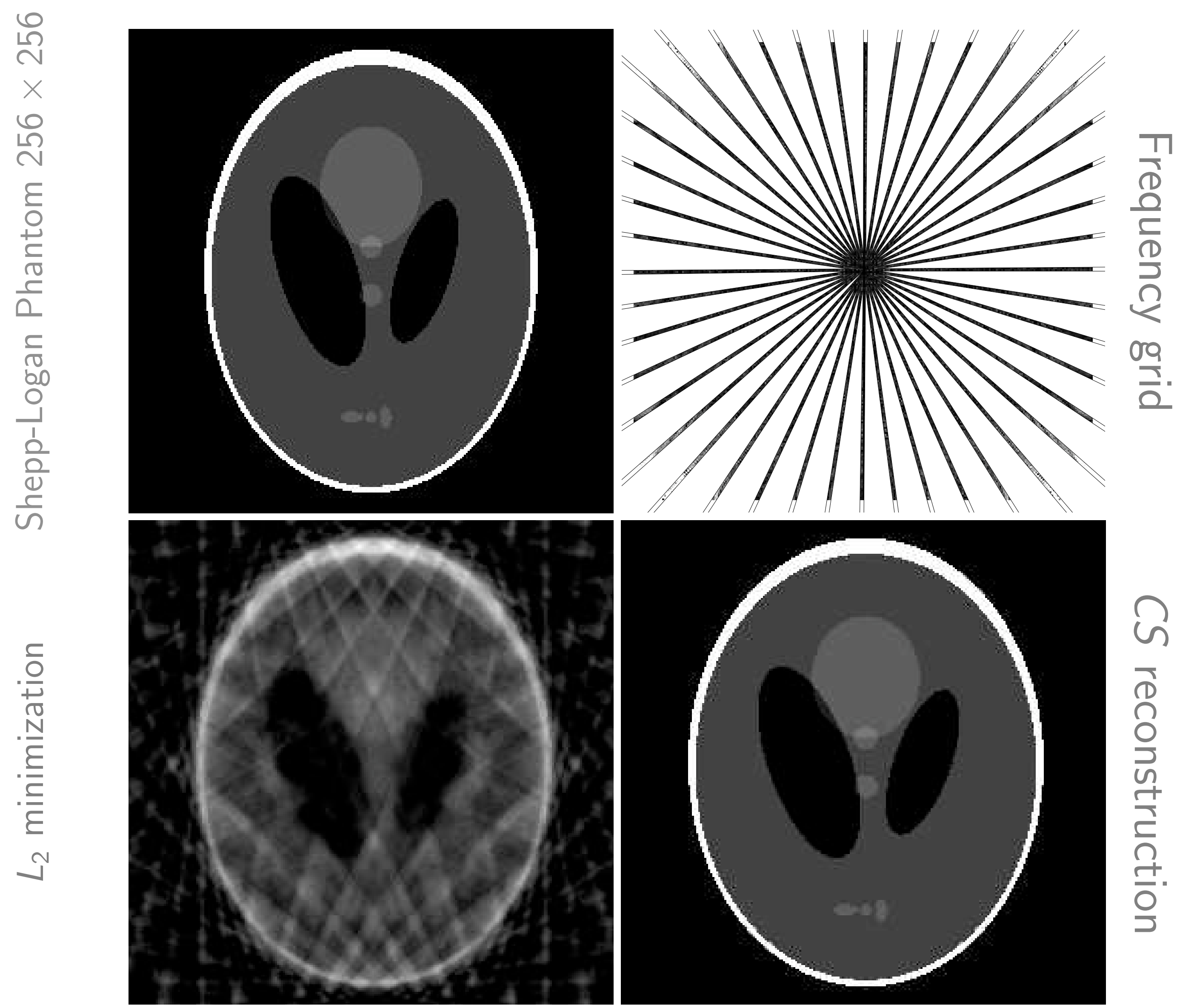


4096 Pixels
m=800



4096 Pixels
m=1600

Magnetic Resonance Imaging



We can recover MRI with 10 % measurements

Why CS ?

- No extraordinary gain in photography
- CCDs are cheap (because silicon responds to visible light)
- Real gain is in non-visible spectrum image acquisition for which CCDs are costly
- Use of exotic detectors (not possible with MP CCDs)

