

# Optimal WiFi Sensing Via Dynamic Programming

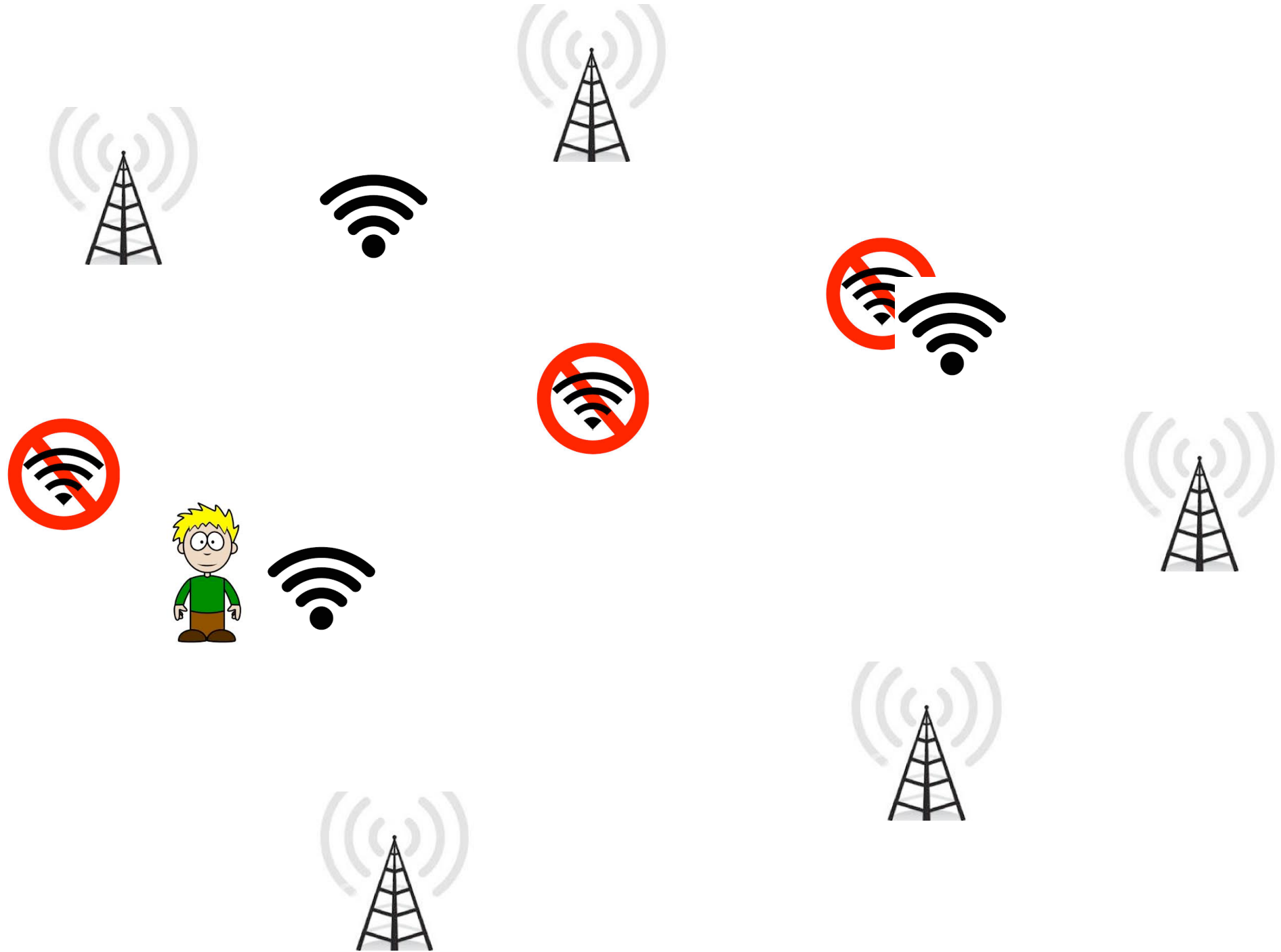
Rahul Vaze



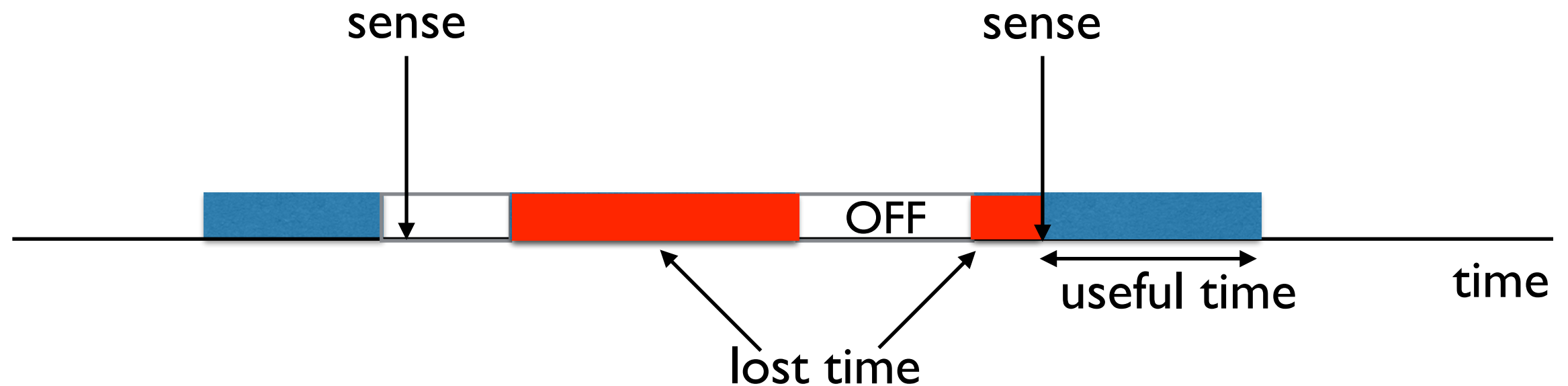
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IIT-Bombay