Simple Geometry

Simple Geometry

Let image be

$$x_1$$
$$x_2$$

Simple Geometry

Let image be $\begin{matrix} x_1 \\ x_2 \end{matrix}$

Two Measurements

Simple Geometry

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Two Measurements

$$\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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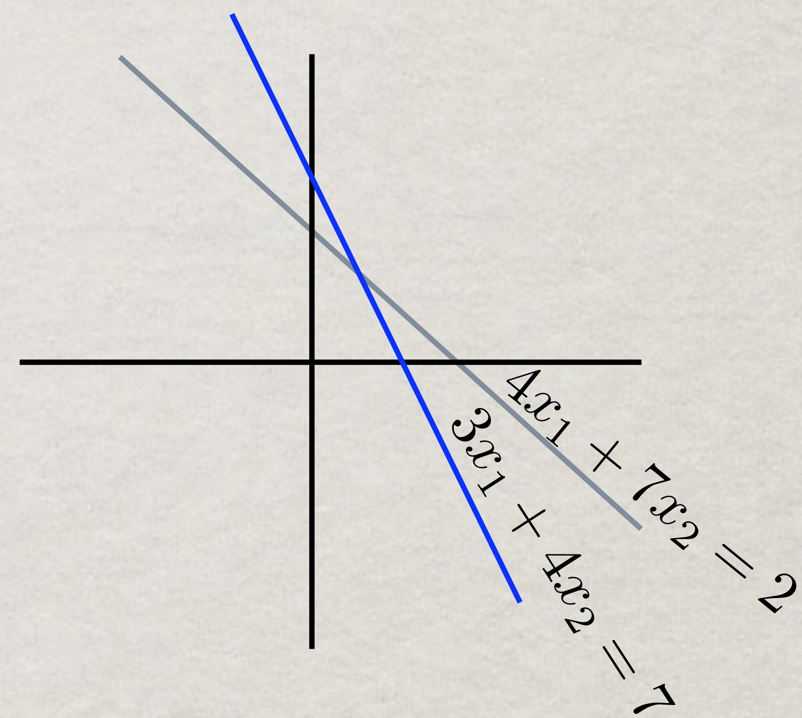

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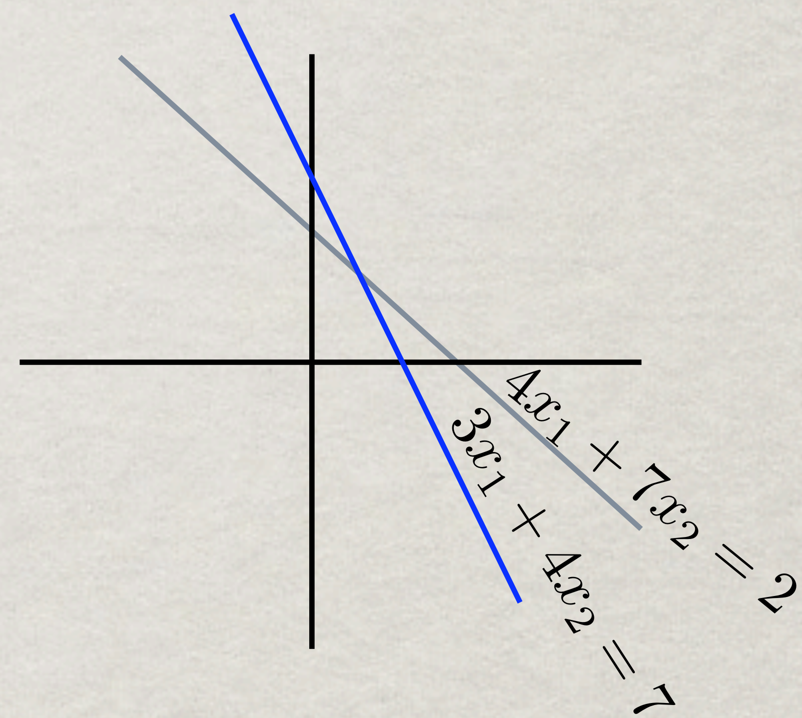

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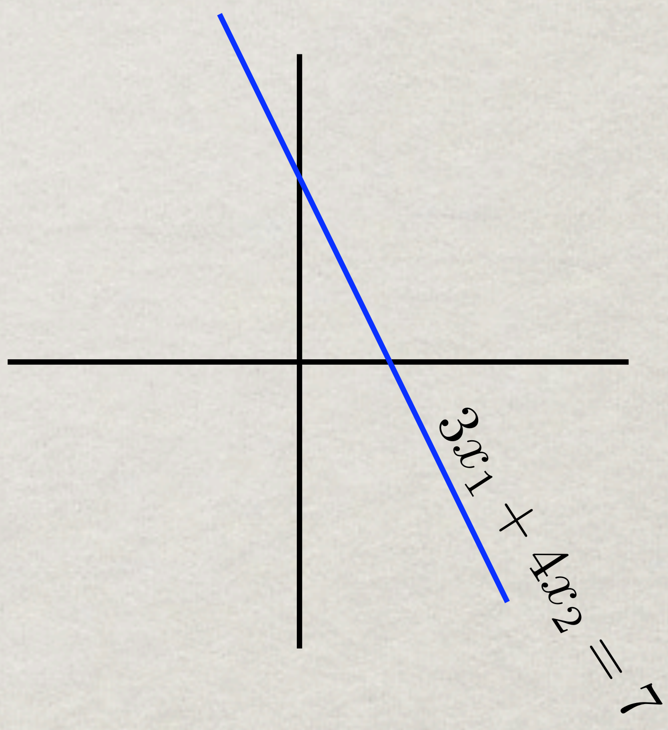