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IISc Bangalore

# Roaming Users



# Setup

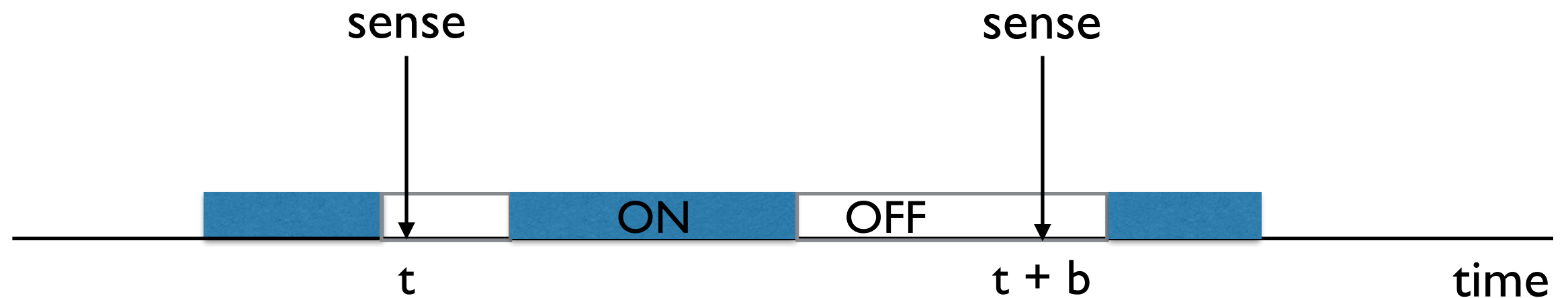


## cost

- sensing
- lost time

**Problem:** find optimal sensing durations to min cost

# Issue



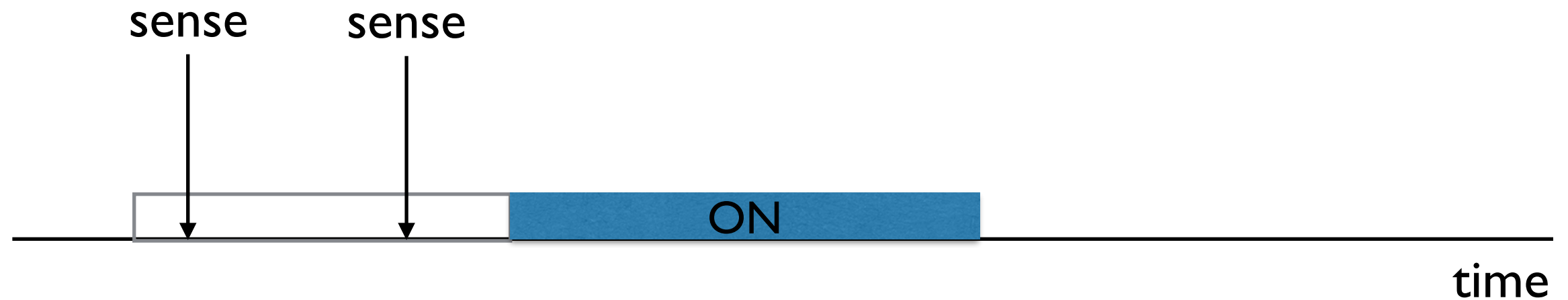
## Non-Memoryless OFF time distribution

- algorithm may assume that OFF period is  $b$ , which is false

For general problem assume that OFF time  $\sim \text{EXP}$

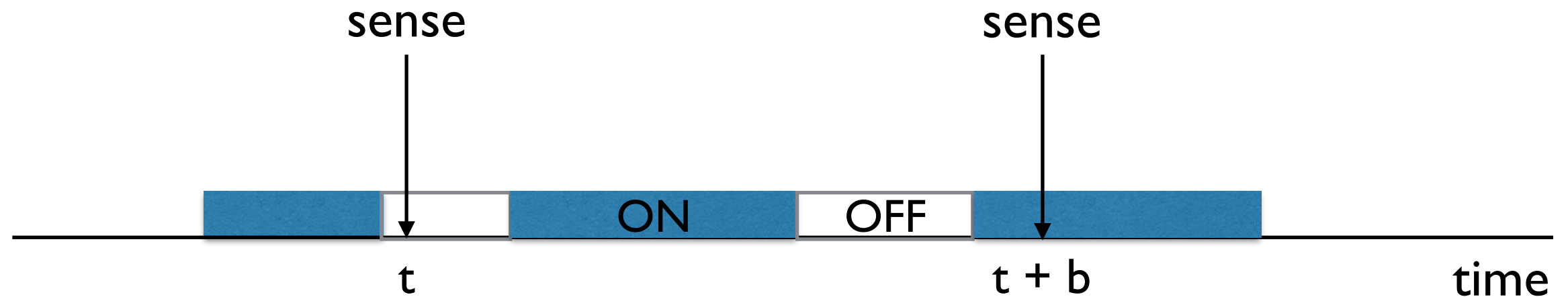


# Context



General case of problem studied by *Azad et al 2011*  
where ON period never expires

# Earlier work



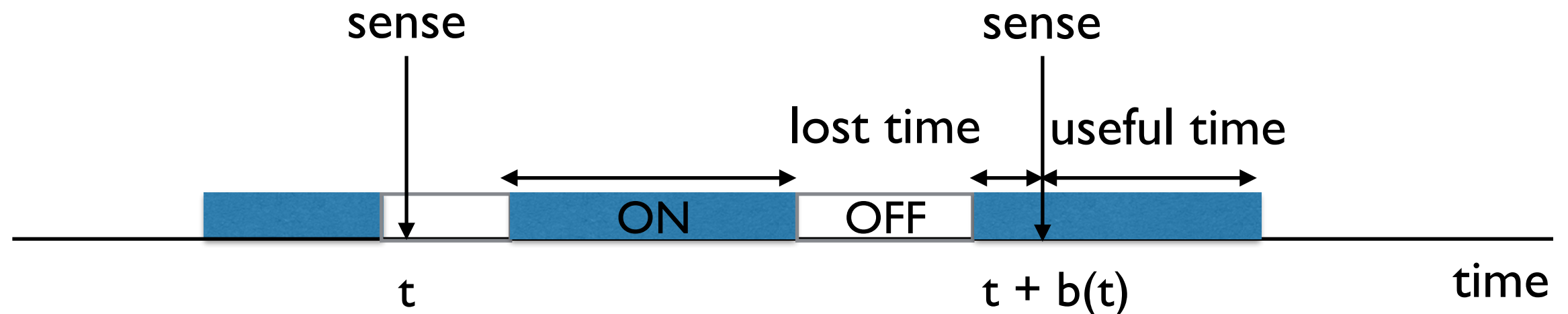
Exp OFF and Exp ON time distribution

- Kim et al, Infocom 2011

General OFF and ON time distribution - simpler cost

- Jeong et al, Infocom 2013

# Problem Formulation - OFF time ( $\sim \text{EXP}$ )



**cost**  $C(t, b(t)) = c_s + M(t, b(t))$

- sensing -  $c_s$
- lost time -  $M(t, b(t))$       sensing at  $(t, t + b(t))$

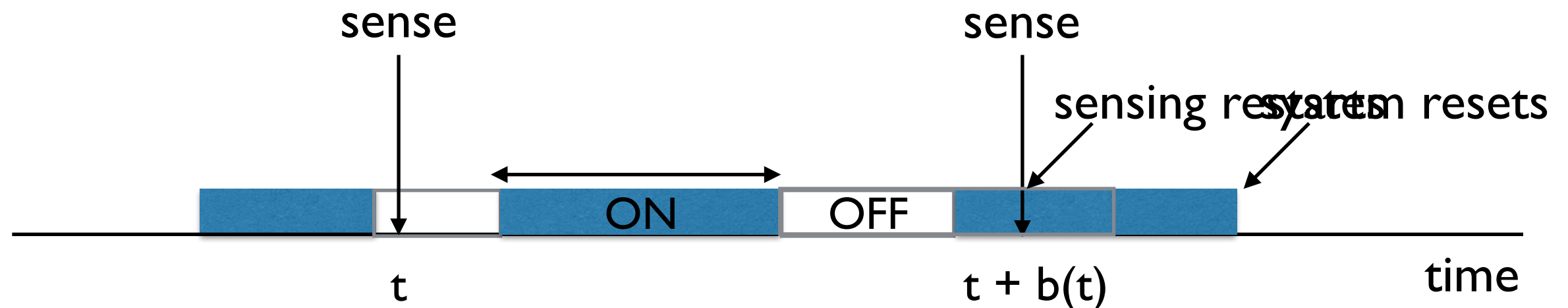
counting restarts

DP

$$V(t) = \max_{b(t) \geq 0} [\mathbb{E}\{C(t, b(t))\} + P_{\text{off}}(t + b(t) | t \in \text{off}) V(t + b(t))]$$

# Problem Formulation - OFF time ( $\sim \text{EXP}$ )

Two cases : **sense** is in **ON**



Two cases : **sense** is in **OFF**

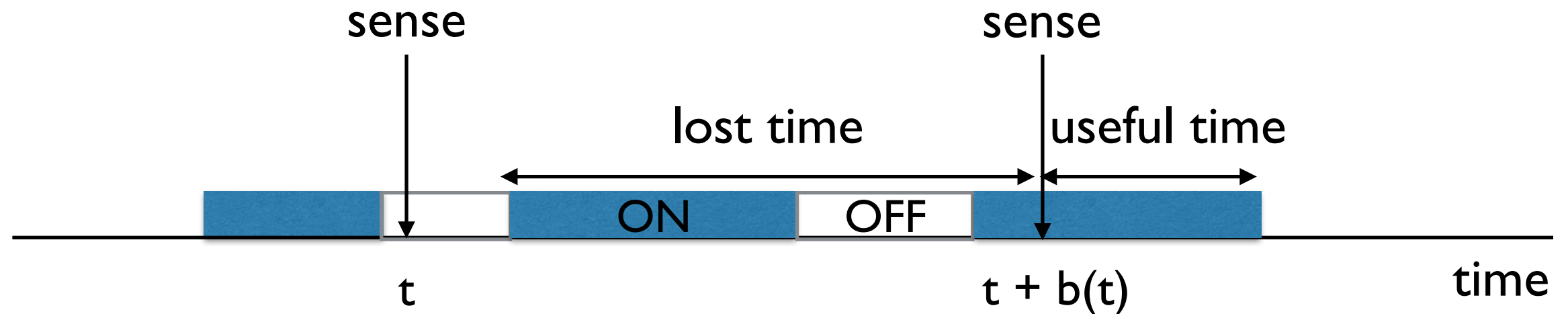
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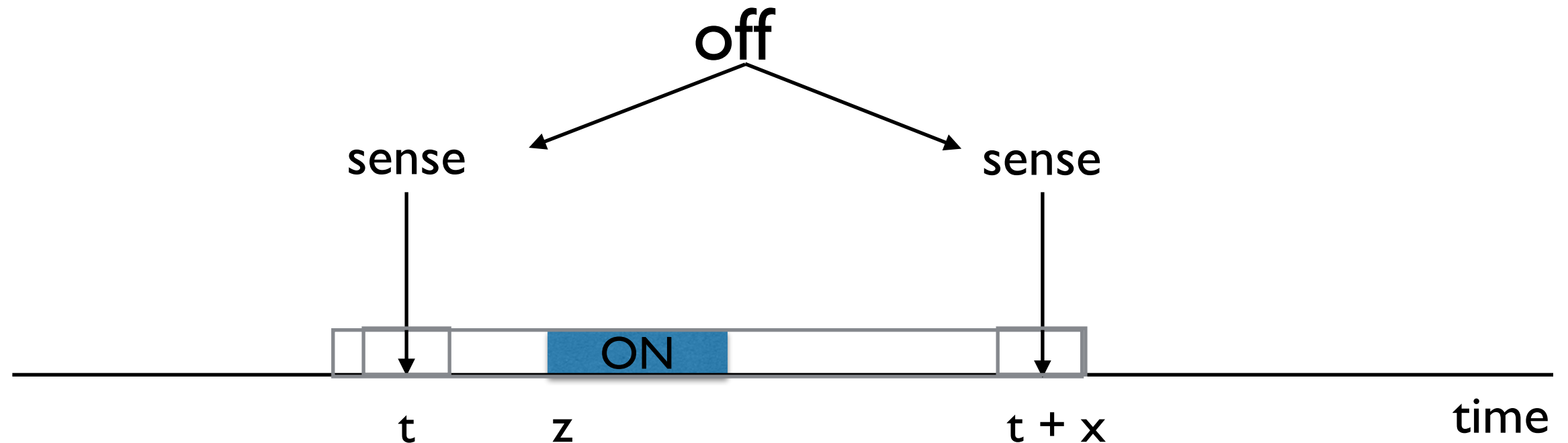
# Simple Property