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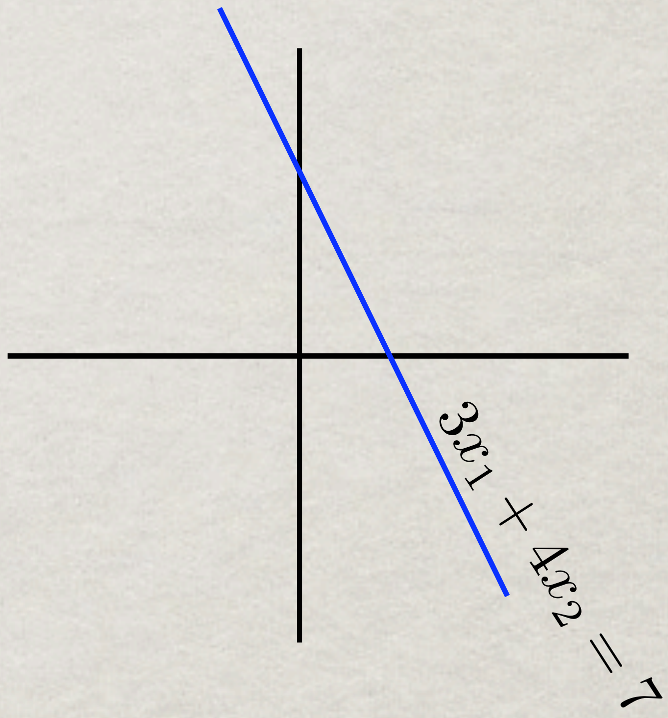

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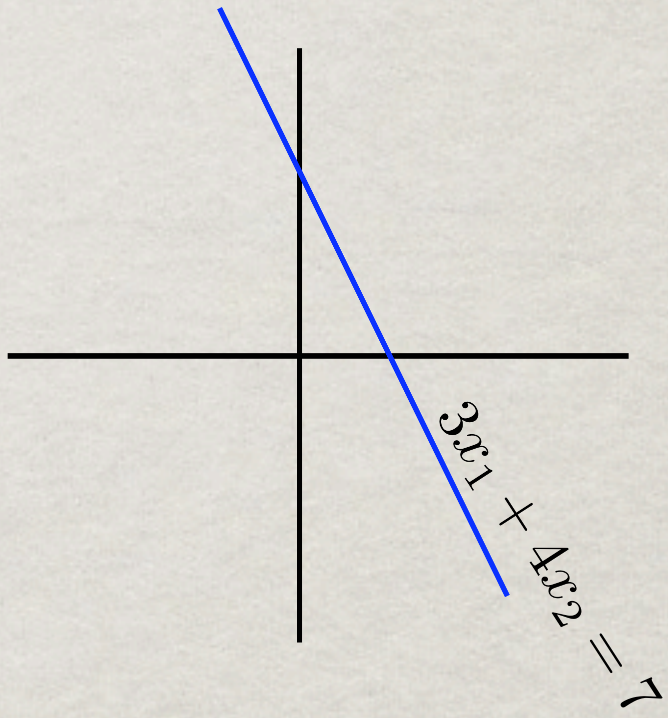

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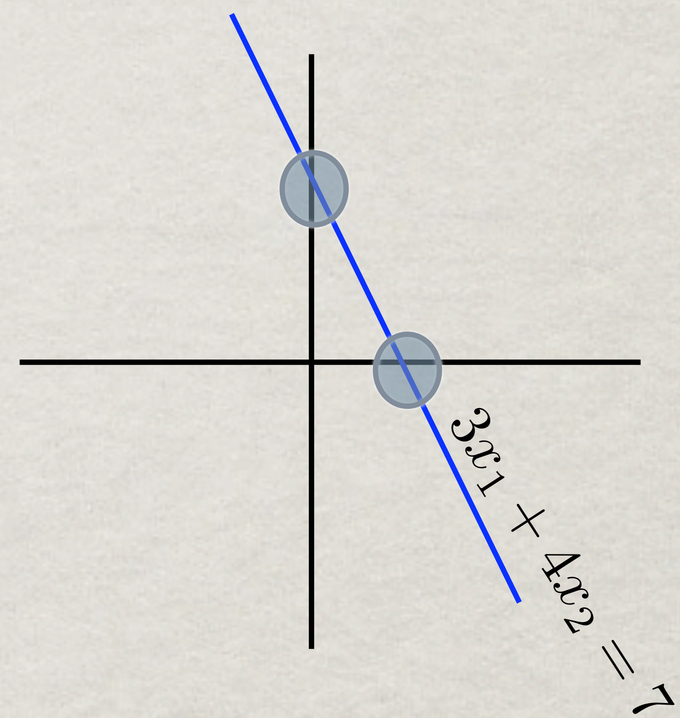

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Two solutions even if soln is 1-sparse

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Let image be $\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$ Let it be 1-sparse (only one coeff. non zero)