The optimal sensing duration  $b(t)$  is independent of  $t$

Because of EXP OFF time, at time  $t$ , know nothing about leftover OFF time

# Useful Recursions - OFF time (Exp)



either OFF is not over at  $t$

or OFF  $\rightarrow$  ON happened at  $z > t$

$$P_{\text{off}}(t + x | t \in \text{off}) = P(\text{OFF}(t) > x) + \int_0^x f_d(z) P_{\text{off}}(t + x | z \uparrow) dz, z \geq t$$

$$P_{\text{off}}(t + x | t \uparrow) = \int_0^x f_c(w) P_{\text{off}}(t + x | w \in \text{off}) dw, w \geq t$$

# Useful Recursions - OFF time (Exp)



Assume that  $\text{ON} \rightarrow \text{OFF}$  happens at 0,

then expected ON time between  $[0, t]$  is  $M_{\downarrow}(t) = \int_0^t f_d(x) M_{\uparrow}(t - x) dx$

If  $\text{OFF} \rightarrow \text{ON}$  happens at 0, then expected ON time between  $[0, t]$  is  $M_{\uparrow}(t)$

$$M_{\uparrow}(t) = t \int_t^{\infty} f_c(x) dx +$$

# Solution

## Use Laplace Transforms

Assume that ON  $\gg$  OFF happens at 0,

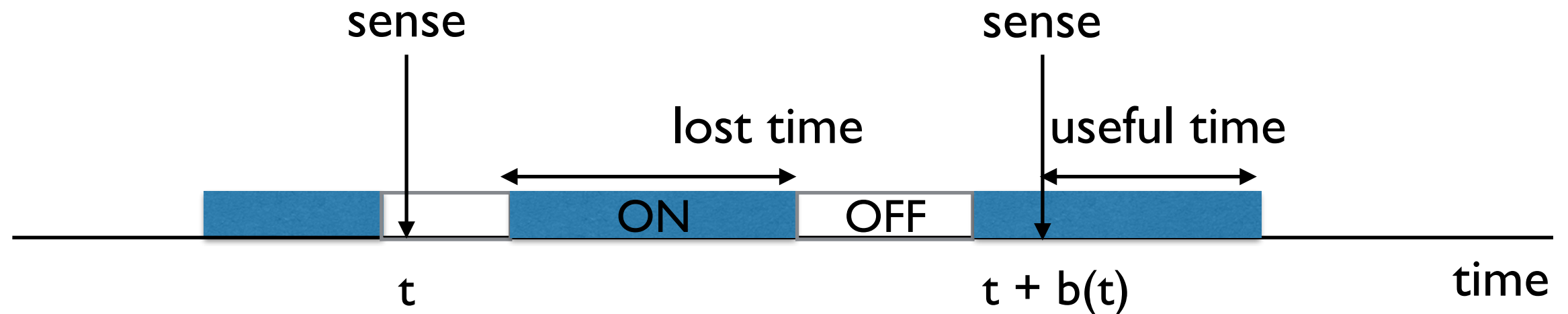
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$$M_{\uparrow}(t) = t \int_t^{\infty} f_c(x) + \int_0^x f_d(x) M_{\downarrow}(t-x) dx$$



# Solution - OFF time (Exp)



**cost**  $C(t, b(t)) = c_s + M(t, b(t))$

- sensing -  $c_s$
- lost time -  $M(t, b(t)) = M_{\downarrow}(b(t))$

counting restarts

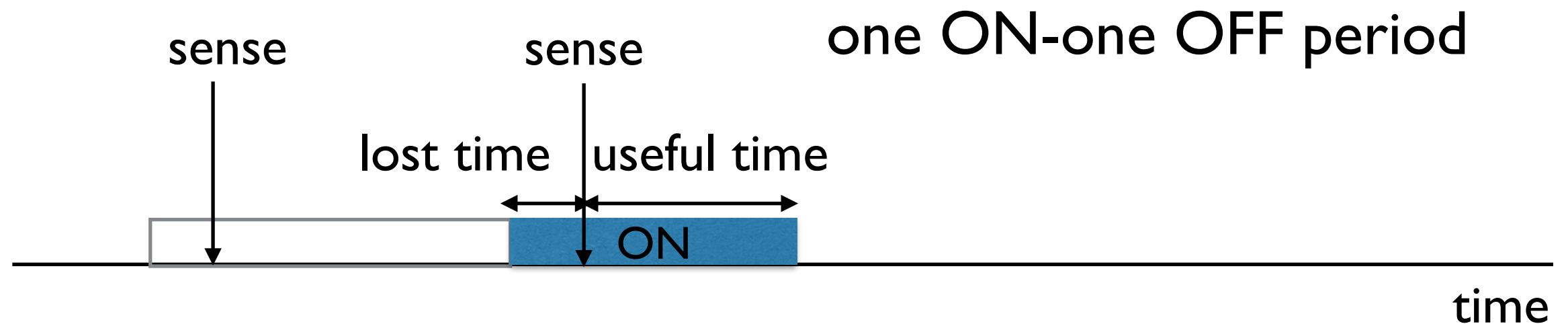
DP

$$V(t) = \max_{b(t) \geq 0} [\mathbb{E}\{C(t, b(t))\} + P_{\text{off}}(t + b(t) | t \in \text{off}) V(t + b(t))]$$