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$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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Not linear multiples of each other

Simple Geometry

Let image be x_1 x_2 x_3 Let it be 1-sparse (only one coeff. non zero)

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Real Image

Simple Geometry

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Not linear multiples of each other

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \qquad \underbrace{\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix}}_{x'} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

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Real Image

Return To CS

$$\text{find } x$$
$$y = Ax$$
$$|x|_0 \leq s$$

equivalent to

$$min \ ||x||_0$$
$$s.t. \ y = Ax$$

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$|x|_0 \leq s$ means all vectors with at most s non-zero entries

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far fewer measurements y (m) than dimension of x (N)

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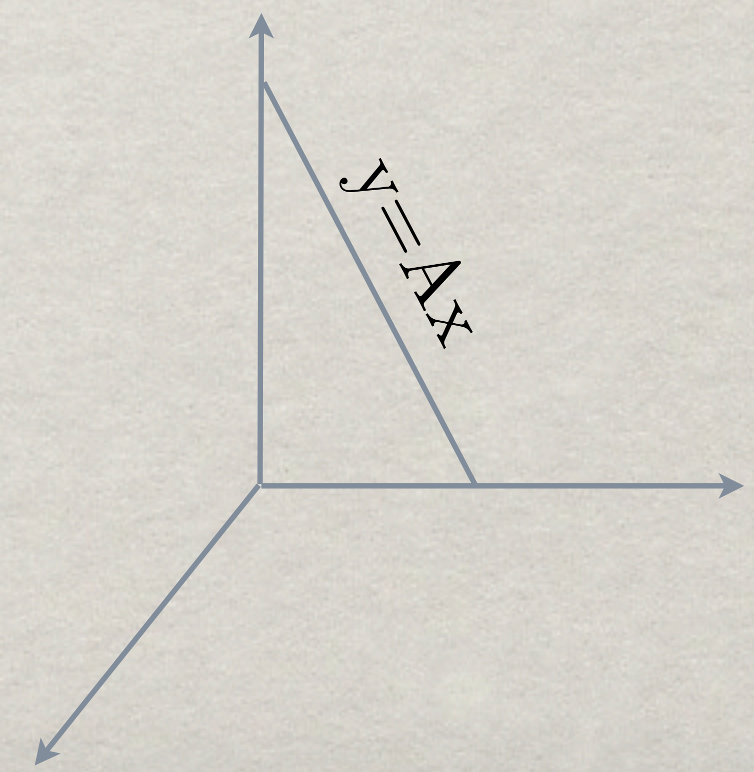

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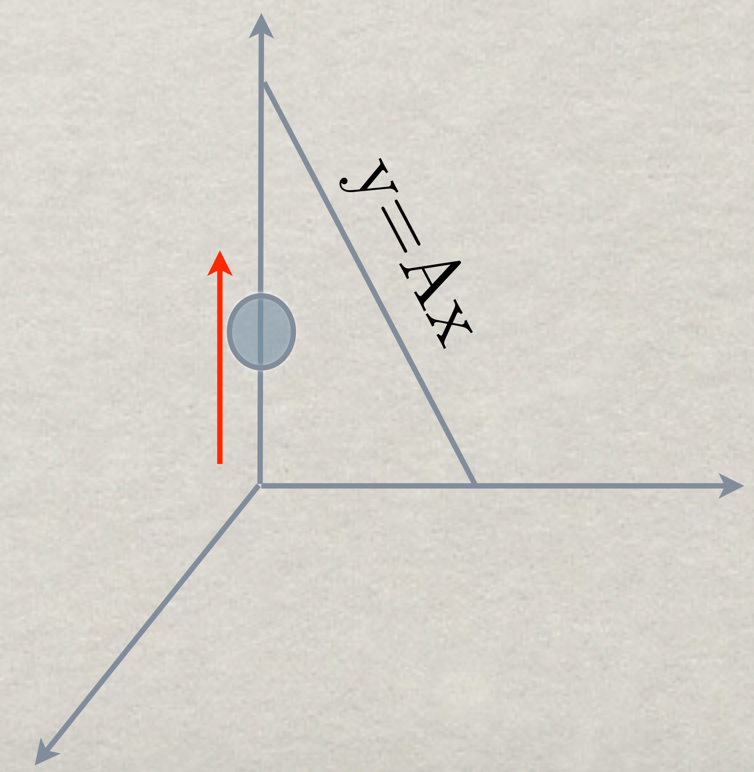