What about Non-EXP  
OFF time

# Solution - OFF time (Non-Exp)



## Critical Assumption

As soon as one full ON period is missed algorithm is notified

counting restarts

DP

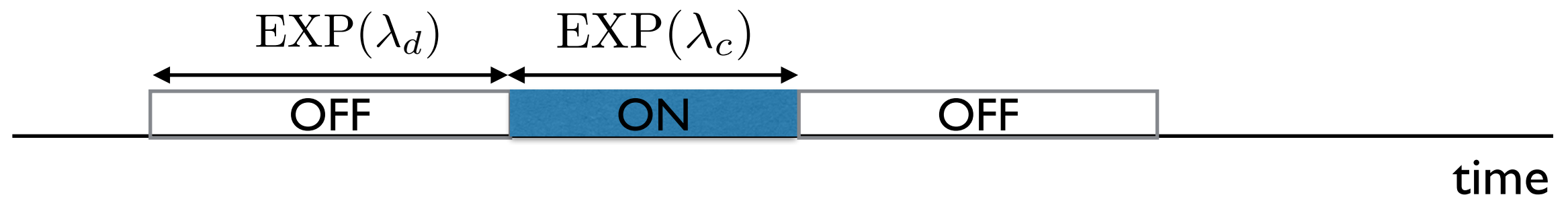
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# Results - OFF time (Non-Exp)

- Optimal Policy is deterministic
- Value/Policy Iterations converge to optimal
- If residual OFF period distribution converges in time, then the optimal sensing duration converges to a constant

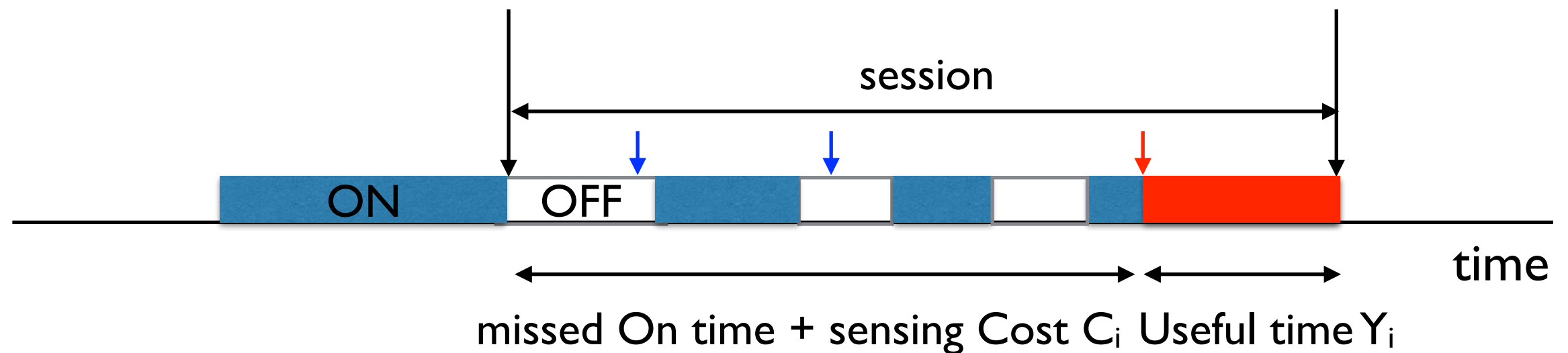
**Who gave you the distributions!**

# Learning Framework



- ON-OFF is EXP with unknown parameters

# Session Definition

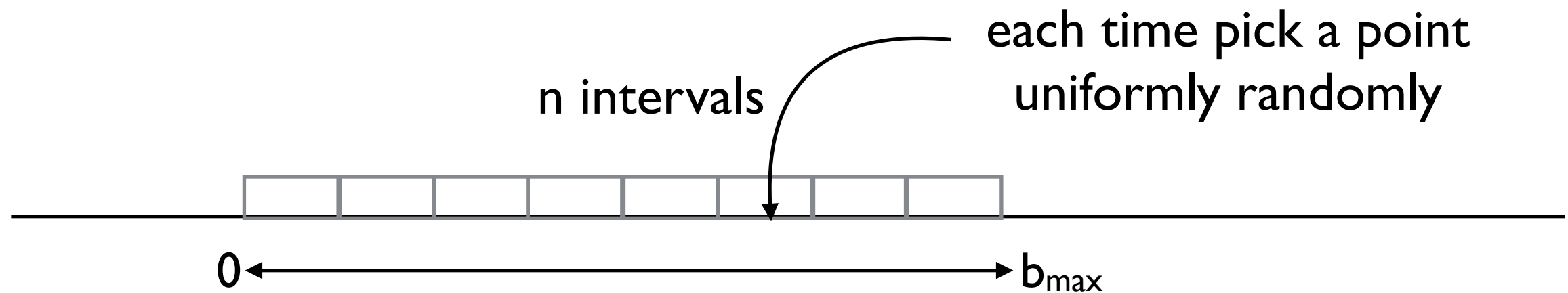


- session reward  $U_i = Y_i - C_i$
- optimal reward  $U_i^* = Y_i^* - C_i^*$

- **Regret** 
$$R(T) = \sum_{i=1}^T U_i^* - \sum_{i=1}^T U_i$$

- **Objective** 
$$\min_b \mathbb{E}\{R(T)\}$$

# Learning Algorithm



- Empirical reward for interval  $k$  until time  $t$  is  $E_k(t)$

$$\text{Find } k^* = \arg \max_{k=1, \dots, n} E_k(t) + \sqrt{\frac{2 \ln t}{t_k}}$$