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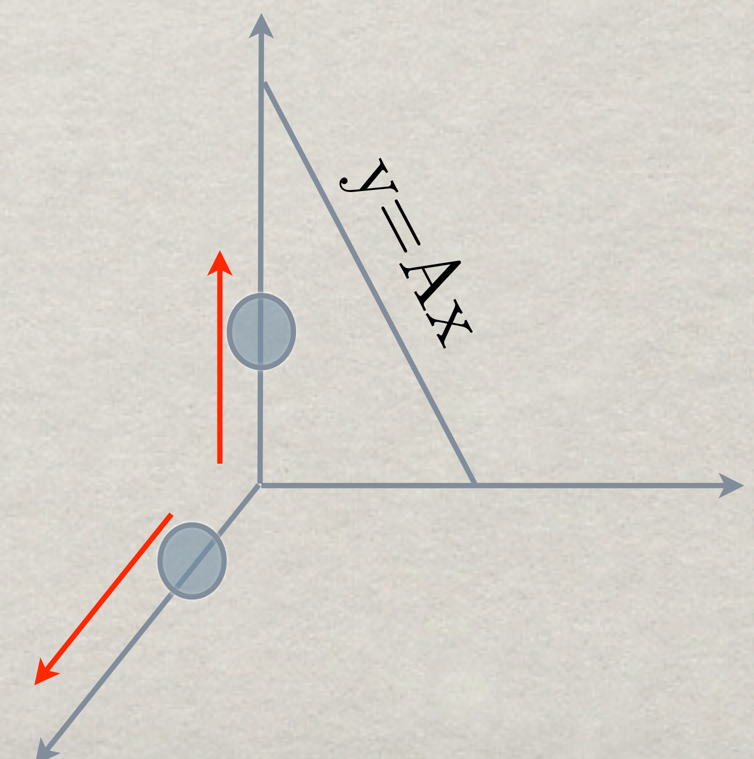

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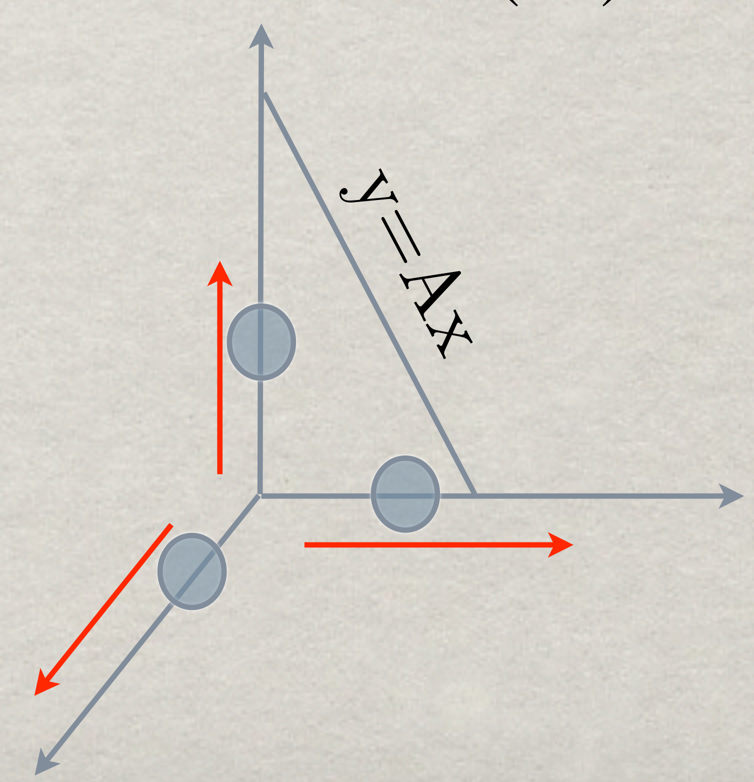