$t_k$  is the number of times interval  $k$  is chosen until time  $t$

use a uniformly random point from interval  $k^*$



# Result

If the number of intervals is  $n = \left(\frac{T}{\ln T}\right)^{1/4}$

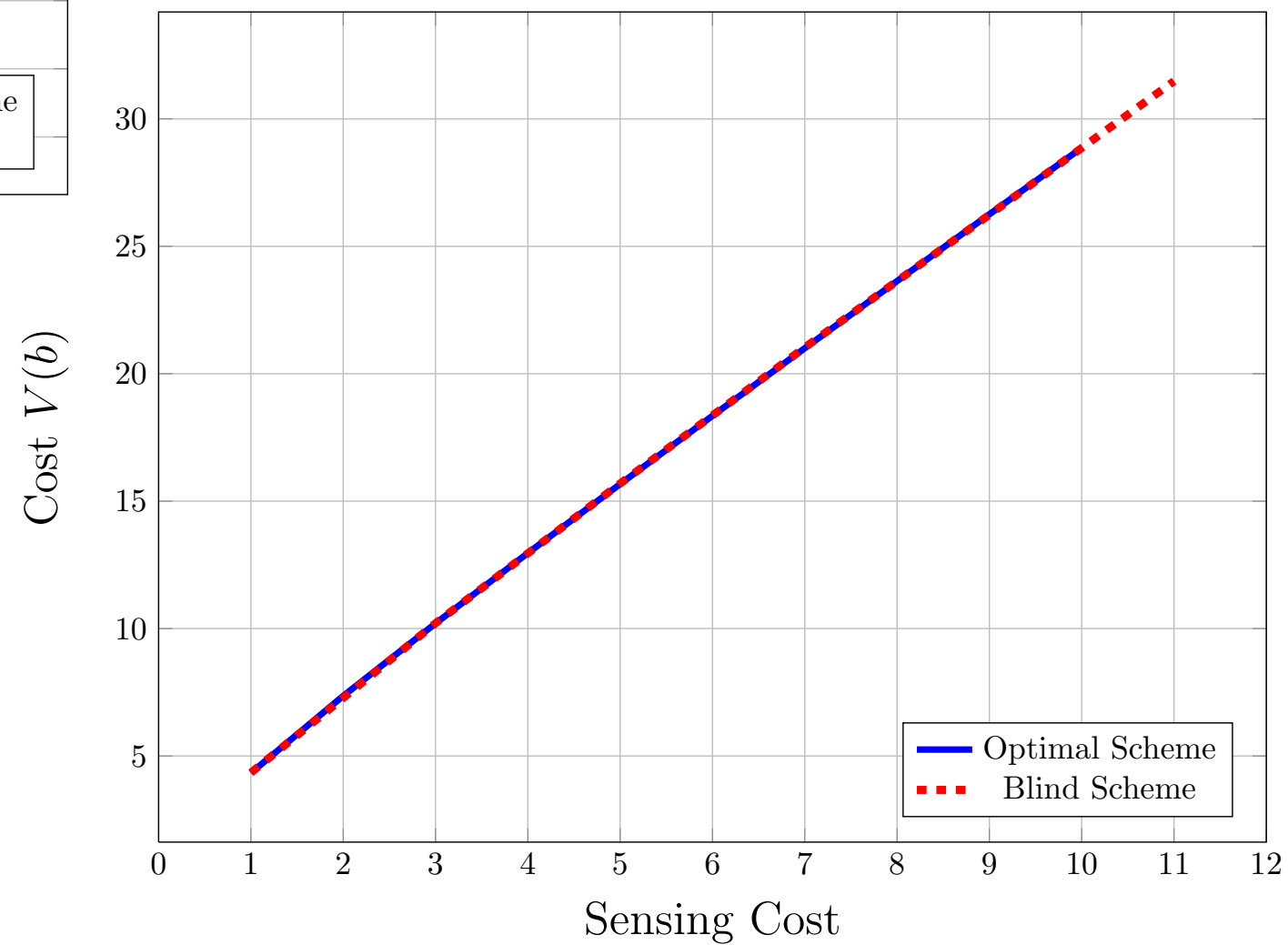
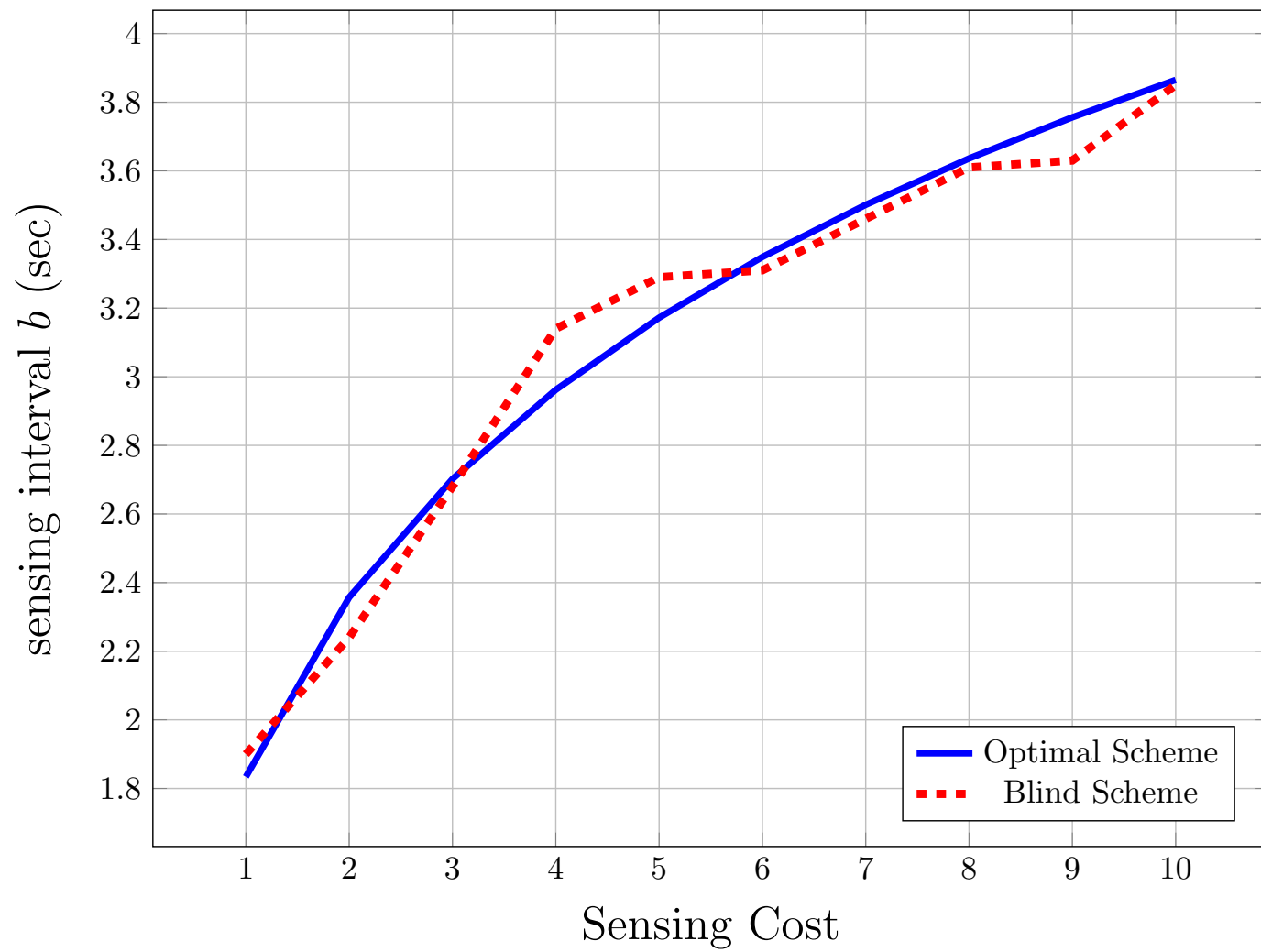
$$\mathbb{E}\{R(T)\} \leq \mathcal{O}(\sqrt{T \ln T})$$