Relaxation

$$min\ ||x||_2$$
$$s.t.\ y = Ax$$

$||x||_2 = r$ - sphere of radius r

Relaxation

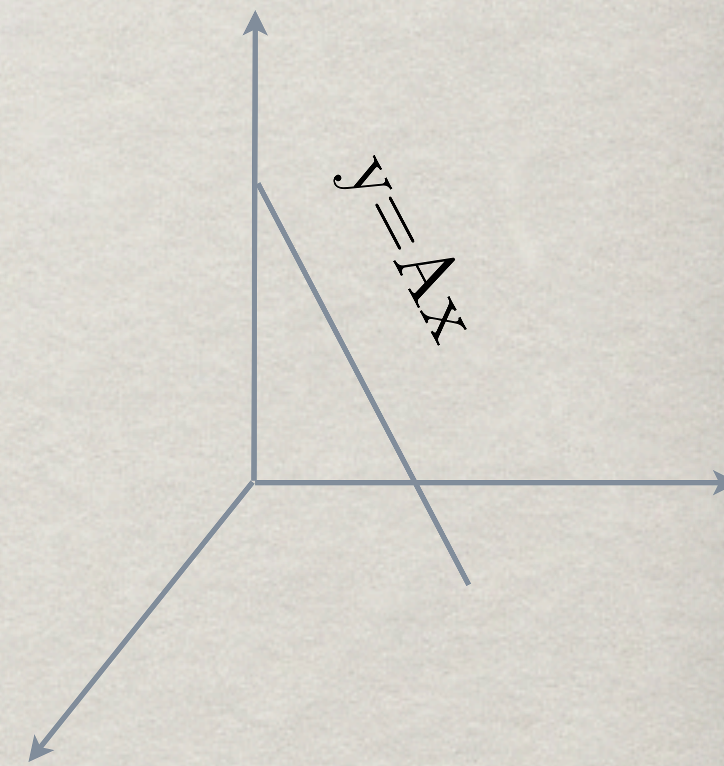
$$min \ ||x||_2$$
$$s.t. \ y = Ax$$

$||x||_2 = r$ - sphere of radius r

$$||x||_2 = \sqrt{\sum_{i=1}^{N} x_i^2}$$

Relaxation

$$min \; ||x||_2$$
$$s.t. \; y = Ax$$

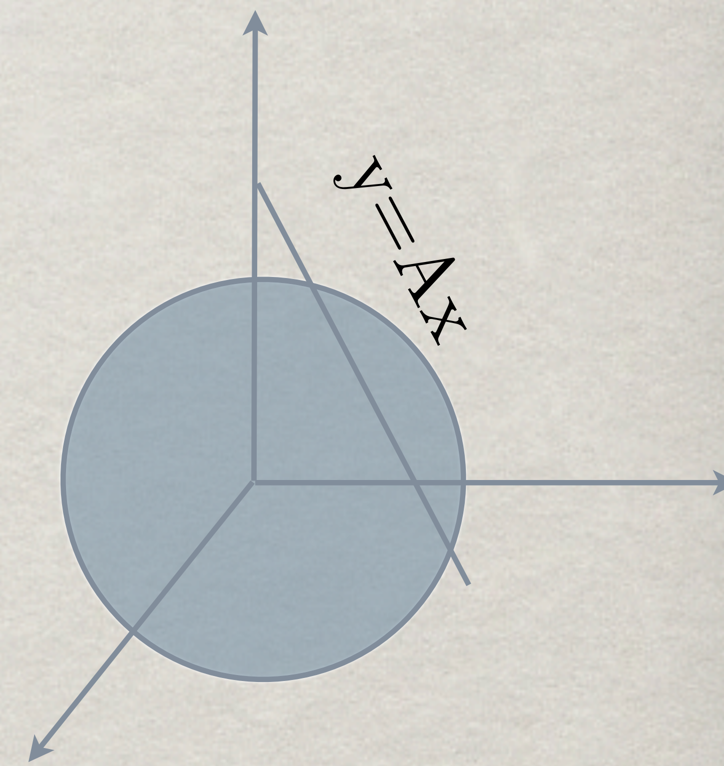


$||x||_2 = r$ - sphere of radius r

$$||x||_2 = \sqrt{\sum_{i=1}^{N} x_i^2}$$

Relaxation

$$min \ ||x||_2$$
$$s.t. \ y = Ax$$



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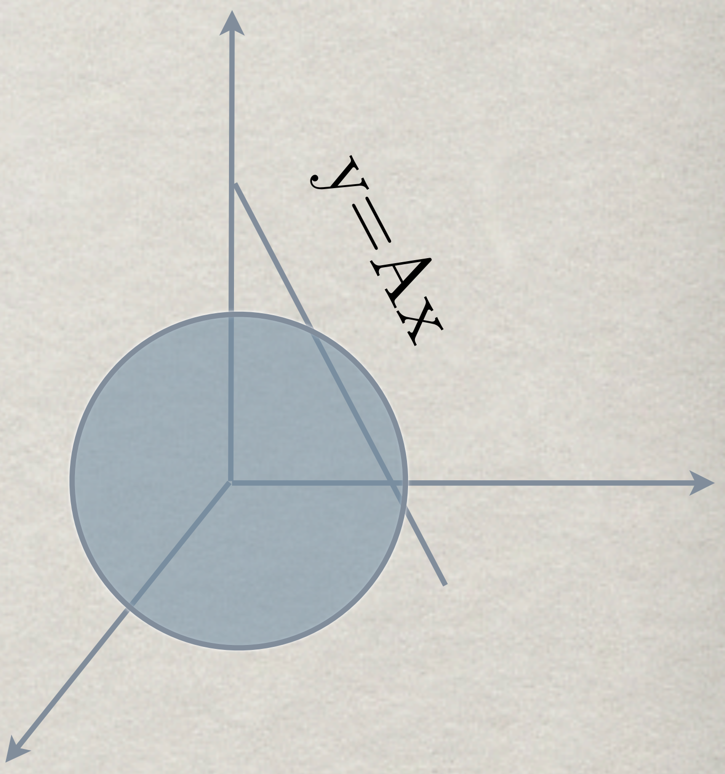

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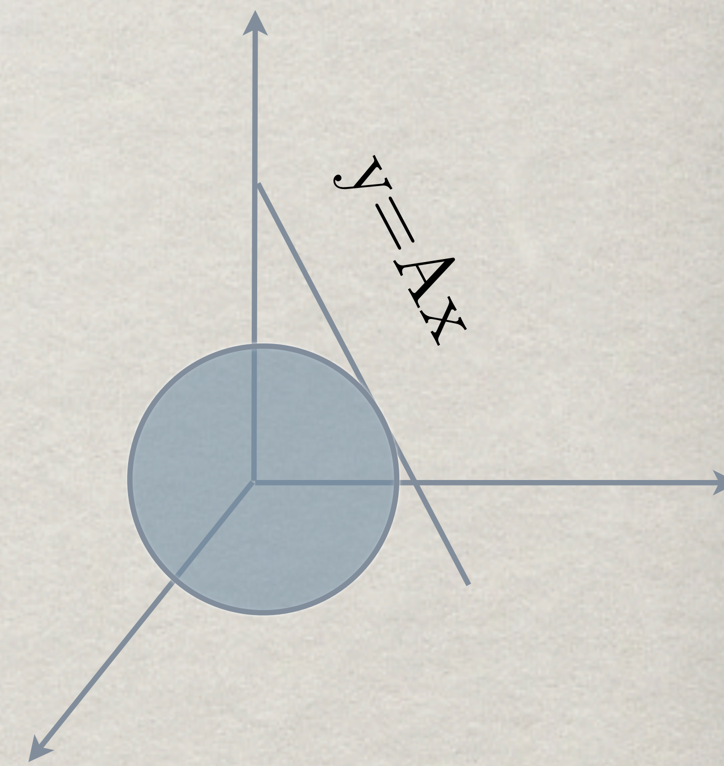