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[Klienbergr 2004]

[Auer, Ortner, Szepesvari 2007]

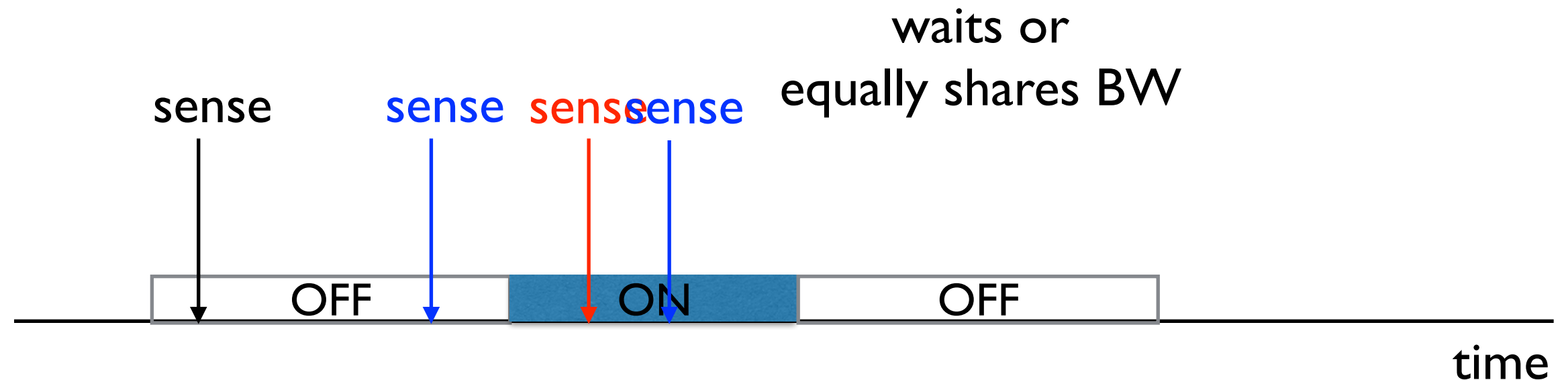
# Result



# What next !

- Multi-user problem with contention

- Game Formulation



**Rahul Vaze** teaches at the School of Technology and Computer Science, Tata Institute of Fundamental Research, Mumbai. He obtained his PhD from the University of Texas at Austin. His research interests are in the areas of multiple antenna communication, ad hoc networks, and combinatorial resource allocation. He is the recipient of the Indian National Science Academy's young scientist award for the year 2013.

This book discusses the theoretical limits of information transfer in random wireless networks or ad hoc networks, where nodes are distributed uniformly random in space and there is no centralized control. Examples of ad hoc networks include sensor networks, military networks, and vehicular networks that have widespread applications. Decentralized nature of these networks makes them easily configurable, scalable, and inherently robust.

The author provides a detailed analysis of the two relevant notions of capacity for random wireless networks – transmission capacity and throughput capacity. The book starts with the transmission capacity framework that is first presented for the single-hop model and later extended to the multi-hop model with retransmissions. By reusing some of the tools developed for analysis of transmission capacity, few key long-standing questions about the performance analysis of cellular networks are also addressed for the benefit of students. To complete the throughput capacity characterization, the author finally discusses the concept of hierarchical cooperation that allows the throughput capacity to scale linearly with the number of nodes.