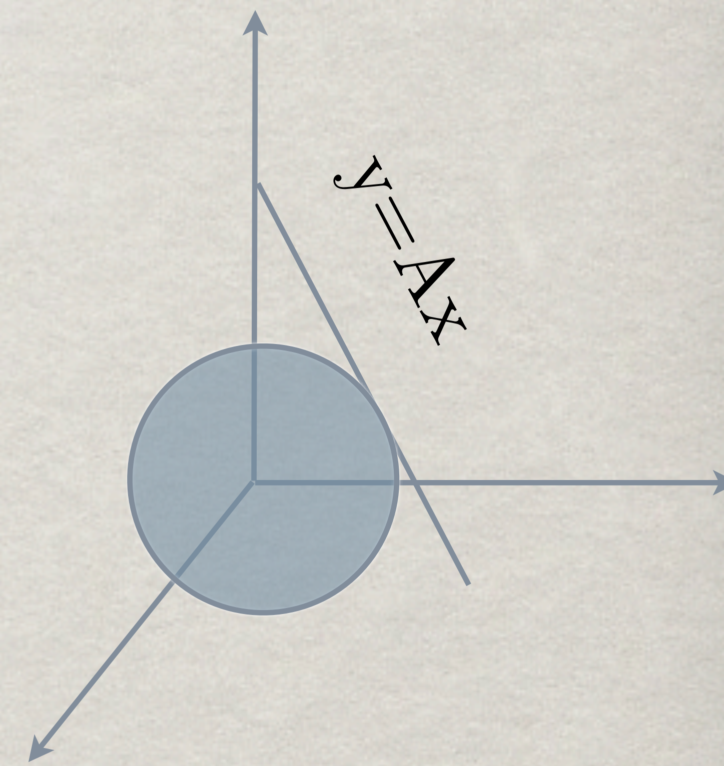
Non-Sparse Solution

Relaxation

$$min\ ||x||_2$$
$$s.t.\ y = Ax$$

$||x||_2 = r$ - sphere of radius r

$$||x||_2 = \sqrt{\sum_{i=1}^{N} x_i^2}$$



Non-Sparse Solution

- Simple to solve but does not serve the purpose

Relaxation

$$min\ ||x||_1$$
$$s.t.\ y = Ax$$

$||x||_1 = r$ - polyhedron

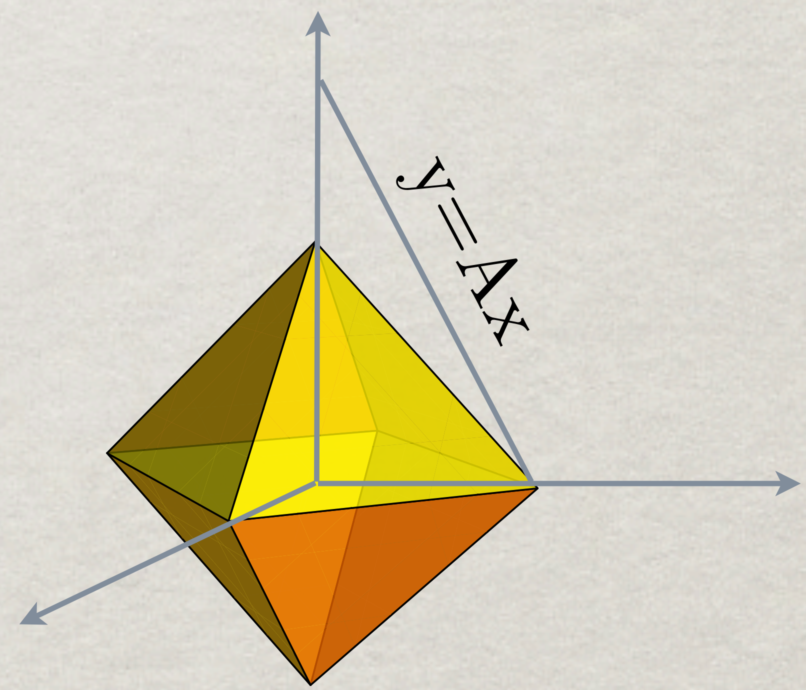
$$||x||_1 = \sum_{i=1}^{N} |x_i|$$

Relaxation

$$min \ ||x||_1$$
$$s.t. \ y = Ax$$

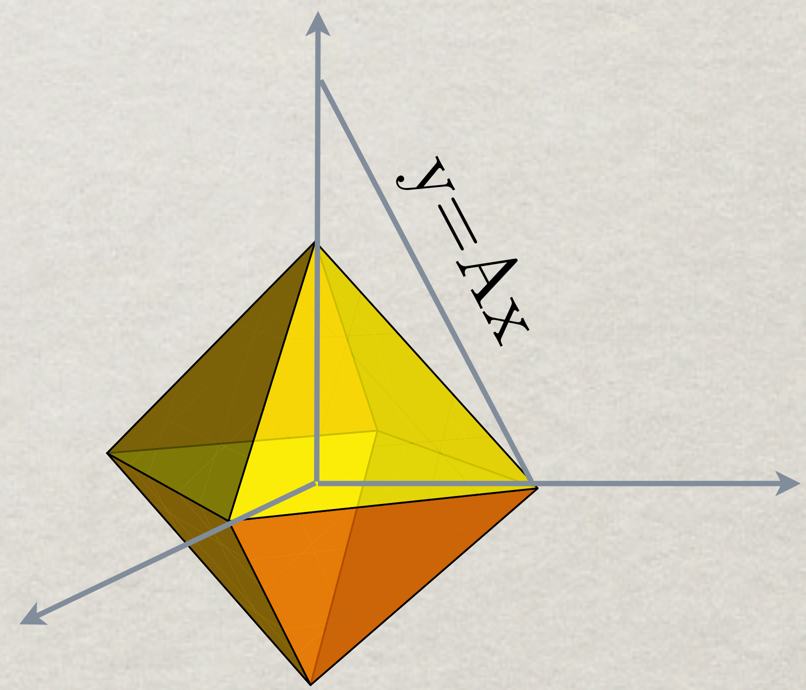
$||x||_1 = r$ - polyhedron

$$||x||_1 = \sum_{i=1}^{N} |x_i|$$



Relaxation

$$min \ ||x||_1$$
$$s.t. \ y = Ax$$



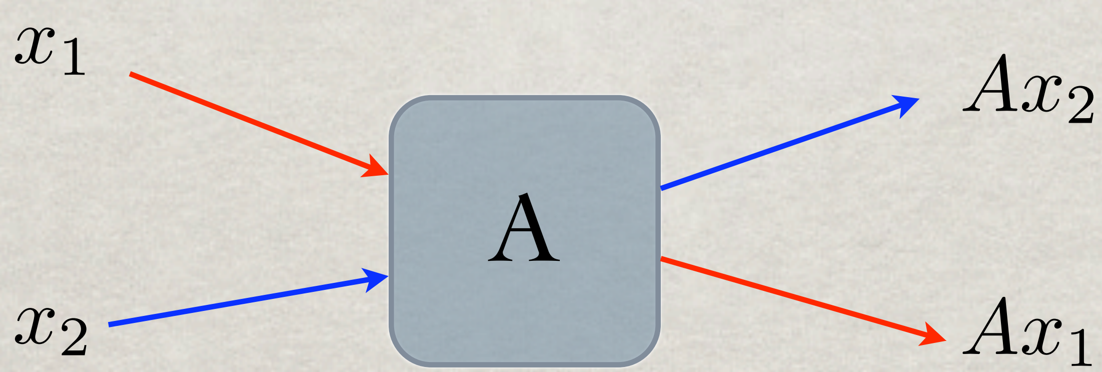
$||x||_1 = r$ - polyhedron

$$||x||_1 = \sum_{i=1}^{N} |x_i|$$

No competing sparse solutions

Technical Conditions

- Restricted Isometry Property: Any 2s columns of A are nearly orthogonal [CandesTao'05]



2s RIP $\Rightarrow$ Pairwise distances are preserved for s-sparse signals

Examples of A-matrices

- Each entry independent coin flips
- Columns dist. uniformly on unit sphere
- Independent Gaussian entries
- Exact recovery if $m \sim s \log N/s$

Conclusions

- Questioned the conventional wisdom
- Exciting area of research

Thank You

Simplification

$$min\ ||x||_2$$
$$s.t.\ y = Ax$$

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$$min\ ||x||_2$$
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