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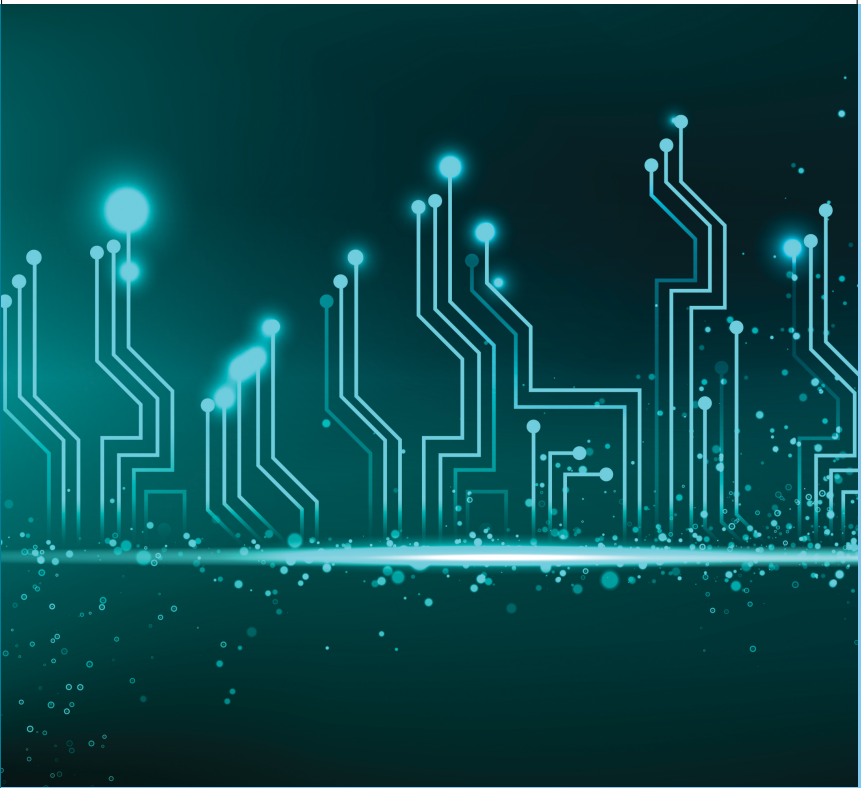
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Vaze

Random Wireless Networks

CAMBRIDGE



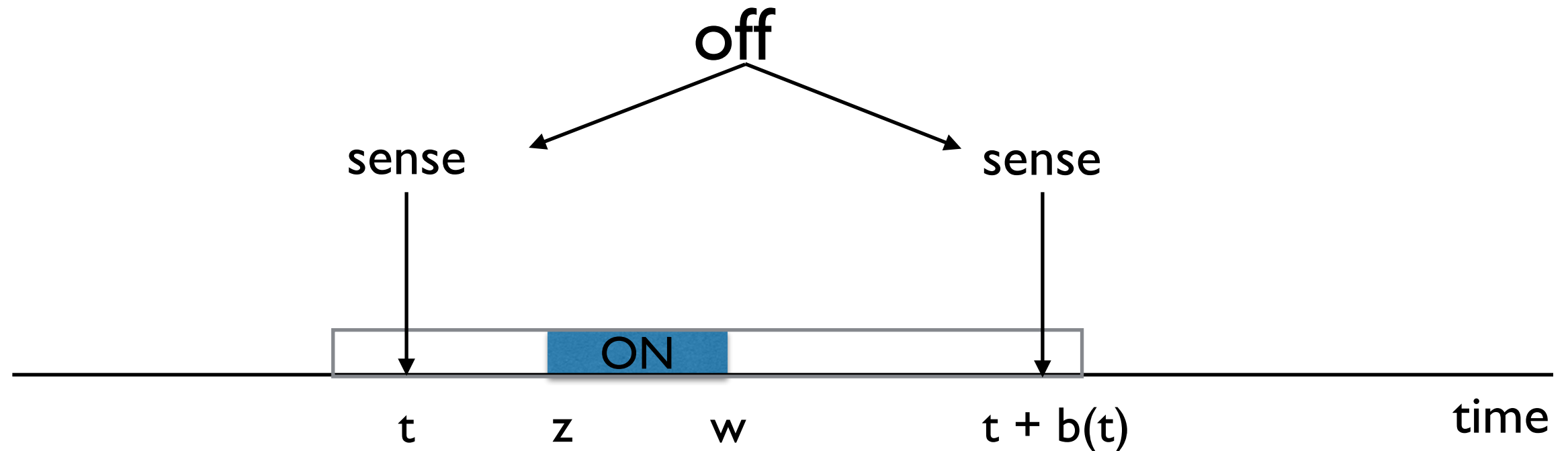
The optimal role of multiple antennas, ARQ protocols, and scheduling protocols in random wireless networks is identified using the transmission capacity paradigm. This book provides a holistic view of all relevant tools and concepts used to analyse random wireless networks. A conscious attempt is made to bring out the connections between transmission and throughput capacity, between percolation theory and throughput capacity, and stochastic geometry and cellular networks. For effective understanding, an extensive effort is made to explain the physical interpretation of all results.

# Random Wireless Networks

An Information Theoretic Perspective

**Rahul Vaze**

# Useful Recursions - OFF time (Exp)



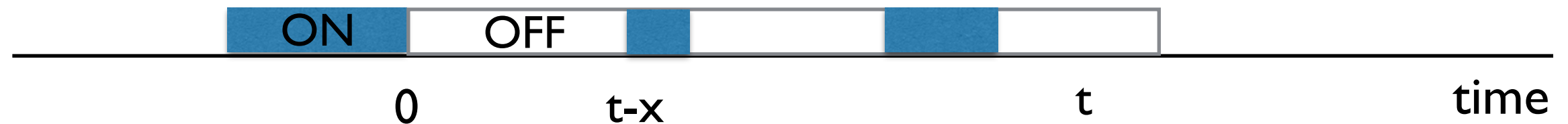
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